Key Concepts Review

In Chapter 8, you examined how the algebraic description of a straight line could be represented using vectors in both two and three dimensions. The form of the vector equation of a line, $\vec{r} = \vec{r_0} + t\vec{d}$, is the same whether the line lies in a two-dimensional plane or a three-dimensional space. The following table summarizes the various forms of the equation of a line, where the coordinates of a point on the line are known as well as a direction vector. t is a parameter where $t \in \mathbf{R}$.

Form	R^2	R^3
Vector equation	$(x, y) = (x_0, y_0) + t(a, b)$	$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$
Parametric equations	$x = x_0 + at, y = y_0 + bt$	$x = x_0 + at, y = y_0 + bt, z = z_0 + ct$
Symmetric equations	$\frac{x - x_0}{a} = \frac{y - y_0}{b}$	$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$
Cartesian equation	$Ax + BY + C = 0$: $\vec{n} = (A, B)$	not applicable

This concept was then extended to planes in R^3. The following table summarizes the various forms of the equation of a plane, where the coordinates of a point on the plane are known as well as two direction vectors. s and t are parameters where $s, t \in \mathbf{R}$.

Form	R^3
Vector equation	$(x, y, z) = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$
Parametric equations	$x = x_0 + sa_1 + tb_1, y = y_0 + sa_2 + tb_2, z = z_0 + sa_3 + tb_3$
Cartesian equation	$Ax + BY + Cz + D = 0$: $\vec{n} = (A, B, C)$

You also saw that when lines and planes intersect, angles are formed between them. Both lines and planes have normals, which are vectors that run perpendicular to the respective line or plane. The size of an angle can be determined using the normal vectors and the following formula:

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$

Sketching the graph of a plane in R^3 can be accomplished by examining the Cartesian equation of the plane. Determine whether the equation contains one, two, or three variables and whether it contains a constant. This information helps to narrow down which case you need to consider to sketch the graph. Once you have determined the specific case (see the In Summary table in Section 8.6), you can determine the appropriate points and lines to help you sketch a representation of the plane.