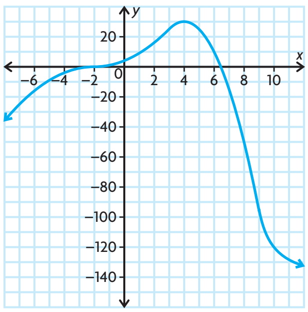
4. hole at $x = -2$; large and negative to left of asymptote, large and positive to right of asymptote;
$y = 1$;
Domain: $\{x \in \mathbf{R} | x \neq -2, x \neq 3\}$

5.



6. There are discontinuities at $x = -3$ and $x = 3$.

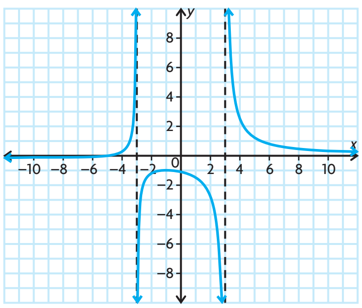
$$\left.\begin{array}{l}\lim_{x \to 3^-} f(x) = \infty \\ \lim_{x \to 3^+} f(x) = -\infty\end{array}\right\} x = -3 \text{ is a vertical asymptote.}$$

$$\left.\begin{array}{l}\lim_{x \to 3^-} f(x) = -\infty \\ \lim_{x \to 3^+} f(x) = \infty\end{array}\right\} x = 3 \text{ is a vertical asymptote.}$$

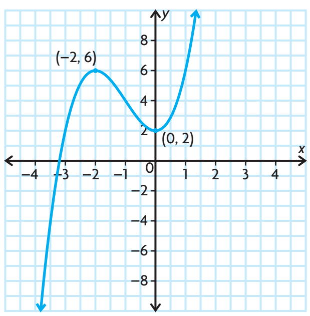
The y-intercept is $-\frac{10}{9}$ and x-intercept is -5.

$\left(-9, -\frac{1}{9}\right)$ is a local minimum and $(-1, -1)$ is a local maximum.

$y = 0$ is a horizontal asymptote.



7. $b = 3, c = 2$



Chapter 5

Review of Prerequisite Skills, pp. 224–225

1. **a.** $\frac{1}{9}$ **c.** $\frac{1}{9}$

b. 4 **d.** $\frac{9}{4}$

2. **a.** $\log_5 625 = 4$

b. $\log_4 \frac{1}{16} = -2$

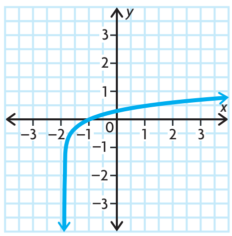
c. $\log_x 3 = 3$

d. $\log_{10} 450 = w$

e. $\log_3 z = 8$

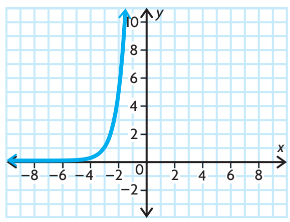
f. $\log_a T = b$

3. **a.**



x-intercept: $(-1, 0)$

b.



no x-intercept

4. **a.** $\frac{y}{r}$ **b.** $\frac{x}{r}$ **c.** $\frac{y}{x}$

5. **a.** 2π **e.** $\frac{3\pi}{2}$

b. $\frac{\pi}{4}$ **f.** $-\frac{2\pi}{3}$

c. $-\frac{\pi}{2}$ **g.** $\frac{5\pi}{4}$

d. $\frac{\pi}{6}$ **h.** $\frac{11\pi}{6}$

6. **a.** b **c.** a **e.** b

b. $\frac{b}{a}$ **d.** a **f.** $-b$

7. **a.** $\cos \theta = -\frac{12}{13}$,

$\tan \theta = -\frac{5}{12}$

b. $\sin \theta = -\frac{\sqrt{5}}{3}$,

$\tan \theta = \frac{\sqrt{5}}{2}$

c. $\sin \theta = -\frac{2}{\sqrt{5}}$,

$\cos \theta = \frac{1}{\sqrt{5}}$

d. $\cos \frac{\pi}{2} = 0$,

$\tan \frac{\pi}{2}$ is undefined

8. **a.** period: π,
amplitude: 1

b. period: 4π,
amplitude: 2

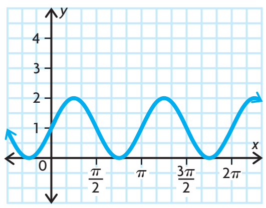
c. period: 2,
amplitude: 3

d. period: $\frac{\pi}{6}$,
amplitude: $\frac{2}{7}$

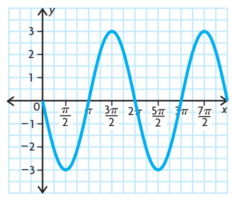
e. period: 2π,
amplitude: 5

f. period: 2π,
amplitude: $\frac{3}{2}$

9. **a.**



b.



10. **a.** $\tan x + \cot x = \sec x \csc x$

$$\begin{aligned}
\text{LS} &= \tan x + \cot x \\
&= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\
&= \frac{\sin^2 x + \cos^2 x}{\cos x + \sin x} \\
&= \frac{1}{\cos x + \sin x} \\
\text{RS} &= \sec x \csc x \\
&= \frac{1}{\cos x} \times \frac{1}{\sin x} \\
&= \frac{1}{\cos x \sin x}
\end{aligned}$$

Therefore, $\tan x + \cot x = \sec x \csc x$.

b. $\dfrac{\sin x}{1 - \sin^2 x} = \tan x + \sec x$

$$\begin{aligned} \text{LS} &= \frac{\sin x}{1 - \sin^2 x} \\ &= \frac{\sin x}{\cos^2 x} \\ \text{RS} &= \tan x \sec x \\ &= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \\ &= \frac{\sin x}{\cos^2 x} \end{aligned}$$

Therefore, $\dfrac{\sin x}{1 - \sin^2 x} = \tan x \sec x$.

11. **a.** $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

b. $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

Section 5.1, pp. 232–234

1. You can only use the power rule when the term containing variables is in the base of the exponential expression. In the case of $y = e^x$, the exponent contains a variable.

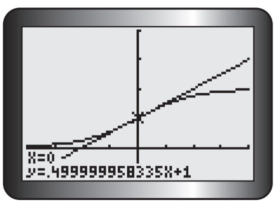
2. **a.** $3e^{3x}$
b. $3e^{3t - 5}$
c. $20e^{10t}$
d. $-3e^{-3x}$
e. $(-6 + 2x)e^{5 - 6x + x^2}$
f. $\dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}$

3. **a.** $6x^2e^{x^3}$
b. $e^{3x}(3x + 1)$
c. $\dfrac{-3x^2e^{-x^3}(x) - e^{-x^3}}{x^2}$
d. $\sqrt{x}e^x + e^x\left(\dfrac{1}{2\sqrt{x}}\right)$
e. $2te^{t^2} - 3e^{-t}$
f. $\dfrac{2e^{2t}}{(1 + e^{2t})^2}$

4. **a.** $e^3 - e^{-3}$
b. $\dfrac{1}{e}$
c. $-2 - 3e$

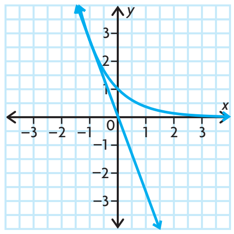
5. **a.** $y = \dfrac{1}{2}x + 1$

b.



c. The answers agree very well; the calculator does not show a slope of exactly 0.5, due to internal rounding.

6. $ex + y = 0$



7. $y = \dfrac{1}{e}$

8. $(0, 0)$ and $\left(2, \dfrac{4}{e^2}\right)$

9. If $y = \dfrac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)$, then

$$y' = \frac{5}{2}\left(\frac{1}{5}e^{\frac{x}{5}} - \frac{1}{5}e^{-\frac{x}{5}}\right), \text{ and}$$

$$\begin{aligned} y'' &= \frac{5}{2}\left(\frac{1}{25}e^{\frac{x}{5}} + \frac{1}{25}e^{-\frac{x}{5}}\right) \\ &= \frac{1}{25}\left[\frac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)\right] \\ &= \frac{1}{25}y \end{aligned}$$

10. **a.** $\dfrac{dy}{dx} = -3e^{-3x}$,

$\dfrac{d^2y}{dx^2} = 9e^{-3x}$,

$\dfrac{d^3y}{dx^3} = -27e^{-3x}$

b. $\dfrac{d^n y}{dx^n} = (-1)^n(3^n)e^{-3x}$

11. **a.** $\dfrac{dy}{dx} = -3e^{3x}$,

$\dfrac{d^2y}{dx^2} = -3e^x$

b. $\dfrac{dy}{dx} = e^{2x}(2x + 1)$,

$\dfrac{d^2y}{dx^2} = 4xe^{2x} + 4e^{2x}$

c. $\dfrac{dy}{dx} = e^x(3 - x)$,

$\dfrac{d^2y}{dx^2} = e^x(2 - x)$

12. **a.** 31 000
b. $-\dfrac{100}{3}e^{-\frac{t}{30}}$
c. -17 bacteria/h
d. 31 000 at time $t = 0$
e. The number of bacteria is constantly decreasing as time passes.

13. **a.** $40\left(1 - e^{-\frac{t}{4}}\right)$

b. $a = \dfrac{dv}{dt} = 40\left(\dfrac{1}{4} - e^{-\frac{t}{4}}\right) = 10e^{-\frac{t}{4}}$

From **a,** $v = 40\left(1 - e^{-\frac{t}{4}}\right)$, which gives $e^{\frac{t}{4}} = 1 - \dfrac{v}{40}$. Thus,

$a = 10\left(1 - \dfrac{v}{40}\right) = 10 - \dfrac{1}{4}v$.

c. 40 m/s
d. about 12 s, about 327.3 m

14. **a.** **i.** e
ii. e
b. The limits have the same value because as $x \to \infty, \frac{1}{x} \to 0$.

15. **a.** 1
b. e^2

16. $m = -3$ or $m = 2$

17. **a.** $$\begin{aligned} \frac{d}{dx}(\sinh x) &= \frac{d}{dx}\left[\frac{1}{2}(e^x - e^{-x})\right] \\ &= \frac{1}{2}(e^x + e^{-x}) \\ &= \cosh x \end{aligned}$$

b. $$\begin{aligned} \frac{d}{dx}(\cosh x) &= \frac{1}{2}(e^t - e^{-t}) \\ &= \sinh x \end{aligned}$$

c. Since $\tanh x = \dfrac{\sinh x}{\cosh x}$,

$$\begin{aligned} \frac{d}{dx}(\tanh x) &= \frac{\left(\frac{d}{dx}\sinh x\right)(\cosh x)}{(\cosh x)^2} - \frac{(\sinh x)\left(\frac{d}{dx}\cosh x\right)}{(\cosh x)^2} \\ &= \frac{\frac{1}{2}(e^x + e^{-x})\left(\frac{1}{2}\right)(e^x + e^{-x})}{(\cosh x)^2} - \frac{\frac{1}{2}(e^x - e^{-x})\left(\frac{1}{2}\right)(e^x - e^{-x})}{(\cosh x)^2} \\ &= \frac{\frac{1}{4}[(e^{2x} + 2 + e^{-2x})}{(\cosh x)^2} - \frac{(e^{2t} - 2 + e^{-2t})]}{(\cosh x)^2} \\ &= \frac{\frac{1}{4}(4)}{(\cosh x)^2} \\ &= \frac{1}{(\cosh x)^2} \end{aligned}$$

18. **a.** Four terms: $2.666\ 66\overline{6}$
Five terms: $2.708\ 33\overline{3}$
Six terms: $2.716\ 66\overline{6}$
Seven terms: $2.718\ 05\overline{5}$

b. The expression for e in part a is a special case of $e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ in that it is the case when $x = 1$. Then $e^x = e^1 = e$ is in fact $e^1 = e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$. The value of x is 1.

Section 5.2, p. 240

1. **a.** $3(2^{3x})\ln 2$
b. $\ln 3.1(3.1)^x + 3x^2$
c. $3(10^{3t-5})\ln 10$
d. $(-6 + 2n)(10^{5-6n+n^2})\ln 10$
e. $2x(3^{x^2+2})\ln 3$
f. $400(2)^{x+3}\ln 2$

2. **a.** $5^x[(x^5 \times \ln 5) + 5x^4]$
b. $(3)^{x^2}[(2x^2 \ln 3) + 1]$
c. $-\frac{2^t}{t^2} + \frac{2^t \ln 2}{t}$
d. $\frac{3^{\frac{x}{2}}[x \ln 3 - 4]}{x^3}$

3. $-\frac{3 \ln 10}{4}$

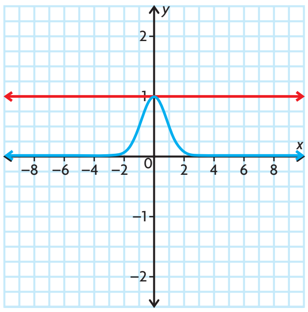
4. $-16.64x + y + 25.92 = 0$

5. $-23.03x + y + 13.03 = 0$

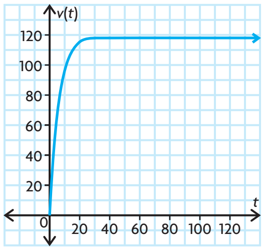
6. **a.** about 3.80 years
b. about -9.12%/year

7. **a.** In 1978, the rate of increase of debt payments was \$904 670 000/annum compared to \$122 250 000/annum in 1968. The rate of increase for 1978 is 7.4 times larger than that for 1968.
b. The rate of increase for 1998 is 7.4 times larger than that for 1988.
c. Answers may vary. For example, data from the past are not necessarily good indicators of what will happen in the future. Interest rates change, borrowing may decrease, principal may be paid off early.

8. $y = 1$



9.



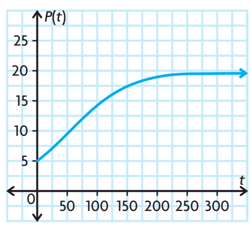
From the graph, the values of $v(t)$ quickly rise in the range of about $0 \le t \le 15$. The slope for these values is positive and steep. Then as the graph nears $t = 20$, the steepness of the slope decreases and seems to get very close to 0. One can reason that the car quickly accelerates for the first 20 units of time. Then, it seems to maintain a constant acceleration for the rest of the time. To verify this, one could differentiate and look at values where $v'(t)$ is increasing.

Section 5.3, pp. 245–247

1. **a.** absolute max: about 0.3849, absolute min: 0
b. absolute max: about 10.043, absolute min: about -5961.916

2. **a.** $f(x)$: max: 0.3849,
min: 0;
$m(x)$: max: about 10,
min: about -5961
b. The graphing approach seems to be easier to use for the functions. It is quicker and it gives the graphs of the functions in a good viewing rectangle. The only problem may come in the second function, $m(x)$, because for $x < 1.5$, the function quickly approaches values in the negative thousands.

3. **a.** 500 squirrels
b. 2000 squirrels
c. (54.9, 10)
d.



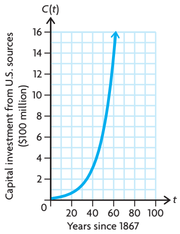
e. P grows exponentially until the point of inflection, then the growth rate decreases and the curve becomes concave down.

4. **a.** 1001 items
b. 500 items

5. 500 units

6. 47.2%; 0.462 h

7. **a.**



b. The growth rate of capital investment grew from 468 million dollars per year in 1947 to 2.112 billion dollars per year in 1967.
c. 7.5%
d. $C = 59.537 \times 10^9$ dollars,
$\frac{dC}{dt} = 4.4849 \times 10^9$ dollars/year
e. Statistics Canada data shows the actual amount of U.S. investment in 1977 was 62.5×10^9 dollars. The error in the model is 3.5%.
f. $C = 570.490 \times 10^9$ dollars,
$\frac{dC}{dt} = 42.975 \times 10^9$ dollars/year

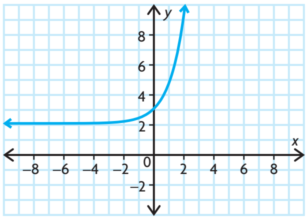
8. **a.** 478 158; 38.2 min after the drug was introduced
b. 42.72 min after the drug was introduced

9. 10 h of study should be assigned to the first exam and 20 h of study for the second exam.

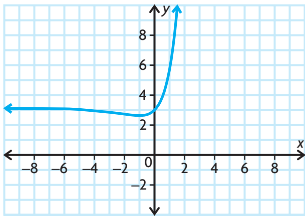
10. Use the algorithm for finding extreme values. First, find the derivate $f'(x)$. Then find any critical points by setting $f'(x) = 0$ and solving for x. Also find the values of x for which $f'(x)$ is undefined. Together these are the critical values. Now evaluate $f(x)$ for the critical values and the endpoints 2 and -2. The highest value will be the absolute maximum on the interval, and the lowest value will be the absolute minimum on the interval.

11. **a.** $f(x)$ is increasing on the intervals $(-\infty, -2)$ and $(0, \infty)$.
Also, $f(x)$ is decreasing on the interval $(-2, 0)$.
b. 0

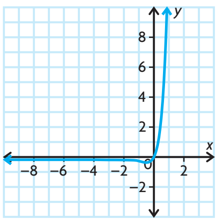
12. **a.** no maximum or minimum value



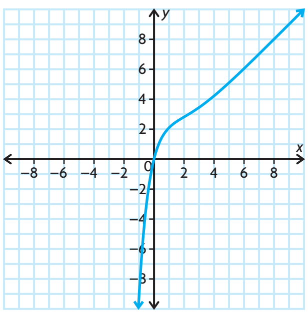
b. min: $-e^{-1} + 3 \doteq 2.63$, no max



c. min: $-e^{-1} \doteq -0.37$, no max

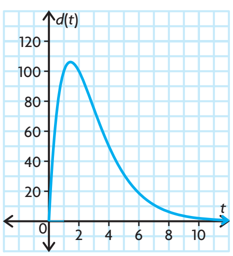


d. no max or min



13. about 0.61

14. **a.**



b. increasing when $0 < t < \frac{1}{\ln 2}$ and decreasing when $t > \frac{1}{\ln 2}$

c. about 106.15°/s

d. $t > 10$ s

15. The solution starts in a similar way to that of question 9. The effectiveness function is

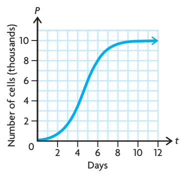
$$E(t) = 0.5\left(10 + te^{-\frac{t}{10}}\right) + 0.6\left(9 + (25 - t)e^{-\frac{25-t}{20}}\right)$$

The derivative simplifies to

$$E'(t) = 0.05e^{-\frac{t}{10}}(10 - t) + 0.03e^{-\frac{25-t}{10}}(5 - t)$$

This expression is very difficult to solve analytically. By calculation on a graphing calculator, we can determine that the maximum effectiveness occurs when $t = 8.16$ h.

16. **a.**



b. after 4.6 days, 5012

c. The rate of growth is slowing down as the colony is getting closer to its limiting value.

Mid-Chapter Review, pp. 248–249

1. **a.** $-15e^{-3x}$

b. $e^{\frac{1}{7}x}$

c. $e^{-2x}(-2x^3 + 3x^2)$

d. $(e^x)(x^2 - 1)$

e. $2(x + xe^{-x} - e^{-x} - e^{-2x})$

f. $\dfrac{4}{(e^x + e^{-x})^2}$

2. **a.** $-500e^{-5t}$

b. -2.5

3. $x + y - 2 = 0$

4. **a.** $y' = -3e^x$,

$y'' = -3e^x$

b. $y' = 2xe^{2x} + e^{2x}$,

$y'' = 4xe^{2x} + 4e^{2x}$

c. $y' = 3e^x - xe^x$,

$y'' = 2e^x - xe^x$

5. **a.** $2(\ln 8)(8^{2x+5})$

b. $0.64(\ln 10)((10)^{2x})$

c. $2^x((\ln 2)(x^2) + 2x)$

d. $900(\ln 5)(5)^{3x-1}$

e. $(\ln 1.9)(1.9)^x + 1.9x^{0.9}$

f. $4^x((\ln 4)(x - 2)^2 + 2x - 4)$

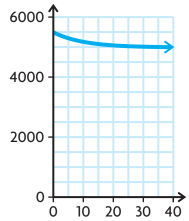
6. **a.** 5500

b. $-50\left(e^{-\frac{t}{10}}\right)$

c. decreasing by about 15 rabbits/month

d. 5500

e.



The graph is constantly decreasing. The y-intercept is (0, 5500). Rabbit populations normally grow exponentially, but this population is shrinking exponentially. Perhaps a large number of rabbit predators, such as snakes, recently began to appear in the forest. A large number of predators would quickly shrink the rabbit population.

7. at about 0.41 h

8. The original function represents growth when $ck > 0$, meaning that c and k must have the same sign. The original function represents decay when c and k have opposite signs.

9. **a.** 5000

b. 5751

c. 9111

10. **a.** 406.80 mm Hg

b. 316.82 mm Hg

c. 246.74 mm Hg

11. 15% per year

12. $f(x) = xe^x$

$f'(x) = xe^x + (1)e^x$

$= e^x(x + 1)$

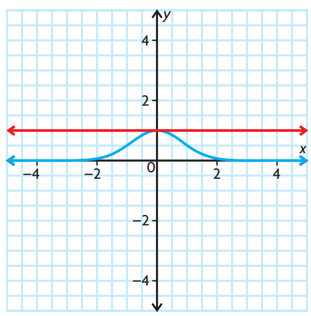
So, $e^x > 0$

$x + 1 > 0$

$x > -1$

This means that the function is increasing when $x > -1$.

13.



14. **a.** $A(t) = 1000(1.06)^t$

b. $A'(t) = 1000(1.06)^t \ln 1.06$

c. $A'(2) = \$65.47$,

$A'(5) = \$77.98$,

$A'(10) = \$104.35$

d. No

e. $\frac{A'(2)}{A(2)} = \ln 1.06,$

$\frac{A'(5)}{A(5)} = \ln 1.06,$

$\frac{A'(10)}{A(10)} = \ln 1.06$

f. All the ratios are equivalent (they equal ln 1.06, which is about 0.058 27), which means that $\frac{A'(t)}{A(t)}$ is constant.

15. $y = ce^x$

$y' = c(e^x) + (0)(e^x)$

$= ce^x$

$y = y' = ce^x$

Section 5.4, pp. 256–257

1. a. $2\cos 2x$

b. $-6\sin 3x$

c. $(3x^2 - 2)(\cos(x^3 - 2x + 4))$

d. $8\sin(-4x)$

e. $3\cos(3x) + 4\sin(4x)$

f. $2^x(\ln 2) + 2\cos x + 2\sin x$

g. $e^x\cos(e^x)$

h. $9\cos(3x + 2\pi)$

i. $2x - \sin x$

j. $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

2. a. $2\cos(2x)$

b. $\frac{2\sin 2x}{x} - \frac{\cos 2x}{x^2}$

c. $-\sin(\sin 2x) \times 2\cos 2x$

d. $\frac{1}{1 + \cos x}$

e. $e^x(2\cos x)$

f. $2x^3\cos x + 6x^2\sin x + 3x\sin x - 3\cos x$

3. a. $-x + 2y + \left(\frac{\pi}{3} - \sqrt{3}\right) = 0$

b. $-2x + y = 0$

c. $y = -1$

d. $y = -3\left(x - \frac{\pi}{2}\right)$

e. $y + \frac{\sqrt{3}}{2} = -\left(x - \frac{\pi}{4}\right)$

f. $2x + y - \pi = 0$

4. a. One could easily find $f'(x)$ and $g'(x)$ to see that they both equal $2(\sin x)(\cos x)$. However, it is easier to notice a fundamental trigonometric identity. It is known that $\sin^2 x + \cos^2 x = 1$. So, $\sin^2 x = 1 - \cos^2 x$. Therefore, $f(x)$ is in fact equal to $g(x)$. So, because $f(x) = g(x)$, $f'(x) = g'(x)$.

b. $f'(x)$ and $g'(x)$ are each others' negative. That is, $f'(x) = (\sin x)(\cos x)$, while $g'(x) = -2(\sin x)(\cos x)$.

5. a. $v'(t) = \frac{\sin(\sqrt{t})\cos(\sqrt{t})}{\sqrt{t}}$

b. $v'(t) = \frac{-\sin t + 2(\sin t)(\cos t)}{2\sqrt{1 + \cos t + \sin^2 t}}$

c. $h'(x) = 3\sin x\sin 2x\cos 3x + 2\sin x\sin 3x\cos 2x + \sin 2x\sin 3x\cos x$

d. $m'(x) = 3(x^2 + \cos^2 x)^2 \times (2x - 2\sin x\cos x)$

6. a. absolute max: $\sqrt{2}$, absolute min: $-\sqrt{2}$

b. absolute max: 2.26, absolute min: −5.14

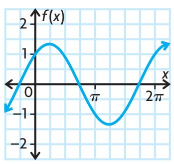
c. absolute max: $\sqrt{2}$, absolute min: $-\sqrt{2}$

d. absolute max: 5, absolute min: −5

7. a. $t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k$ for positive integers k

b. 8

8. a.



b. $\left(\frac{\pi}{4}, \sqrt{2}\right)$

9. $(\csc x)' = -\csc x\cot x,$

$(\sec x)' = \sec x\tan x$

10. $-\sqrt{3}$

11. a. $t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k$ for positive integers k

b. 4

c. minimum: 0 maximum: 4

12. $\theta = \frac{\pi}{3}$

13. $\theta = \frac{\pi}{6}$

14. First find y''.

$y = A\cos kt + B\sin kt$

$y' = -kA\sin kt + kB\cos kt$

$y'' = -k^2A\cos kt - k^2B\sin kt$

So, $y'' + k^2y$

$= -k^2A\cos kt - k^2B\sin kt + k^2(A\cos kt + B\sin kt)$

$= -k^2A\cos kt - k^2B\sin kt + k^2A\cos kt + k^2B\sin kt$

$= 0$

Therefore, $y'' + k^2y = 0$.

Section 5.5, p. 260

1. a. $3\sec^2 3x$

b. $2\sec^2 x - 2\sec 2x$

c. $6x^2\tan(x^2)\sec^2(x^3)$

d. $\frac{x(2\tan \pi x - \pi x\sec^2 \pi x)}{\tan^2 \pi x}$

e. $2x\sec^2(x^2) - 2\tan x\sec^2 x$

f. $15(\tan 5x\cos 5x + \sin 5x\sec^2 5x)$

2. a. $y = 2\left(x - \frac{\pi}{4}\right)$

b. $y = -2x$

3. a. $\cos x\sec^2(\sin x)$

b. $-4x[\tan(x^2 - 1)]^{-3}\sec^2(x^2 - 1)$

c. $-2\tan(\cos x)\sec^2(\cos x)\sin x$

d. $2(\tan x + \cos x)(\sec^2 x - \sin x)$

e. $\sin^2 x(3\tan x\cos x + \sin x\sec^2 x)$

f. $\frac{1}{2\sqrt{x}}e^{\tan\sqrt{x}}\sec^2 2\sqrt{x}$

4. a. $\cos x + \sec x + \frac{2\sin^2 x}{\cos^3 x}$

b. $2\sec^2 x(1 + 3\tan^2 x)$

5. $x = 0, \pi$, and 2π

6. $\left(\frac{\pi}{4}, 0.57\right)$

7. $y = \sec x + \tan x$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{1 + \sin x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos^2 x - (1 + \sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x - (-\sin x - \sin^2 x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$

$= \frac{1 + \sin x}{\cos^2 x}$

The denominator is never negative. $1 + \sin x > 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, since $\sin x$ reaches its minimum of -1 at $x = \frac{\pi}{2}$. Since the derivative of the original function is always positive in the specified interval, the function is always increasing in that interval.

8. $-4x + y - (2 - \pi) = 0$

9. Write $\tan x = \frac{\sin x}{\cos x}$ and use the quotient rule to derive the derivative of the tangent function.

10. $-\csc^2 x$

11. $f''(x) = 8\csc^2 x\cot x$

Review Exercise, pp. 263–265

1. a. $-e^x$
   b. $2 + 3e^x$
   c. $2e^{2x+3}$
   d. $(-6x + 5)e^{-3x^2+5x}$
   e. $e^x(x + 1)$
   f. $\dfrac{2e^t}{(e^t + 1)^2}$
2. a. $10^x \ln 10$
   b. $6x(4^{3x^2})\ln 4$
   c. $5 \times 5^x(x \ln 5 + 1)$
   d. $x^3 \times 2^x(x \ln 2 + 4)$
   e. $\dfrac{4 - 4x \ln 4}{4^x}$
   f. $5^{\sqrt{x}}\left(-\dfrac{1}{x^2} + \dfrac{\ln 5}{2x\sqrt{x}}\right)$
3. a. $6 \cos(2x) + 8 \sin(2x)$
   b. $3 \sec^2(3x)$
   c. $-\dfrac{\sin x}{(2 - \cos x)^2}$
   d. $2x \sec^2(2x) + \tan 2x$
   e. $e^{3x}(3 \sin 2x + 2 \cos 2x)$
   f. $-4 \cos(2x)\sin(2x)$
4. a. $x = 1$
   b. The function has a horizontal tangent at $(1, e)$. So this point could be possible local max or min.
5. a. 0
   b. The slope of the tangent to $f(x)$ at the point with x-coordinate $\frac{1}{2}$ is 0.
6. a. $e^x(x + 1)$
   b. $20e^{10x}(5x + 1)$
7. $y = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

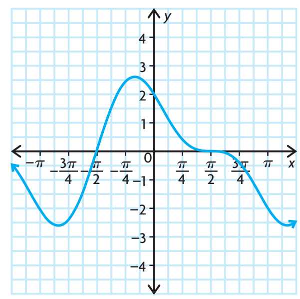
$$\frac{dy}{dx} = \frac{2e^{2x}(e^{2x} + 1) - (e^{2x} - 1)(2e^{2x})}{(e^{2x} + 1)^2}$$
$$= \frac{2e^{4x} + 2e^{2x} - 2e^{4x} + 2e^{2x}}{(e^{2x} + 1)^2}$$
$$= \frac{4e^{2x}}{(e^{2x} + 1)^2}$$

Now, $1 - y^2 = 1 - \dfrac{e^{4x} - 2e^{2x} + 1}{(e^{2x} + 1)^2}$
$$= \frac{e^{4x} + 2e^{2x} + 1 - e^{4x} + 2e^{2x} - 1}{(e^{2x} + 1)^2}$$
$$= \frac{4e^{2x}}{(3^{2x} + 1)^2} = \frac{dy}{dx}$$

8. $3x - y + 2 \ln 2 - 2 = 0$
9. $-x + y = 0$
10. about 0.3928 m per unit of time
11. a. $t = 20$
    b. After 10 days, about 0.1156 mice are infected per day. Essentially, almost 0 mice are infected per day when $t = 10$.
12. a. c_2
    b. c_1
13. a. $-9e^{-x}(2 + 3e^{-x})^2$
    b. ex^{e-1}
    c. e^{x+e^x}
    d. $-25e^{5x}(1 - e^{5x})^4$
14. a. $5^x \ln 5$
    b. $(0.47)^x \ln(0.47)$
    c. $2(52)^{2x} \ln 52$
    d. $5(2)^x \ln 2$
    e. $4e^x$
    f. $-6(10)^{3x} \ln 10$
15. a. $2^x \ln 2 \cos 2^x$
    b. $x^2 \cos x + 2x \sin x$
    c. $-\cos\left(\dfrac{\pi}{2} - x\right)$
    d. $\cos^2 x - \sin^2 x$
    e. $-2 \cos x \sin x$
    f. $2 \sin x \cos^2 x - \sin^3 x$
16. $x + y - \dfrac{\pi}{2} = 0$
17. $v = \frac{ds}{dt}$;
    Thus, $v = 8(\cos(10\pi t))(10\pi)$
    $= 80\pi \cos(10\pi t)$
    The acceleration at any time t is
    $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.
    Hence, $a = 80\pi(-\sin(10\pi t))(10\pi)$
    $= -800\pi^2 \sin(10\pi t)$. Now,
    $\frac{d^2s}{dt^2} + 100\pi^2 s = -800\pi^2 \sin(10\pi t)$
    $+ 100\pi^2(8 \sin(10\pi t)) = 0$.
18. displacement: 5,
    velocity: 10,
    acceleration: 20
19. each angle $\frac{\pi}{4}$ rad, or 45°
20. 4.5 m
21. 2.5 m
22. 5.19 ft
23. a. $f''(x) = -8 \sin^2(x - 2) + 8 \cos^2(x - 2)$
    b. $f''(x) = (4 \cos x)(\sec^2 x \tan x) - 2 \sin x(\sec x)^2$

Chapter 5 Test, p. 266

1. a. $-4xe^{-2x^2}$
   b. $3e^{x^2+3x} \cdot \ln 3 \cdot (2x + 3)$
   c. $\frac{3}{2}[e^{3x} - e^{-3x}]$
   d. $2 \cos x + 15 \sin 5x$
   e. $6x \sin^2(x^2)\cos(x^2)$
   f. $-\dfrac{\sec^2\sqrt{1 - x}}{2\sqrt{1 - x}}$
2. $-6x + y = 2$,
   The tangent line is the given line.
3. $-2x + y = 1$
4. a. $a(t) = v'(t) = -10ke^{-kt}$
   $= -k(10e^{-kt})$
   $= -kv(t)$
   Thus, the acceleration is a constant multiple of the velocity. As the velocity of the particle decreases, the acceleration increases by a factor of k.
   b. 10 cm/s
   c. $\dfrac{\ln 2}{k}$; $-5k$
5. a. $f''(x) = 2(\sin^2 x - \cos^2 x)$
   b. $f''(x) = \csc x \cot^2 x + \csc^3 x + \sin x$
6. absolute max: 1,
   absolute min: 0
7. 40.24
8. minimum: $\left(-4, -\frac{1}{e^4}\right)$, no maximum
9. a. $x = -\dfrac{\pi}{6}, -\dfrac{5\pi}{6}, \dfrac{\pi}{2}$
   b. increasing: $-\frac{5\pi}{6} < x < -\frac{\pi}{6}$;
   decreasing: $-\pi \le x < -\frac{5\pi}{6}$ and $-\frac{\pi}{6} < x < \pi$
   c. local maximum at $x = -\frac{\pi}{6}$; local minimum at $x = -\frac{5\pi}{6}$
   d.



Cumulative Review of Calculus, pp. 267–270

1. a. 16
   b. −2
   c. $\frac{1}{6}$
   d. $160 \ln 2$
2. a. 13 m/s
   b. 15 m/s
3. $f(x) = x^3$
4. a. 19.6 m/s
   b. 19.6 m/s
   c. 53.655 m/s

5. **a.** 19 000 fish/year
b. 23 000 fish/year

6. **a.** **i.** 3
ii. 1
iii. 3
iv. 2

b. No, $\lim_{x \to 4} f(x)$ does not exist. In order for the limit to exist, $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$ must exist and they must be the same. In this case, $\lim_{x \to 4^-} f(x) = \infty$, but $\lim_{x \to 4^+} f(x) = -\infty$, so $\lim_{x \to 4} f(x)$ does not exist.

7. $f(x)$ is discontinuous at $x = 2$. $\lim_{x \to 2^-} f(x) = 5$, but $\lim_{x \to 2^+} f(x) = 3$.

8. **a.** $-\frac{1}{5}$ **d.** $\frac{4}{3}$
b. 6 **e.** $\frac{1}{12}$
c. $-\frac{1}{9}$ **f.** $\frac{1}{2}$

9. **a.** $6x + 1$
b. $-\frac{1}{x^2}$

10. **a.** $3x^2 - 8x + 5$
b. $\frac{3x^2}{\sqrt{2x^3 + 1}}$
c. $\frac{6}{(x + 3)^2}$
d. $4x(x^2 + 3)(4x^5 + 5x + 1) + (x^2 + 3)^2(20x^4 + 5)$
e. $\frac{(4x^2 + 1)^4(84x^2 - 80x - 9)}{(3x - 2)^4}$
f. $5[x^2 + (2x + 1)^3]^4 \times [2x + 6(2x + 1)^2]$

11. $4x + 3y - 10 = 0$

12. 3

13. **a.** $p'(t) = 4t + 6$
b. 46 people per year
c. 2006

14. **a.** $f'(x) = 5x^4 - 15x^2 + 1$; $f''(x) = 20x^3 - 30x$
b. $f'(x) = \frac{4}{x^3}$; $f''(x) = -\frac{12}{x^4}$
c. $f'(x) = -\frac{2}{\sqrt{x^3}}$; $f''(x) = \frac{3}{\sqrt{x^5}}$
d. $f'(x) = 4x^3 + \frac{4}{x^5}$; $f''(x) = 12x^2 - \frac{20}{x^6}$

15. **a.** maximum: 82, minimum: 6
b. maximum: $9\frac{1}{3}$, minimum: 2
c. maximum: $\frac{e^4}{1 + e^4}$, minimum: $\frac{1}{2}$
d. maximum: 5, minimum: 1

16. **a.** $v(t) = 9t^2 - 81t + 162$, $a(t) = 18t - 81$
b. stationary when $t = 6$ or $t = 3$, advancing when $v(t) > 0$, and retreating when $v(t) < 0$
c. $t = 4.5$
d. $0 \le t < 4.5$
e. $4.5 < t \le 8$

17. 14 062.5 m^2

18. $r \doteq 4.3$ cm, $h \doteq 8.6$ cm

19. $r = 6.8$ cm, $h = 27.5$ cm

20. **a.** $140 - 2x$
b. 101 629.5 cm^3; 46.7 cm by 46.7 cm by 46.6 cm

21. $x = 4$

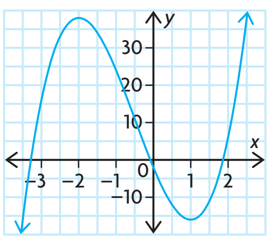
22. \$70 or \$80

23. \$1140

24. **a.** $\frac{dy}{dx} = -10x + 20$,
$x = 2$ is critical number,
Increase: $x < 2$,
Decrease: $x > 2$
b. $\frac{dy}{dx} = 12x + 16$,
$x = -\frac{4}{3}$ is critical number,
Increase: $x > -\frac{4}{3}$,
Decrease: $x < -\frac{4}{3}$
c. $\frac{dy}{dx} = 6x^2 - 24$,
$x = \pm 2$ are critical numbers,
Increase: $x < -2$, $x > 2$,
Decrease: $-2 < x < 2$
d. $\frac{dy}{dx} = -\frac{2}{(x - 2)^2}$. The function has no critical numbers. The function is decreasing everywhere it is defined, that is, $x \neq 2$.

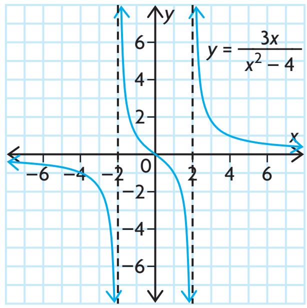
25. **a.** $y = 0$ is a horizontal asymptote.
$x = \pm 3$ are the vertical asymptotes.
There is no oblique asymptote.
$\left(0, -\frac{8}{9}\right)$ is a local maximum.
b. There are no horizontal asymptotes.
$x = \pm 1$ are the vertical asymptotes.
$y = 4x$ is an oblique asymptote.
$(-\sqrt{3}, -6\sqrt{3})$ is a local maximum, $(\sqrt{3}, 6\sqrt{3})$ is a local minimum.

26. **a.**



$y = 4x^3 + 6x^2 - 24x - 2$

b.



27. **a.** $(-20)e^{5x + 1}$
b. $e^{3x}(3x + 1)$
c. $(3 \ln 6)6^{3x - 8}$
d. $(\cos x)e^{\sin x}$

28. $y = 2e(x - 1) + e$

29. **a.** 5 days
b. 27

30. **a.** $2 \cos x + 15 \sin 5x$
b. $8 \cos 2x(\sin 2x + 1)^3$
c. $\frac{2x + 3 \cos 3x}{2\sqrt{x^2 + \sin 3x}}$
d. $\frac{1 + 2 \cos x}{(\cos x + 2)^2}$
e. $2x \sec^2 x^2 - 2 \tan x \sec^2 x$
f. $-2x \sin x^2 \cos(\cos x^2)$

31. about 4.8 m

32. about 8.5 m

Chapter 6

Review of Prerequisite Skills, p. 273

1. **a.** $\frac{\sqrt{3}}{2}$ **d.** $\frac{\sqrt{3}}{2}$
b. $-\sqrt{3}$ **e.** $\frac{\sqrt{2}}{2}$
c. $\frac{1}{2}$ **f.** 1

2. $\frac{4}{3}$

3. **a.** $AB \doteq 29.7$, $\angle B \doteq 36.5°$, $\angle C \doteq 53.5°$
b. $\angle A \doteq 97.9°$, $\angle B \doteq 29.7°$, $\angle C \doteq 52.4°$

4. $\angle Z \doteq 50°$, $XZ \doteq 7.36$, $YZ \doteq 6.78$

5. $\angle R \doteq 44°$, $\angle S \doteq 102°$, $\angle T \doteq 34°$

6. 5.82 km

7. 8.66 km

8. 21.1 km

9. 59.4 cm^2