Chapter 8 Test

1. a. Given the points $A(1, 2, 4)$, $B(2, 0, 3)$, and $C(4, 4, 4)$,
   i. determine the vector and parametric equations of the plane that contains these three points
   ii. determine the corresponding Cartesian equation of the plane that contains these three points

   b. Does the point with coordinates $\left(1, -1, -\frac{1}{2}\right)$ lie on this plane?
2. The plane π intersects the coordinate axes at $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$.
   a. Write an equation for this plane, expressing it in the form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
   b. Determine the coordinates of a normal to this plane.
3. a. Determine a vector equation for the plane containing the origin and the line with equation $\vec{r} = (2, 1, 3) + t(1, 2, 5), t \in \mathbf{R}$.
   b. Determine the corresponding Cartesian equation of this plane.
4. a. Determine a vector equation for the plane that contains the following two lines:
   $L_1: \vec{r} = (4, -3, 5) + t(2, 0, -3), t \in \mathbf{R}$, and
   $L_2: \vec{r} = (4, -3, 5) + s(5, 1, -1), s \in \mathbf{R}$

   b. Determine the corresponding Cartesian equation of this plane.
5. a. A line has $\frac{x - 2}{4} = \frac{y - 4}{-2} = z$ as its symmetric equations. Determine the coordinates of the point where this line intersects the yz-plane.
   b. Write a second symmetric equation for this line using the point you found in part a.
6. a. Determine the angle between π_1 and π_2 where the two planes are defined as $\pi_1: x + y - z = 0$ and $\pi_2: x - y + z = 0$.
   b. Given the planes $\pi_3: 2x - y + kz = 5$ and $\pi_4: kx - 2y + 8z = 9$,
      i. determine a value of k if these planes are parallel
      ii. determine a value of k if these planes are perpendicular

   c. Explain why the two given equations that contain the parameter k in part b cannot represent two identical planes.
7. a. Using a set of coordinate axes in R^2, sketch the line $x + 2y = 0$.
   b. Using a set of coordinate axes in R^3, sketch the plane $x + 2y = 0$.
   c. The equation $Ax + By = 0$, $A, B \neq 0$, represents an equation of a plane in R^3. Explain why this plane must always contain the z-axis.