Glossary

A

absolute extrema: the largest or smallest value of a function over its entire domain.

acceleration: the rate of change of velocity with respect to time $\left(\frac{dv}{dt}\right)$ or the second derivative of displacement with respect to time $\left(\frac{d^2s}{dt^2}\right)$.

algebraic vectors: vectors that are considered with respect to coordinate axes.

asymptote: a line having the property that the distance from a point P on a curve to the line approaches zero as the distance from P to the origin increases indefinitely. The line and the curve get closer and closer but never touch. See **horizontal**, **vertical**, and **slant asymptote**.

augmented matrix: a matrix made up of the coefficient matrix and one additional column containing the constant terms of the equations to be solved.

average rate of change: given by the difference quotient $\frac{\Delta y}{\Delta x} = \frac{f(a + h) - f(a)}{h}$. The average rate of change of the function $f(x)$ over the interval $x = a$ to $x = a + h$.

B

bearing: a way of specifying the direction from one object to another, often stated in terms of compass directions.

C

calculus: a branch of mathematics, discovered independently by Sir Isaac Newton and Gottfried Wilhelm von Leibniz, that deals with the instantaneous rate of change of a function (differential calculus) and the area under a function (integral calculus).

Cartesian coordinate system: a reference system in two-space, consisting of two axes at right angles, or three-space (three axes) in which any point in the plane is located by its displacements from these fixed lines (axes). The origin is the common point from which each displacement is measured. In two-space, a set of two numbers or coordinates is required to uniquely define a position; in three-space, three coordinates are required.

Cartesian equation of a plane: Cartesian (or scalar) equation of a plane in R^3 is of the form $Ax + By + Cz + D = 0$ with a normal $\vec{n} = (A, B, C)$. The normal n is a nonzero vector perpendicular to all of vectors in the plane.

chain rule: if $f(x)$ and $g(x)$ are continuous and differentiable functions, then the composite function $h(x) = f[g(x)]$ has a derivative given by $h'(x) = f'[g(x)]g'(x)$. In Leibniz notation, if $y = f(u)$ where $u = g(x)$, then y is a composite function and $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

chord: in a circle, the portion of the secant inside the circle.

coefficient matrix: a matrix whose elements are the coefficients of the unknown terms in the equations to be solved by matrix methods.

coincident: Two or more congruent geometric figures (vectors, lines, or planes) which can be translated to lie on top of each other.

collinear vectors: vectors that are parallel and that lie on the same straight line.

composition of forces: the process of finding the resultant of all the forces acting on an object.

composite function: given two functions, $f(x)$ and $g(x)$, the composite function $f \circ g = f[g(x)]$. $g(x)$ is called the inner function and $f(x)$ is the outer function.

composition: the process of combining functions.

concave up/down: $f(x)$ is concave up at x_0 if and only if $f'(x)$ is increasing at x_0. $f(x)$ is concave down at x_0 if and only if $f'(x)$ is decreasing at x_0. If $f''(x)$ exists at x_0 and is positive, then $f(x)$ is concave up at x_0. If $f''(x)$ exists and is negative, then $f(x)$ is concave down at x_0. If $f''(x)$ does not exist or is zero, then the test fails.

conjugate radical: for an expression of the form $\sqrt{a} + \sqrt{b}$, the conjugate radical is $\sqrt{a} - \sqrt{b}$.

constant function rule: if $f(x) = k$, where k is a constant, then $f'(x) = 0$. In Leibniz notation, $\frac{d}{dx}(k) = 0$.

constant multiple rule: if $f(x) = kg(x)$ where k is a constant, then $f'(x) = kg'(x)$. In Leibniz notation: $\frac{df}{dx} = k\frac{dg}{dx}$.

consistent system of equations: a system of equations that has either one solution or an infinite number of solutions.

continuity: the condition of being uninterrupted, without break or irregularity.

continuous function: a function $f(x)$ is continuous at a particular point $x = a$, if $f(a)$ is defined and if $\lim_{x \to a} f(x) = f(a)$. If this property is true for all points in the domain of the function, then the function is said to be continuous over the domain.

coplanar: the description given to two or more geometric objects that lie in the same plane.

critical points (of a function): a critical point on $f(x)$ occurs at x_0 if and only if either $f'(x_0) = 0$ or the derivative doesn't exist.

critical numbers: numbers in the domain of a function that cause its derivative to be zero or undefined.

cross product: the cross product of two vectors $\vec{a}$ and $\vec{b}$ in R^3 (three-space) is the vector that is perpendicular to these factors and has a magnitude of $|\vec{a}||\vec{b}|\sin\theta$ such that the vectors $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ form a right-handed system.

cusp: a type of double point. A cusp is a point on a continuous curve where the tangent line reverses sign.

D

decreasing function: a function $f(x)$ is decreasing at a point x_0 if and only if there exists some interval I containing x_0, such that $f(x_0) < f(x)$ for all x in I to the left of x_0 and $f(x_0) > f(x)$ for all x in I to the right of x_0.

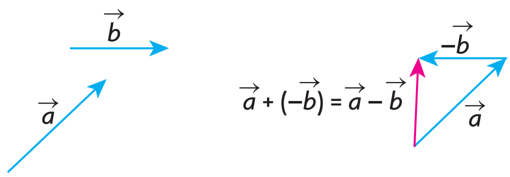
delta: the fourth letter of the Greek alphabet: lower case $[\delta]$; upper case $[\Delta]$.

dependent variable: in a relation, the variable whose value depends upon the value of the independent variable. On a coordinate grid, the values of the independent variable are usually plotted on the horizontal axis, and the values of the dependent variable on the vertical axis.

derivative: the instantaneous rate of change of a function with respect to the variable. The derivative at a particular point is equal to the slope of the tangent line drawn to the curve at that point. The derivative of $f(x)$ at the point $x = a$: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ provided the limit exists.

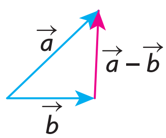
derivative function: for a function $f(x)$, the derivative function is $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ for all x for which the limit exists.

difference of two vectors:



In the diagram above, the difference between vectors $\vec{a}$ and $\vec{b}$ is found by adding the opposite vector $\vec{b}$ to $\vec{a}$ using the triangle law of addition.

Another way to think about $\vec{a} - \vec{b}$ is to arrange vectors tail to tail. In this case, $\vec{a} - \vec{b}$ is the vector that must be added to $\vec{b}$ to get $\vec{a}$. This is illustrated in the following diagram. Using the vectors above, the difference vector is the same as the one produced by adding the opposite.



difference quotient: the slope of the secant drawn to a curve $f(x)$ between the points on the curve $(a, f(a))$ and $(a + h, f(a + h))$: $\frac{\Delta y}{\Delta x} = \frac{f(a + h) - f(a)}{h}$.

difference rule: if functions $p(x)$ and $q(x)$ are differentiable and $f(x) = p(x) - q(x)$, then $f'(x) = p'(x) - q'(x)$. In Leibniz notation: $\frac{df}{dx} = \frac{dp}{dx} - \frac{dq}{dx}$.

differentiability: a real function is said to be differentiable at a point if its derivative exists at that point.

differential calculus: that portion of calculus dealing with derivatives.

direction angles: the angles that a vector makes with each of the coordinate axes.

direction cosines: the cosines of the direction angles.

direction numbers: the components of a direction vector; for the vector (a, b), the direction numbers are a and b.

direction vector: a vector that determines the direction of a particular line.

discontinuity: an interrupted or broken connection. A value for x, on an $x - y$ grid, for which a value for y is not defined. A formal mathematical definition: a function $f(x)$ is discontinuous at a particular point $x = a$ if $f(a)$ is not defined and/or if $\lim_{x \to a} f(x) \neq f(a)$.

displacement: a translation from one position to another, without consideration of any intervening positions. The minimal distance between two points.

dot product: for two vectors, the dot product is the product of the magnitudes of the vectors and the cosine of the angle between the two vectors.

E

***e*:** the base of the natural logarithm, whose symbol "e" honours Euler. It can be defined as the $\lim_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$ and is equal to 2.718 281 828 6… (a non-repeating, infinite decimal).

elementary operations: operations that produce equivalent systems:

1. multiplication of an equation by a nonzero constant
2. interchanging any pair of equations
3. adding a nonzero multiple of one equation to a second equation to replace the second equation

equivalent systems: two systems of equations are equivalent if every solution of one system is also a solution to the second system.

exponent laws:

Action	Result
$a^m \times a^n$	a^{m+n}
$\frac{a^m}{a^n}$	a^{m-n}
$(a^m)^n$	$a^{m \times n}$
$(ab)^m$	$a^m b^m$
$\left(\frac{a}{b}\right)^m, b \neq 0$	$\frac{a^m}{b^m}$

Action	Result
a^0	1
a^{-n}	$\frac{1}{a^n}$
$\left(\frac{a}{b}\right)^{-n}$	$\frac{b^n}{a^n}$
$a^{\frac{p}{q}}$	$(\sqrt[q]{a})^p$ or $\sqrt[q]{a^p}$

equilibrant: equal in magnitude but acting in the opposite direction to the resultant force, resulting in a state of equilibrium.

extended power rule: a symmetric expression that extends the power rule for the product of two functions to three functions and beyond. For example, if $f(x) = g(x)h(x)k(x)$, then $f'(x) = g'(x)h(x)k(x) + g(x)h'(x)k(x) + g(x)h(x)k'(x)$ Note the symmetry.

extreme values (of a function): the maximum and minimum values of a function over a particular interval of values (domain).

F

first derivative test: if $f'(x)$ changes sign from negative to positive at $x = c$, then $f(x)$ has a local minimum at this point; if $f'(x)$ changes sign from positive to negative at $x = c$, then $f(x)$ has a local maximum at this point.

force: something that either pushes or pulls an object.

G

geometric vectors: vectors that are considered without reference to coordinate axes.

H

horizontal asymptote: the line $y = y_0$ is a horizontal asymptote of $f(x)$ if and only if $f(x)$ approaches y_0 as $x \to \pm\infty$.

I

identity matrix: the matrix that consists entirely of a diagonal of 1's with all other numbers in the matrix 0.

implicit differentiation: a method for differentiating an implicit function, utilizing the chain rule and ultimately solving for the derivative desired $\left(\frac{dy}{dx}\right)$.

implicit function: a function in which the dependent variable is not directly stated as a function of the independent variable.

inconsistent system of equations: a system of equations that has no solutions.

increasing function: a function $f(x)$ is increasing at a point x_0 if and only if there exists some interval I containing x_0, such that $f(x_0) > f(x)$ for all x in I to the left of x_0, and $f(x_0) < f(x)$ for all x in I to the right of x_0.

independent variable: in a relation, the variable whose value determines the value of the dependent variable. See **dependent variable.**

indeterminate form: a quotient $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ both approach 0 or $\pm\infty$ as x approaches a is an indeterminate form: $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$.

inflection point: an inflection point occurs on $f(x)$ at x_0 if and only if $f(x)$ has a tangent line at x_0 and there exists an interval I containing x_0 such that $f(x)$ is concave up on one side of x_0 and concave down on the other side.

instantaneous rate of change: the rate of change of $y = f(x)$ at a particular point $x = a$ is given by $\lim\limits_{x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{x \to 0} \frac{f(a + h) - f(a)}{h}$ provided the limit exists.

L

Leibniz notation: for example, $\frac{dy}{dx}$ is Leibniz notation for the derivative of y with respect to x. The notation we use in everyday calculus is attributable primarily to Leibniz.

limit (of a function): the notation $\lim\limits_{x \to a} f(x) = L$ implies that, as x approaches closer and closer to the value a, the value of the function approaches a limiting value of L.

linear combination: the sum of nonzero multiples of two or more vectors or equations.

local maximum: a function $f(x)$ has a local maximum at x_0 if and only if there exists some interval I containing x_0 such that $f(x_0) \geq f(x)$ for all x in I.

local minimum: a function $f(x)$ has a local minimum at x_0 if and only if there exists some interval I containing x_0 such that $f(x_0) \leq f(x)$ for all x in I.

logarithm (natural): logarithms of numbers using a base of e. Usually written as $\ln x$.

logarithmic differentiation: a process using logarithms to differentiate algebraically complicated functions or functions for which the ordinary rules of differentiation do not apply.

logarithmic function: the inverse of the exponential function. If $y = b^x$ represents the exponential function, then $y = \log_b y$ is the logarithmic function. Usually written as $y = \log_b x$.

logistic model: a mathematical model that describes a population that grows exponentially at the beginning and then levels off to a limiting value. There are several different forms of equations representing this model.

M

magnitude: the absolute value of a quantity.

maximum: the largest value of a function on a given interval of values.

minimum: the smallest value of a function on a given interval of values.

N

natural logarithm function: the logarithm function with base e, written $y = \log_e x$ or $y = \ln x$.

normal axis: for a given line, the normal axis is the only line that can be drawn from the origin perpendicular to the given line.

normal line: the line drawn at a point on a graph of $f(x)$, perpendicular to the tangent line drawn at that point.

O

oblique asymptote: See **slant asymptote**.

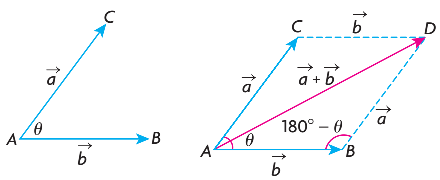
opposite vectors: vectors that have the same magnitude but point in opposite directions.

optimization: a procedure to determine the best possible outcome of a situation. If the situation can be modelled as a function, it may involve finding either the maximum or minimum value of the function over a set of values (domain).

optimize: to realize the best possible outcome for a situation, subject to a set of restrictions.

P

parallelogram law for adding two vectors: To determine the sum of the two vectors $\vec{a}$ and $\vec{b}$, complete the parallelogram formed by these two vectors when placed tail to tail. Their sum is the vector $\overrightarrow{AD}$, the diagonal of the constructed parallelogram, $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$.



parameter: a variable that permits the description of a relation among other variables (two or more) to be expressed in an indirect manner using that variable.

parametric equations of a line: derived from vector equation; in $R^2, x = x_0 + ta, y = y_0 + tb, t \in \mathbf{R}$.

parametric equations for a plane: in R^3, $\vec{r}_0 = (x_0, y_0, z_0)$ is determined by a point on a plane and $a = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ are vectors that lie on the same plane as the point.
$x = x_0 + sa_1 + tb_1,$
$y = y_0 + sa_2 + tb_2,$
$z = z_0 + sa_3 + tb_3, s, t \in \mathbf{R}.$

point of inflection: See **inflection point**.

position vector: the position vector $\overrightarrow{OP}$ has its head at the point $P(a, 0)$ and its tail at the origin $O(0, 0)$.

power function: a function of the form $f(x) = x^n$, where n is a real number.

power of a function rule: if u is a function of x and n is a positive integer, then in Leibniz notation $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$. In function notation, if $f(x) = [g(x)]^n$, then $f'(x) = n[g(x)]^{n-1}g(x)$.

power rule: if $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$.

product rule: if $h(x) = f(x)g(x)$, then $h'(x) = f'(x)g(x) + f(x)g'(x)$. See **extended power rule**.

projection: a mapping of a geometric figure formed by dropping a perpendicular from each of the points onto a line or plane.

Q

quotient rule: if $h(x) = \frac{f(x)}{g(x)}$, then
$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, g(x) \neq 0.$

R

rate of change: a measure of how rapidly the dependent variable changes when there is a change in the independent variable.

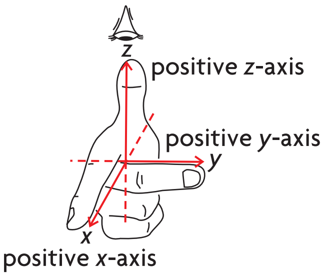
rational function: a function that can be expressed as $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

rationalizing the denominator: the process of multiplying the numerator and denominator of a rational expression by the conjugate radical of the denominator.

resolution: the opposite of composition; taking a single force and decomposing it into two components, often parallel to the vertical and horizontal axes.

resultant: the sum of two or more vectors; represents the combined effect of the vectors.

right-handed system of coordinates: one method of specifying the relative position of the coordinate axes in three dimensions; illustrated in the figure below.



row-echelon form: any matrix that has the following characteristics:

1. All zero rows are at the bottom of the matrix
2. The leading entry of each nonzero row after the first occurs to the right of the leading entry of the previous row.
3. The leading entry in any nonzero row is 1.
4. All entries in the column above and below a leading 1 are zero.

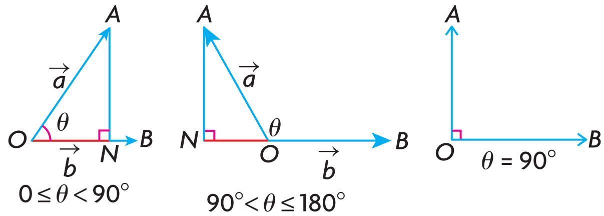
reduced row-echelon form: a matrix derived by the method of Gauss-Jordan elimination that permits the solution of a system of linear equations.

S

scalar: a quantity whose magnitude can be completely specified by just one number.

scalar product: See **dot product**.

scalar projection: the scalar projection of vector $\vec{a}$ onto $\vec{b}$ is ON where $ON = |\vec{a}|\cos\theta$.



second derivative: for a function $f(x)$, the second derivative of $y = f(x)$ is the derivative of $y = f'(x)$.

second derivative test: if $f(x)$ is a function for which $f''(c) = 0$, and the second derivative of $f(x)$ exists on an interval containing c, then

- $f(c)$ is a local minimum value if $f''(x) > 0$
- $f(c)$ is a local maximum value if $f''(x) < 0$
- the test is indeterminate if $f'(x) = 0$, and the first derivative test must be used

secant: a line through two points on a curve.

slant asymptote: the line $y = ax + b$ is a slant or oblique asymptote of $f(x)$ if and only if $\lim_{x\to\pm\infty} f(x) = ax + b$.

slope of tangent: the slope of the tangent to the function $y = f(x)$ at the point $(a, f(a))$ on the curve is given by $\lim_{h\to 0}\frac{\Delta y}{\Delta x} = \lim_{x\to 0}\frac{f(a+h) - f(a)}{h}$.

speed: distance travelled per unit of time. The absolute value of velocity.

spanning set: a set of two vectors forms a spanning set for R^2 if every vector in R^2 can be written as a linear combination of these two vectors; a spanning set for R^3contains three vectors.

standard basis vectors: unit vectors that lie along the axes; $\vec{i}$ and $\vec{j}$ for R^2 and $\vec{i}$, $\vec{j}$, and $\vec{k}$ for R^3.

sum rule: if functions $p(x)$ and $q(x)$ are differentiable and $f(x) = p(x) + q(x)$, then $f'(x) = p'(x) + q'(x)$. In Leibniz notation: $\frac{df}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$.

symmetric equation: for a line in R^3,

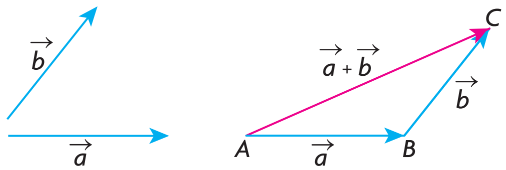
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} =, a \neq 0, b \neq 0, c \neq 0.$$

(x_0, y_0, z_0) is the vector from the origin to a point on the line and is a direction vector of the line.

T

tangent: the straight line that most resembles the graph near that point.

triangle law of addition: in the diagram, the sum of the vectors $\vec{a}$ and $\vec{b}$, $\vec{a} + \vec{b}$, is found by translating the tail of vector $\vec{b}$ to the head of vector $\vec{a}$. This could also have been done by translating $\vec{a}$ so that its tail was at the head of $\vec{b}$. In either case, the sum of the vectors $\vec{a}$ and $\vec{b}$ is $\overrightarrow{AC}$.



U

unit vector: a vector with magnitude 1.

V

vector: a quantity that requires both a magnitude and a direction for a complete description.

vector equation of a line: equation of a line written in terms of a position vector for a point on the line and a vector specifying the direction of the vector; in R^2, $\vec{r} = \vec{r_0} + t\vec{m}, t \in \mathbf{R}$.

vector equation of a plane: in R_3, $\vec{r_0} = (x_0, y_0, z_0)$ is determined by a point on a plane and $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ are vectors that lie on the same plane as the point: $\vec{r} = \vec{r_0} + s\vec{a} + t\vec{b}, s, t \in \mathbf{R}$.

velocity: the rate of change of displacement with respect to time: $\left(\frac{ds}{dt}\right)$.

vertical asymptote: the line $x = x_0$ is a vertical asymptote of $f(x)$ if and only if $f(x) \rightarrow \pm\infty$ as $x \rightarrow x_0$ from the left or from the right.

vector product: See **cross product**.

vector projection: the vector projection of $\vec{a}$ on $\vec{b}$ is the product of the dot product for the two vectors and a unit vector in the direction of $\vec{b}$.

Z

zero vector: the vector with a magnitude of 0 and no defined direction.