1. Determine vector and parametric equations of the plane that contains the points $A(1, 2, -1)$, $B(2, 1, 1)$, and $C(3, 1, 4)$.

2. In question 1, there are a variety of different answers possible, depending on the points and direction vectors chosen. Determine two Cartesian equations for this plane using two different vector equations, and verify that these two equations are identical.

3. a. Determine the vector, parametric, and symmetric equations of the line passing through points $A(-3, 2, 8)$ and $B(4, 3, 9)$.

   b. Determine the vector and parametric equations of the plane containing the points $A(-3, 2, 8)$, $B(4, 3, 9)$, and $C(-2, -1, 3)$.

   c. Explain why a symmetric equation cannot exist for a plane.

4. Determine the vector, parametric, and symmetric equations of the line passing through the point $A(7, 1, -2)$ and perpendicular to the plane with equation $2x - 3y + z - 1 = 0$.

5. Determine the Cartesian equation of each of the following planes:

   a. through the point $P(0, 1, -2)$, with normal $\vec{n} = (-1, 3, 3)$

   b. through the points $(3, 0, 1)$ and $(0, 1, -1)$, and perpendicular to the plane with equation $x - y - z + 1 = 0$

   c. through the points $(1, 2, 1)$ and $(2, 1, 4)$, and parallel to the $x$-axis

6. Determine the Cartesian equation of the plane that passes through the origin and contains the line $\vec{r} = (3, 7, 1) + t(2, 2, 3)$, $t \in \mathbf{R}$.

7. Find the vector and parametric equations of the plane that is parallel to the $yz$-plane and contains the point $A(-1, 2, 1)$.

8. Determine the Cartesian equation of the plane that contains the line $\vec{r} = (2, 3, 2) + t(1, 1, 4)$, $t \in \mathbf{R}$, and the point $(4, -3, 2)$.

9. Determine the Cartesian equation of the plane that contains the following lines:
   $L_1 : \vec{r} = (4, 4, 5) + t(5, -4, 6)$, $t \in \mathbf{R}$, and
   $L_2 : \vec{r} = (4, 4, 5) + s(2, -3, -4)$, $s \in \mathbf{R}$

10. Determine an equation for the line that is perpendicular to the plane $3x - 2y + z = 1$ passing through $(2, 3, -3)$. Give your answer in vector, parametric, and symmetric form.

11. A plane has $3x + 2y - z + 6 = 0$ as its Cartesian equation. Determine the vector and parametric equations of this plane.

12. Determine an equation for the line that has the same $x$- and $z$-intercepts as the plane with equation $2x + 5y - z + 7 = 0$. Give your answer in vector, parametric, and symmetric form.

13. Determine the vector, parametric, and Cartesian forms of the equation of the plane containing the lines $L_1 : \vec{r} = (3, -4, 1) + s(1, -3, -5)$, $s \in \mathbf{R}$, and $L_2 : \vec{r_2} = (7, -1, 0) + t(2, -6, -10)$, $t \in \mathbf{R}$.

14. Sketch each of the following planes:

   a. $\pi_1 : 2x + 3y - 6z - 12 = 0$

   b. $\pi_2 : 2x + 3y - 12 = 0$

   c. $\pi_3 : x - 3z - 6 = 0$

   d. $\pi_4 : y - 2z - 4 = 0$

   e. $\pi_5 : 2x + 3y - 6z = 0$

15. Determine the vector, parametric, and Cartesian equations of each of the following planes:

   a. passing through the points $P(1, -2, 5)$ and $Q(3, 1, 2)$ and parallel to the line with equation $L : \vec{r} = 2t\vec{i} + (4t + 3)\vec{j} + (t + 1)\vec{k}, t \in \mathbf{R}$

   b. containing the point $A(1, 1, 2)$ and perpendicular to the line joining the points $B(2, 1, -6)$ and $C(-2, 1, 5)$

   c. passing through the points $(4, 1, -1)$ and $(5, -2, 4)$ and parallel to the $z$-axis

   d. passing through the points $(1, 3, -5)$, $(2, 6, 4)$, and $(3, -3, 3)$

16. Show that $L_1 : \vec{r} = (1, 2, 3) + s(-3, 5, 21) + t(0, 1, 3)$, s, $t \in \mathbf{R}$, and $L_2 : \vec{r} = (1, -1, -6) + u(1, 1, 1) + v(2, 5, 11)$, $u$, $v \in \mathbf{R}$, are equations for the same plane.

17. The two lines $L_1 : \vec{r} = (-1, 1, 0) + s(2, 1, -1)$, $s \in \mathbf{R}$, and $L_2 : \vec{r} = (2, 1, 2) + t(2, 1, -1)$, $t \in \mathbf{R}$, are parallel but do not coincide. The point $A(5, 4, -3)$ is on $L_1$. Determine the coordinates of a point $B$ on $L_2$ such that $\overrightarrow{AB}$ is perpendicular to $L_2$.

18. Write a brief description of each plane.

   a. $\pi_1 : 2x - 3y = 6$

   b. $\pi_2 : x - 3z = 6$

   c. $\pi_3 : 2y - z = 6$

19. a. Which of the following points lies on the line $x = 2t$, $y = 3 + t$, $z = 1 + t$?

   $A(2, 4, 2)$ \qquad $B(-2, 2, 1)$ \qquad $C(4, 5, 2)$ \qquad $D(6, 6, 2)$

   b. If the point $(a, b, -3)$ lies on the line, determine the values of $a$ and $b$.

20. Calculate the acute angle that is formed by the intersection of each pair of lines.

a. $L_1: \dfrac{x-1}{1} = \dfrac{y-3}{5}$ and $L_2: \dfrac{x-2}{2} = \dfrac{1-y}{3}$

b. $y = 4x + 2$ and $y = -x + 3$

c. $L_1: x = -1 + 3t,\ y = 1 + 4t,\ z = -2t$ and
$L_2: x = -1 + 2s,\ y = 3s,\ z = -7 + s$

d. $L_1: (x, y, z) = (4, 7, -1) + t(4, 8, -4)$ and
$L_2: (x, y, z) = (1, 5, 4) + s(-1, 2, 3)$

21. Calculate the acute angle that is formed by the intersection of each pair of planes.

a. $2x + 3y - z + 9 = 0$ and $x + 2y + 4 = 0$

b. $x - y - z - 1 = 0$ and $2x + 3y - z + 4 = 0$

22. a. Which of the following lines is parallel to the plane $4x + y - z - 10 = 0$?

i. $\vec{r} = (3, 0, 2) + t(1, -2, 2)$

ii. $x = -3t,\ y = -5 + 2t,\ z = -10t$

iii. $\dfrac{x-1}{4} = \dfrac{y+6}{-1} = \dfrac{z}{1}$

b. Do any of these lines lie in the plane in part a.?

23. Does the point $(4, 5, 6)$ lie in the plane
$(x, y, z) = (4, 1, 6) + p(3, -2, 1) + q(-6, 6, -1)$?
Support your answer with the appropriate calculations.

24. Determine the parametric equations of the plane that contains the following two parallel lines:

$L_1: (x, y, z) = (2, 4, 1) + t(3, -1, 1)$ and

$L_2: (x, y, z) = (1, 4, 4) + t(-6, 2, -2)$

25. Explain why the vector equation of a plane has two parameters, while the vector equation of a line has only one.

26. Explain why any plane with a vector equation of the form
$(x, y, z) = (a, b, c) + s(d, e, f) + t(a, b, c)$ will always pass through the origin.

27. a. Explain why the three points $(2, 3, -1)$, $(8, 5, -5)$ and $(-1, 2, 1)$ do not determine a plane.

b. Explain why the line $\vec{r} = (4, 9, -3) + t(1, -4, 2)$ and the point $(8, -7, 5)$ do not determine a plane.

28. Find a formula for the scalar equation of a plane in terms of $a$, $b$, and $c$, where $a$, $b$, and $c$ are the $x$-intercept, the $y$-intercept, and the $z$-intercept of a plane, respectively. Assume that all intercepts are nonzero.

29. Determine the Cartesian equation of the plane that has normal vector $(6, -5, 12)$ and passes through the point $(5, 8, -3)$.

30. A plane passes through the points $A(1, -3, 2)$, $B(-2, 4, -2)$, and $C(3, 2, 1)$.

    a. Determine a vector equation of the plane.

    b. Determine a set of parametric equations of the plane.

    c. Determine the Cartesian equation of the plane.

    d. Determine if the point $(3, 5, -4)$ lies on the plane.

31. Determine the Cartesian equation of the plane that is parallel to the plane $4x - 2y + 5z - 10 = 0$ and passes through each point below.

    a. $(0, 0, 0)$

    b. $(-1, 5, -1)$

    c. $(2, -2, 2)$

32. Show that the following pairs of lines intersect. Determine the coordinates of the point of intersection and the angles formed by the lines.

    a. $L_1: x = 5 + 2t$    and    $L_2: x = 23 - 2s$
             $y = -3 + t$             $y = 6 - s$

    b. $L_1: \dfrac{x + 3}{3} = \dfrac{y + 1}{4}$    and    $L_2: \dfrac{x - 6}{3} = \dfrac{y - 2}{-2}$

33. Determine the vector equation, parametric equations, and, if possible, symmetric equation of the line that passes through the point $P(1, 3, 5)$ and

    a. has direction vector $(-2, -4, -10)$

    b. also passes through the point $Q(-7, 9, 3)$

    c. is parallel to the line that passes through $R(4, 8, -5)$ and $S(-2, -5, 9)$

    d. is parallel to the $x$-axis

    e. is perpendicular to the line $(x, y, z) = (1, 0, 5) + t(-3, 4, -6)$

    f. is perpendicular to the plane determined by the points $A(4, 2, 1)$, $B(3, -4, 2)$, and $C(-3, 2, 1)$

34. Determine the Cartesian equation of the plane that

    a. contains the point $P(-2, 6, 1)$ and has normal vector $(2, -4, 5)$

    b. contains the point $P(-2, 0, 6)$ and the line $\dfrac{x - 4}{3} = \dfrac{y + 2}{-5} = \dfrac{z - 1}{2}$

    c. contains the point $P(3, 3, 3)$ and is parallel to the $xy$-plane

    d. contains the point $P(-4, 2, 4)$ and is parallel to the plane $3x + y - 4z + 8 = 0$

    e. is perpendicular to the $yz$-plane and has $y$-intercept 4 and $z$-intercept $-2$

    f. is perpendicular to the plane $x - 2y + z = 6$ and contains the line $(x, y, z) = (2, -1, -1) + t(3, 1, 2)$