Mid-Chapter Review

1. Name three points on each of the following lines:
   a. $x = 2t - 5, y = 3t + 1, t \in \mathbf{R}$
   b. $\vec{r} = (2, 3) + s(3, -2), s \in \mathbf{R}$
   c. $3x + 5y - 8 = 0$
   d. $\frac{x-1}{3} = \frac{y+2}{2} = \frac{z-5}{1}$
2. Find x- and y-intercepts for each of the following lines:
   a. $\vec{r} = (3, 1) + t(-3, 5), t \in \mathbf{R}$
   b. $x = -6 + 2s$ and $y = 3 - 2s, s \in \mathbf{R}$
3. Two lines $L_1: \vec{r} = (5, 3) + p(-4, 7), p \in \mathbf{R}$, and $L_2: \vec{r} = (5, 3) + q(2, 1), q \in \mathbf{R}$, intersect at the point with coordinates $(5, 3)$. What is the angle between L_1 and L_2?
4. Determine the angle that the line with equation $\vec{r} = t(4, -5), t \in \mathbf{R}$, makes with the x-axis and y-axis.
5. Determine a Cartesian equation for the line that passes through the point $(4, -3)$ and is perpendicular to the line $\vec{r} = (2, -3) + t(5, -7), t \in \mathbf{R}$.
6. Determine an equation in symmetric form of a line parallel to $\frac{x-3}{3} = \frac{y-5}{-4} = \frac{z+7}{4}$ and passing through $(0, 0, 2)$.
7. Determine parametric equations of the line passing through $(1, 2, 5)$ and parallel to the line passing through $K(2, 4, 5)$ and $L(3, -5, 6)$.
8. Determine direction angles (the angles the direction vector makes with the x-axis, y-axis, and z-axis) for the line with parametric equations $x = 5 + 2t$, $y = 12 - 8t$, $z = 5 + 7t$, $t \in \mathbf{R}$.
9. Determine an equation in symmetric form for the line passing through $P(3, -4, 6)$ and having direction angles 60°, 90°, and 30°.
10. Write an equation in parametric form for each of the three coordinate axes in R^3.
11. The two lines with equations $\vec{r} = (1, 2, -4) + t(k + 1, 3k + 1, k - 3)$, $t \in \mathbf{R}$, and $x = 2 - 3s$, $y = 1 - 10s$, $z = 3 - 5s$, $s \in \mathbf{R}$, are given.
    a. Determine a value for k if these lines are parallel.
    b. Determine a value for k if these lines are perpendicular.
12. Determine the perimeter and area of the triangle whose vertices are the origin and the x- and y-intercepts of the line $\frac{x-6}{3} = \frac{y+8}{-2}$.

13. The Cartesian equation of a line is given by $3x + 4y - 24 = 0$.
    a. Determine a vector equation for this line.
    b. Determine the parametric equations of this line.
    c. Determine the acute angle that this line makes with the x-axis.
    d. Determine a vector equation of the line that is perpendicular to the given line and passes through the origin.
14. Determine the scalar, vector, and parametric equations of the line that passes through points $A(-4, 6)$ and $B(8, 4)$.
15. Determine a unit vector normal to the line defined by the parametric equations $x = 1 + 2t$ and $y = -5 - 4t$.
16. Determine the parametric equations of each line.
    a. the line that passes through $(-5, 10)$ and has a slope of $-\frac{2}{3}$
    b. the line that passes through $(1, -1)$ and is perpendicular to the line $(x, y) = (4, -6) + t(2, -2)$
    c. the line that passes through $(0, 7)$ and $(0, 10)$
17. Given the line $(x, y, z) = (12, -8, -4) + t(-3, 4, 2)$,
    a. determine the intersections with the coordinate planes, if any
    b. determine the intercepts with the coordinate axes, if any
    c. graph the line in an x-, y-, z-coordinate system.
18. For each of the following, determine vector, parametric, and, if possible, symmetric equations of the line that passes through P_0 and has direction vector $\vec{d}$.
    a. $P_0 = (1, -2, 8)$, $\vec{d} = (-5, -2, 1)$
    b. $P_0 = (3, 6, 9)$, $\vec{d} = (2, 4, 6)$
    c. $P_0 = (0, 0, 6)$, $\vec{d} = (-1, 5, 1)$
    d. $P_0 = (2, 0, 0)$, $\vec{d} = (0, 0, -2)$
19. Determine a vector equation of the line that passes through the origin and is parallel to the line through the points $(-4, 5, 6)$ and $(6, -5, 4)$.
20. Determine the parametric equations of the line through $(0, -8, 1)$ and which passes through the midpoint of the segment joining $(2, 6, 10)$ and $(-4, 4, -8)$.
21. The symmetric equations of two lines are given. Show that these lines are parallel.
    $L_1 : \frac{x - 2}{1} = \frac{y + 3}{3} = \frac{z - 4}{-5}$ and $L_2 : \frac{x + 1}{-3} = \frac{y - 2}{-9} = \frac{z + 1}{15}$
22. Does the point $D(7, -1, 8)$ lie on the line with symmetric equations $\frac{x - 4}{3} = \frac{y + 2}{1} = \frac{z - 6}{2}$? Explain.