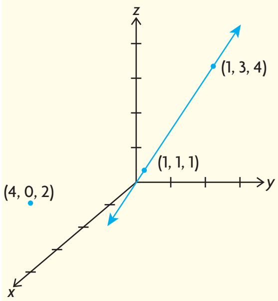
CAREER LINK WRAP-UP Investigate and Apply

CHAPTER 9: RELATIONSHIPS BETWEEN POINTS, LINES, AND PLANES



A pipeline engineer needs to find the line that will allow a new pipeline to intersect and join an existing pipeline at a right angle. The existing line has a pathway determined by the equation L_2: $r = (1, 1, 1) + d(0, 2, 3)$, $d \in \mathbf{R}$. The new pipeline will also need to be exactly 2 units away from the point (4, 0, 2).

a. Determine the vector and parametric equations of L_3, the line that passes through (4, 0, 2) and is perpendicular to L_2.

b. Determine the vector and parametric equations of L_1, the line that is parallel to L_3 and 2 units away from (4, 0, 2). There will be exactly two lines that fulfill this condition.

c. Plot each line on the coordinate axes.

Key Concepts Review

In this chapter, you learned how to solve systems of linear equations using elementary operations. The number of equations and the number of variables in the system are directly related to the geometric interpretation that each system represents.

System of Equations	Geometric Interpretation	Possible Points of Intersection
Two equations and two unknowns	two lines in R^2	zero, one, or an infinite number
Two equations and three unknowns	two planes in R^3	zero or an infinite number
Three equations and three unknowns	three planes in R^3	zero, one, or an infinite number

To make a connection between the algebraic equations and the geometric position and orientation of lines or planes in space, draw graphs or diagrams and compare the direction vectors of the lines and the normals of the planes. This will help you decide whether the system is consistent or inconsistent and which case you are dealing with.

Distances between points, lines and planes can be determined using the formulas developed in this chapter.

Distance between a point and a line in R^2	$d = \dfrac{\lvert Ax_0 + By_0 + C\rvert}{\sqrt{A^2 + B^2}}$
Distance between a point and a line in R^3	$d = \dfrac{\lvert\vec{m} \times \overrightarrow{QP}\rvert}{\lvert\vec{m}\rvert}$
Distance between a point and a plane in R^3	$d = \dfrac{\lvert Ax_0 + By_0 + Cz_0 + D\rvert}{\sqrt{A^2 + B^2 + C^2}}$