Mid-Chapter Review

1. Determine the point of intersection between the line $\vec{r} = (4, -3, 15) + t(2, -3, 5), t \in \mathbf{R}$, and each of the following planes:
   a. the xy-plane
   b. the xz-plane
   c. the yz-plane
2. $A(2, 1, 3)$, $B(3, -2, 5)$, and $C(-8, -5, 7)$ are three points in R^3 that form a triangle.
   a. Determine the parametric equations for any two of the three medians. (A median is a line drawn from one vertex to the midpoint of the opposite side.)
   b. Determine the point of intersection of the two medians you found in part a.
   c. Determine the equation of the third median for this triangle.
   d. Verify that the point of intersection you found in part b. is a point on the line you found in part c.
   e. State the coordinates of the point of intersection of the three medians.
3. a. Determine an equation for the line of intersection of the planes $5x + y + 2z + 15 = 0$ and $4x + y + 2z + 8 = 0$.
   b. Determine an equation for the line of intersection of the planes $4x + 3y + 3z - 2 = 0$ and $5x + 2y + 3z + 5 = 0$.
   c. Determine the point of intersection between the line you found in part a. and the line you found in part b.
4. a. Determine the line of intersection of the planes $\pi_1: 3x + y + 7z + 3 = 0$ and $\pi_2: x - 13y - 3z - 38 = 0$.
   b. Determine the line of intersection of the planes $\pi_3: x - 3y + z + 11 = 0$ and $\pi_4: 6x - 13y + 8z - 28 = 0$.
   c. Show that the lines you found in parts a. and b. do not intersect.
5. Consider the following system of equations:

   ① $x + ay = 9$

   ② $ax + 9y = -27$

   Determine the value(s) of a for which the system of equations has
   a. no solution
   b. an infinite number of solutions
   c. one solution

6. Show that $\frac{x - 11}{2} = \frac{y - 4}{-4} = \frac{z - 27}{5}$ and $x = 0, y = 1 - 3t, z = 3 + 2t$, $t \in \mathbf{R}$, are skew lines.

7. a. Determine the intersection of the lines
   $(x - 3, y - 20, z - 7) = t(2, -4, 5), t \in \mathbf{R}$, and $\frac{x - 5}{2} = y - 2 = \frac{z + 4}{-3}$.
   b. What conclusion can you make about these lines?

8. Determine the point of intersection between the lines
   $x = 1 + 2s, y = 4 - s, z = -3s, s \in \mathbf{R}$, and
   $x = -3, y = t + 3, z = 2t, t \in \mathbf{R}$.

9. Determine the point of intersection for each pair of lines.
   a. $\vec{r} = (5, 1, 7) + s(2, 0, 5), s \in \mathbf{R}$, and
      $\vec{r} = (-1, -1, 3) + t(4, 2, -1), t \in \mathbf{R}$
   b. $\vec{r} = (2, -1, 3) + s(5, -1, 6), s \in \mathbf{R}$, and
      $\vec{r} = (-8, 1, -9) + t(5, -1, 6), t \in \mathbf{R}$

10. You are given a pair of vector equations that both represent lines in R^3.
    a. Explain all the possible ways that these lines could be related to each other. Support your explanation with diagrams.
    b. Explain how you could use the equations you are given to help you identify which of the situations you described in part a. you are dealing with.

11. a. Explain when a line and a plane can have an infinite number of points of intersection.
    b. Give an example of a pair of vector equations (one for a line and one for a plane) that have an infinite number of points of intersection.

12. Use elementary operations to solve each system of equations.
    a. ① $2x + 3y = 30$
       ② $x - 2y = -13$
    b. ① $x + 4y - 3z + 6 = 0$
       ② $2x + 8y - 6z + 11 = 0$
    c. ① $x - 3y - 2z = -9$
       ② $2x - 5y + z = 3$
       ③ $-3x + 6y + 2z = 8$

13. For the system of equations given in parts a. and b. of question 12, describe the corresponding geometrical representation.

14. L is the line of intersection of planes $x - y = 1$ and $y + z = -3$, and L_1 is the line of intersection of the planes $y - z = 0$ and $x = -\frac{1}{2}$.
    a. Determine the point of intersection of L and L_1.
    b. Determine the angle between the lines of intersection.
    c. Determine the Cartesian equation of the plane that contains the point you found in part a. and the two lines of intersection.