Review Exercise

1. The lines $2x - y = 31$, $x + 8y = -34$, and $3x + ky = 38$ all pass through a common point. Determine the value of k.
2. Solve the following system of equations:

   ① $x - y = 13$

   ② $3x + 2y = -6$

   ③ $x + 2y = -19$
3. Solve each system of equations.

   a. ① $x - y + 2z = 3$

   ② $2x - 2y + 3z = 1$

   ③ $2x - 2y + z = 11$

   b. ① $x + y + z = 300$

   ② $x + y - z = 98$

   ③ $x - y + z = 100$
4. a. Show that the points $(1, 2, 6)$, $(7, -5, 1)$, $(1, 1, 4)$, and $(-3, 5, 6)$ all lie on the same plane.

   b. Determine the distance from the origin to the plane you found in part a.
5. Determine the following distances:

   a. the distance from $A(-1, 1, 2)$ to the plane with equation $3x - 4y - 12z - 8 = 0$

   b. the distance from $B(3, 1, -2)$ to the plane with equation $8x - 8y + 4z - 7 = 0$
6. Determine the intersection of the plane $3x - 4y - 5z = 0$ with $\vec{r} = (3, 1, 1) + t(2, -1, 2), t \in \mathbf{R}$.
7. Solve the following systems of equations:

   a. ① $3x - 4y + 5z = 9$

   ② $6x - 9y + 10z = 9$

   ③ $9x - 12y + 15z = 9$

   b. ① $2x + 3y + 4z = 3$

   ② $4x + 6y + 8z = 4$

   ③ $5x + y - z = 1$

   c. ① $4x - 3y + 2z = 2$

   ② $8x - 6y + 4z = 4$

   ③ $12x - 9y + 6z = 1$

8. Solve each system of equations.

a. ① $3x + 4y + z = 4$
② $5x + 2y + 3z = 2$
③ $6x + 8y + 2z = 8$

b. ① $4x - 8y + 12z = 4$
② $2x + 4y + 6z = 4$
③ $x - 2y - 3z = 4$

c. ① $x - 3y + 3z = 7$
② $2x - 6y + 6z = 14$
③ $-x + 3y - 3z = -7$

9. Solve each of the following systems:

a. ① $3x - 5y + 2z = 4$
② $6x + 2y - z = 2$
③ $6x - 3y + 8z = 6$

b. ① $2x - 5y + 3z = 1$
② $4x + 2y + 5z = 5$
③ $2x + 7y + 2z = 4$

10. Determine the intersection of each set of planes, and show your answer geometrically.

a. $2x + y + z = 6, x - y - z = -9, 3x + y = 2$

b. $2x - y + 2z = 2, 3x + y - z = 1, x - 3y + 5z = 4$

c. $2x + y - z = 0, x - 2y + 3z = 0, \ 9x + 2y - z = 0$

11. The line $\vec{r} = (2, -1, -2) + s(1, 1, -2), s \in \mathbf{R}$, intersects the xz-plane at point P and the xy-plane at point Q. Calculate the length of the line segment PQ.

12. a. Given the line $\vec{r} = (3, 1, -5) + s(2, 1, 0), s \in \mathbf{R}$, and the plane $x - 2y + z + 4 = 0$, verify that the line lies on the plane.

b. Determine the point of intersection between the line $\vec{r} = (7, 5, -1) + t(4, 3, 2), t \in \mathbf{R}$, and the line given in part a.

c. Show that the point of intersection of the lines is a point on the plane given in part a.

d. Determine the Cartesian equation of the plane that contains the line $\vec{r} = (7, 5, -1) + t(4, 3, 2), t \in \mathbf{R}$ and is perpendicular to the plane given in part a.

13. a. Determine the distance from point $A(-2, 1, 1)$ to the line with equation $\vec{r} = (3, 0, -1) + t(1, 1, 2), t \in \mathbf{R}$.

b. What are the coordinates of the point on the line that produces this shortest distance?

14. You are given the lines $\vec{r} = (1, -1, 1) + t(3, 2, 1), t \in \mathbf{R}$, and $\vec{r} = (-2, -3, 0) + s(1, 2, 3), s \in \mathbf{R}$.
    a. Determine the coordinates of their point of intersection.
    b. Determine a vector equation for the line that is perpendicular to both of the given lines and passes through their point of intersection.
15. a. Determine the equation of the plane that contains $L: \vec{r} = (1, 2, -3) + s(1, 2, -1), s \in \mathbf{R}$, and point $K(3, -2, 4)$.
    b. Determine the distance from point $S(1, 1, -1)$ to the plane you found in part a.
16. Consider the following system of equations:

    ① $x + y - z = 1$

    ② $2x - 5y + z = -1$

    ③ $7x - 7y - z = k$

    a. Determine the value(s) of k for which the solution to this system is a line.
    b. Determine the vector equation of the line.
17. Determine the solution to each system of equations.

    a. ① $x + 2y + z = 1$

    ② $2x - 3y - z = 6$

    ③ $3x + 5y + 4z = 5$

    ④ $4x + y + z = 8$

    b. ① $x - 2y + z = 1$

    ② $2x - 5y + z = -1$

    ③ $3x - 7y + 2z = 0$

    ④ $6x - 14y + 4z = 0$
18. Solve the following system of equations for a, b, and c:

    ① $\dfrac{9a}{b} - 8b + \dfrac{3c}{b} = 4$

    ② $\dfrac{-3a}{b} + 4b + \dfrac{4c}{b} = 3$

    ③ $\dfrac{3a}{b} + 4b - \dfrac{4c}{b} = 3$

    $\left(\textit{Hint}\text{: Let } x = \frac{a}{b}, y = b, \text{ and } z = \frac{c}{b}.\right)$
19. Determine the point of intersection of the line $\frac{x+1}{-4} = \frac{y-2}{3} = \frac{z-1}{-2}$ and the plane with equation $x + 2y - 3z + 10 = 0$.
20. Point $A(1, 0, 4)$ is reflected in the plane with equation $x - y + z - 1 = 0$. Determine the coordinates of the image point.

21. The three planes with equations $3x + y + 7z + 3 = 0$, $4x - 12y + 4z - 24 = 0$, and $x + 2y + 3z - 4 = 0$ do not simultaneously intersect.

   a. Considering the planes in pairs, determine the three lines of intersection.

   b. Show that these three lines are parallel.

22. Solve for a, b, and c in the following system of equations:

$$① \quad \frac{2}{a^2} + \frac{5}{b^2} + \frac{3}{c^2} = 40$$

$$② \quad \frac{3}{a^2} - \frac{6}{b^2} - \frac{1}{c^2} = -3$$

$$③ \quad \frac{9}{a^2} - \frac{5}{b^2} + \frac{4}{c^2} = 67$$

23. Determine the equation of a parabola that has its axis parallel to the y-axis and passes through the points $(-1, 2)$, $(1, -1)$, and $(2, 1)$. (Note that the general form of the parabola that is parallel to the y-axis is $y = ax^2 + bx + c$.)

24. A perpendicular line is drawn from point $X(3, 2, -5)$ to the plane $4x - 5y + z - 9 = 0$ and meets the plane at point M. Determine the coordinates of M.

25. Determine the values of A, B, and C if the following is true:

$$\frac{11x^2 - 14x + 9}{(3x - 1)(x^2 + 1)} = \frac{A}{3x - 1} + \frac{Bx + C}{x^2 + 1}$$

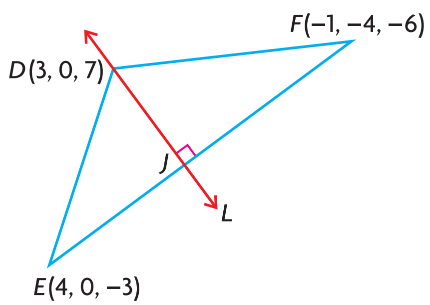
   (*Hint*: Simplify the right side by combining fractions and comparing numerators.)

26. A line L is drawn through point D, perpendicular to the line segment EF, and meets EF at point J.

   a. Determine an equation for the line containing the line segment EF.

   b. Determine the coordinates of point J on EF.

   c. Determine the area of $\triangle DEF$.



27. Determine the equation of the plane that passes through $(5, -5, 5)$ and is perpendicular to the line of intersection of the planes $3x - 2z + 1 = 0$ and $4x + 3y + 7 = 0$.