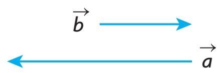
24.



25. **a.** $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$

b. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$

c. $\sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2}$

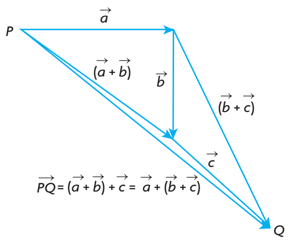
26. **Case 1** If $\vec{b}$ and $\vec{c}$ are collinear, then $2\vec{b} + 4\vec{c}$ is also collinear with both $\vec{b}$ and $\vec{c}$. But $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, so $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Case 2 If $\vec{b}$ and $\vec{c}$ are not collinear, then by spanning sets, $\vec{b}$ and $\vec{c}$ span a plane in R^3, and $2\vec{b} + 4\vec{c}$ is in that plane. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then it is perpendicular to the plane and all vectors in the plane. So, $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Chapter 6 Test, p. 348

1. Let P be the tail of $\vec{a}$ and let Q be the head of $\vec{c}$. The vector sums $[\vec{a} + (\vec{b} + \vec{c})]$ and $[(\vec{a} + \vec{b}) + \vec{c}]$ can be depicted as in the diagram below, using the triangle law of addition.

We see that $\vec{PQ} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$. This is the associative property for vector addition.



2. **a.** (8, 4, 8)

b. 12

c. $\left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

3. $\sqrt{19}$

4. **a.** $\vec{x} = 2\vec{b} - 3\vec{a}, \vec{y} = 3\vec{b} - 5\vec{a}$

b. $a = 1, b = 5, c = -11$

5. **a.** $\vec{a}$ and $\vec{b}$ span R^2, because any vector (x, y) in R^2 can be written as a linear combination of $\vec{a}$ and $\vec{b}$. These two vectors are not multiples of each other.

b. $p = -2, q = 3$

6. **a.** $(1, 12, -29) = -2(3, 1, 4) + 7(1, 2, -3)$

b. $\vec{r}$ cannot be written as a linear combination of $\vec{p}$ and $\vec{q}$. In other words, $\vec{r}$ does not lie in the plane determined by $\vec{p}$ and $\vec{q}$.

7. $\sqrt{13}, \theta \doteq 3.61$; 73.9° relative to x

8. $\vec{DE} = \vec{CE} - \vec{CD}$

$\vec{DE} = \vec{b} - \vec{a}$

Also,

$\vec{BA} = \vec{CA} - \vec{CB}$

$\vec{BA} = 2\vec{b} - 2\vec{a}$

Thus,

$\vec{DE} = \frac{1}{2}\vec{BA}$

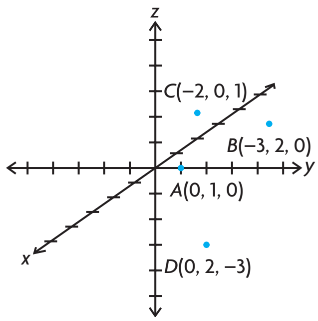
Chapter 7

Review of Prerequisite Skills, p. 350

1. $v \doteq 806$ km/h N 7.1° E

2. 15.93 units W 32.2° N

3.



4. **a.** $(3, -2, 7); l \doteq 7.87$

b. $(-9, 3, 14); l \doteq 16.91$

c. $(1, 1, 0); l \doteq 1.41$

d. $(2, 0, -9); l \doteq 9.22$

5. **a.** $(x, y, 0)$

b. $(x, 0, z)$

c. $(0, y, z)$

6. **a.** $\vec{i} - 7\vec{j}$

b. $6\vec{i} - 2\vec{j}$

c. $-8\vec{i} + 11\vec{j} + 3\vec{k}$

d. $4\vec{i} - 6\vec{j} + 8\vec{k}$

7. **a.** $\vec{i} + 3\vec{j} - \vec{k}$

b. $5\vec{i} + \vec{j} - \vec{k}$

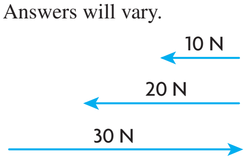
c. $12\vec{i} + \vec{j} - 2\vec{k}$

Section 7.1, pp. 362–364

1. **a.** 10 N is a melon, 50 N is a chair, 100 N is a computer

b. Answers will vary.

2. **a.**



b. 180°

3. a line along the same direction

4. For three forces to be in equilibrium, they must form a triangle, which is a planar figure.

5. **a.** The resultant is 13 N at an angle of N 22.6° W. The equilibrant is 13 N at an angle of S 22.6° W.

b. The resultant is 15 N at an angle of S 36.9° W. The equilibrant is 15 N at N 36.9° E.

6. **a.** yes **b.** yes **c.** no **d.** yes

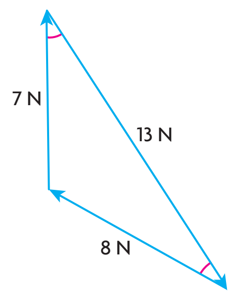
7. Arms 90 cm apart will yield a resultant with a smaller magnitude than at 30 cm apart. A resultant with a smaller magnitude means less force to counter your weight, hence a harder chin-up.

8. The resultant would be 12.17 N at 34.7° from the 6 N force toward the 8 N force. The equilibrant would be 12.17 N at 145.3° from the 6 N force away from the 8 N force.

9. 9.66 N 15° from given force, 2.95 N perpendicular to 9.66 N force

10. 49 N directed up the ramp

11. **a.**



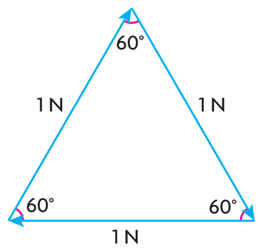
b. 60°

12. approximately 7.1 N 45° south of east

13. **a.** 7

b. The angle between f_1 and the resultant is 16.3°. The angle between $\vec{f_1}$ and the equilibrant is 163.7°.

14. **a.**



For these three equal forces to be in equilibrium, they must form an equilateral triangle. Since the resultant will lie along one of these lines, and since all angles of an equilateral triangle are 60°, the resultant will be at a 60° angle with the other two vectors.

b. Since the equilibrant is directed opposite the resultant, the angle between the equilibrant and the other two vectors is $180° - 60° = 120°$.

15. 7.65 N, 67.5° from $\vec{f_2}$ toward $\vec{f_3}$

16. 45° rope: 175.73 N
30° rope: 143.48

17. 24 cm string: approximately 39.2 N,
32 cm string: approximately 29.4 N

18. 8.5° to the starboard side

19. **a.** magnitude for resultant and equilbrant $\doteq 13.75$ N
b. $\theta_{5N} \doteq 111.3°$, $\theta_{8N} \doteq 125.6°$, $\theta_{10N} \doteq 136.7°$

20. We know that the resultant of these two forces is equal in magnitude and angle to the diagonal line of the parallelogram formed with $\vec{f_1}$ and $\vec{f_2}$ as legs and has diagonal length $|\vec{f_1} + \vec{f_2}|$. We also know from the cosine rule that $|\vec{f_1} + \vec{f_2}|^2$

$$= |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi,$$

where ϕ is the supplement to θ in, our parallelogram. Since we know $\phi = 180 - \theta$, then $\cos\phi = \cos(180 - \theta) = -\cos\theta$. Thus, we have

$$|\vec{f_1} + \vec{f_2}|^2 = |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi$$

$$|\vec{f_1} + \vec{f_2}|^2 = |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi$$

$$|\vec{f_1} + \vec{f_2}| = \sqrt{|\vec{f_1}|^2 + |\vec{f_2}|^2 + 2|\vec{f_1}||\vec{f_2}|\cos\theta}$$

Section 7.2, pp. 369–370

1. **a.** 84 km/h in the direction of the train's movement
b. 76 km/h in the direction of the train's movement

2. **a.** 500 km/h north
b. 700 km/h north

3. 304.14, W 9.5° S

4. 60° upstream

5. **a.** 2 m/s forward
b. 22 m/s in the direction of the car

6. 13 m/s, N 37.6° W

7. **a.** 732.71 km/h, N 5.5° W
b. about 732.71 km

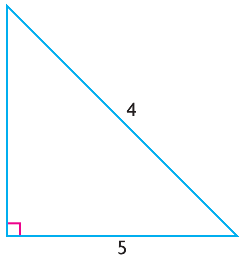
8. **a.** about 1383 km
b. about 12.5° east of north

9. **a.** about 10.4° south of west
b. 2 h, 53.1° downstream to the bank

10. **a.** 5 km/h
b. about 0.67 km
c. 20 min

11. **a.** about 18.4° west of north
b. about 108 km/h

12.



Since her swimming speed is a maximum of 4 km/h, this is her maximum resultant magnitude, which is also the hypotenuse of the triangle formed by her and the river's velocity vector. Since one of these legs is 5 km/h, we have a triangle with a leg larger than its hypotenuse, which is impossible.

13. **a.** about 68 m
b. 100 s

14. **a.** about 58.5°, upstream
b. about 58.6 s

15. 35 h

Section 7.3, pp. 377–378

1. To be guaranteed that the two vectors are perpendicular, the vectors must be nonzero.

2. $\vec{a} \cdot \vec{b}$ is a scalar, and a dot product is only defined for vectors.

3. Answers may vary. For example, let $\vec{a} = \hat{i}, \vec{b} = \hat{j}, \vec{c} = -\vec{i}\ \hat{i} \cdot \vec{a} \cdot \vec{b} \cdot \vec{b} = 0$, $\vec{a} \cdot \vec{c} \cdot \vec{c} = 0$, but $\vec{a} = -\vec{c}$.

4. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = \vec{b} \cdot \vec{c}$ because $\vec{c} = \vec{a}$

5. -1

6. **a.** 16
b. -6.93
c. 0
d. -1
e. 0
f. -26.2

7. **a.** 30°
b. 80°
c. 53°
d. 127°
e. 60°
f. 120°

8. 22.5

9. **a.** $2|\vec{a}|^2 - 15|\vec{b}|^2 + 7\vec{a} \cdot \vec{b}$
b. $6|\vec{x}|^2 - 19\vec{x} \cdot \vec{y} + 3|\vec{y}|^2$

10. 0

11. 1

12. **a.** $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$
$= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$
$= |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + |\vec{b}|^2$
$= |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$

b. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$
$= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$
$= |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2$
$= |\vec{a}|^2 - |\vec{b}|^2$

13. **a.** $|\vec{a}|^2 = \vec{a} \cdot \vec{a}$
$= (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c})$
$= |\vec{b}|^2 + 2\vec{b} \cdot \vec{c} + |\vec{c}|^2$

b. $\vec{b} \cdot \vec{c} = 0$, vectors are perpendicular. Therefore $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$, which is the Pythagorean theory.

14. 14

15. $|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2$
$= (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$
$+ (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v})$
$= |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 + |\vec{u}|^2$
$- 2\vec{u} \cdot \vec{v} + |\vec{v}|^2$
$= 2|\vec{u}|^2 + 2|\vec{v}|^2$

16. 3

17. -7

18. $\vec{d} = \vec{b} - \vec{c}$
$\vec{b} = \vec{d} + \vec{c}$
$\vec{c} \cdot \vec{a} = ((\vec{b} \cdot \vec{a})\vec{a}) \cdot \vec{a}$
$\vec{c} \cdot \vec{a} = (\vec{b} \cdot \vec{a})(\vec{a} \cdot \vec{a})$ because
$\vec{b} \cdot \vec{a}$ is a scalar
$\vec{c} \cdot \vec{a} = (\vec{b} \cdot \vec{a})|\vec{a}|^2$
$\vec{c} \cdot \vec{a} = (\vec{d} + \vec{c}) \cdot \vec{a}$ because $|\vec{a}| = 1$
$\vec{c} \cdot \vec{a} = \vec{d} \cdot \vec{a} + \vec{c} \cdot \vec{a}$
$\vec{d} \cdot \vec{a} = 0$

Section 7.4, pp. 385–387

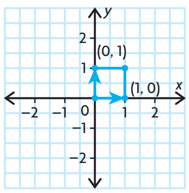
1. Any vector of the form (c, c) is perpendicular to $\vec{a}$. Therefore, there are infinitely many vectors perpendicular to $\vec{a}$. Answers may vary. For example: $(1, 1), (2, 2), (3, 3)$.

2. **a.** 0; 90°
b. $34 > 0$; acute
c. $-3 < 0$; obtuse

3. Answer may vary. For example:
a. $(0, 0, 1)$ is perpendicular to every vector in the xy-plane.
b. $(0, 1, 0)$ is perpendicular to every vector in the xz-plane.
c. $(1, 0, 0)$ is perpendicular to every vector in the yz-plane.

4. **a.** $(1, 2, -1)$ and $(4, 3, 10)$;
$(-4, -5, -6)$ and $\left(5, -3, -\frac{5}{6}\right)$
b. no

5. a. The vectors must be in R^3, which is impossible
b. This is not possible since R^3 does not exist in R^2.
6. a. about 148°
b. about 123°
c. about 64°
d. about 154°
7. a. $k = \frac{2}{3}$
b. $k \geq 0$
8. a.



b. $(1, 1)$ and $(1, -1)$; $(1, 1)$ and $(-1, 1)$
c. $(1, 1) \cdot (1, -1)$
$= 1 - 1$
$= 0$
or
$(1, 1) \cdot (-1, 1)$
$= -1 + 1$
$= 0$
9. a. 90°
b. 30°
10. a. i. $p = \frac{8}{3}$; $q = 3$
ii. Answers may vary. For example, $p = 1, q = -50$.
b. Unique for collinear vectors; not unique for perpendicular vectors
11. $\theta_A = 90°$; $\theta_B \doteq 26.6°$; $\theta_C \doteq 63.4°$
12. a. $O = (0, 0, 0), A = (7, 0, 0),$
$B = (7, 4, 0), C = (0, 4, 0),$
$D = (7, 0, 5), E = (0, 4, 5),$
$F = (0, 0, 5)$
b. 50°
13. a. Answers may vary. For example, $(3, 1, 1)$.
b. Answers may vary. For example, $(1, 1, 1)$.
14. 3 or −1
15. a. $3 + 4p + q = 0$
b. 0
16. Answers may vary. For example, $(1, 0, 1)$ and $(1, 1, 3)$.
$(x, y, z)(1, 2, -1) = 0$
$x + 2y - z = 0$
Let $x = z = 1$.
$(1, 0, 1)$ is perpendicular to $(1, 2, -1)$ and $(-2, -4, 2)$.
Let $x = y = 1$.
$(1, 1, 3)$ is perpendicular to $(1, 2, -1)$ and $(-2, -4, 2)$.

17. 4 or $-\frac{44}{65}$
18. a. $\vec{a} \cdot \vec{b} = 0$
Therefore, since the two diagonals are perpendicular, all the sides must be the same length.
b. $\vec{AB} = (1, 2, -1),$
$\vec{BC} = (2, 1, 1),$
$|\vec{AB}| = |\vec{BC}| = \sqrt{6}$
c. $\theta_1 = 60°$; $\theta_2 = 120°$
19. a. $(6, 18, -4)$
b. 87.4°
20. $\alpha \doteq 109.5°$ or $\theta \doteq 70.5°$

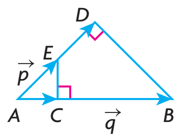
Mid-Chapter Review, pp. 388–389

1. a. 3
b. 81
2. 15 cm cord: 117.60 N;
20 cm cord: 88.20 N
3. 0
4. a. about 575.1 km/h at S 7.06° E
b. about 1.74 h
5. a. about 112.61 N
b. about 94.49 N
6. 4.5
7. a. 34
b. $\frac{34}{63}$
8. a. 0
b. 5
c. $5\vec{i} - 4\vec{j} + 3\vec{k}$
d. 0
e. 34
f. 9
9. a. $x = -3$ or $x = -\frac{1}{3}$
b. no value
10. a. $\vec{i} - 4\vec{j} - \vec{k}$
b. 24
c. $\sqrt{2}$ or 1.41
d. −4
e. −12
11. about 126.9°
12. $\vec{F} \doteq 6.08$ N, 25.3° from the 4 N force towards the 3 N force. $\vec{E} \doteq 6.08$ N, $180° - 25.3° = 154.7°$ from the 4 N force away from the 3 N force.
13. a. about 109.1°
b. about 87.9°
14. a. about N 2.6° E
b. about 2.17 h
15. $\vec{x} = \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ or $\left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

16. a. about 6.12 m
b. about 84.9 s
17. a. when $\vec{x}$ and $\vec{y}$ have the same length
b. Vectors $\vec{a}$ and $\vec{b}$ determine a parallelogram. Their sum $\vec{a} + \vec{b}$ is one diagonal of the parallelogram formed, with its tail in the same location as the tails of $\vec{a}$ and $\vec{b}$. Their difference $\vec{a} - \vec{b}$ is the other diagonal of the parallelogram.
18. about 268.12 N

Section 7.5, pp. 398–400

1. a. scalar projection $= 2$,
vector projection $= 2\vec{i}$
b. scalar projection $= 3$,
vector projection $= 3\vec{j}$
2. Using the formula would cause a division by 0. Generally the $\vec{0}$ has any direction and 0 magnitude. You cannot project onto nothing.
3. You are projecting $\vec{a}$ onto the tail of $\vec{b}$, which is a point with magnitude 0. Therefore, it is $\vec{0}$; the projections of $\vec{b}$ onto the tail of $\vec{a}$ are also 0 and $\vec{0}$.
4. Answers may vary. For example,
$\vec{p} = \vec{AE}, \vec{q} = \vec{AB}$

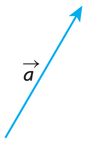


scalar projection $\vec{p}$ on $\vec{q} = |\vec{AC}|$,
vector projection $\vec{p}$ on $\vec{q} = \vec{AC}$,
scalar projection $\vec{q}$ on $\vec{p} = |\vec{AD}|$,
vector projection $\vec{q}$ on $\vec{p} = \vec{AD}$
5. scalar projection of $\vec{a}$ on $\vec{i} = -1$,
vector projection of $\vec{a}$ on $\vec{i} = -\vec{i}$,
scalar projection of $\vec{a}$ on $\vec{j} = 2$,
vector projection of $\vec{a}$ on $\vec{j} = 2\vec{j}$,
scalar projection of $\vec{a}$ on $\vec{k} = -5$,
vector projection of $\vec{a}$ on $\vec{k} = -5\vec{k}$;
Without having to use formulae, a projection of $(-1, 2, 5)$ on $\vec{i}, \vec{j}$, or $\vec{k}$ is the same as a projection of $(-1, 0, 0)$ on $\vec{i}$, $(0, 2, 0)$ on $\vec{j}$, and $(0, 0, 5)$ on $\vec{k}$, which intuitively yields the same result.
6. a. scalar projection: $\frac{\vec{p} \cdot \vec{q}}{|\vec{q}|} = \frac{458}{21}$,
vector projection: $\frac{458}{441}(-4, 5, -20)$
b. about 82.5°, about 74.9°, about 163.0°

7. **a.** scalar projection: 0, vector projection: $\vec{0}$
b. scalar projection: 2, vector projection: $2\vec{i}$
c. scalar projection: $\frac{50}{13}$, vector projection: $\frac{50}{169}(-5, 12)$

8. **a.** The scalar projection of $\vec{a}$ on the x-axis $(X, 0, 0)$ is -1; The vector projection of $\vec{a}$ on the x-axis is $-\vec{i}$; The scalar projection of $\vec{a}$ on the y-axis $(0, Y, 0)$ is 2; The vector projection of $\vec{a}$ on the y-axis is $2\vec{j}$; The scalar projection of $\vec{a}$ on the z-axis $(0, 0, Z)$ is 4; The vector projection of $\vec{a}$ on the z-axis is $4\vec{k}$.
b. The scalar projection of $m\,\vec{a}$ on the x-axis $(X, 0, 0)$ is $-m$; The vector projection of $m\,\vec{a}$ on the x-axis is $-m\vec{i}$; The scalar projection of $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m$; The vector projection $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m\vec{j}$; The scalar projection of $m\,\vec{a}$ on the z-axis $(0, 0, Z)$ is $4m$; The vector projection of $m\,\vec{a}$ on the z-axis is $4m\vec{k}$.

9. **a.**



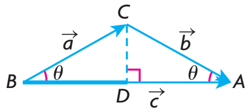
vector projection: $\vec{a}$
scalar projection: $|\vec{a}|$
b. $|\vec{a}|\cos\theta = |\vec{a}|\cos 0$
$= |\vec{a}|(1)$
$= |\vec{a}|.$
The vector projection is the scalar projection multiplied by $\frac{\vec{a}}{|\vec{a}|}$,
$|\vec{a}| \times \frac{\vec{a}}{|\vec{a}|} = \vec{a}.$

10. **a.**

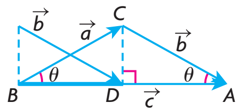
b. $\frac{(-\vec{a})\cdot\vec{a}}{|\vec{a}|} = \frac{-|\vec{a}|^2}{|\vec{a}|}$
$= -|\vec{a}|$
So, the vector projection is
$-|\vec{a}|\left(\frac{\vec{a}}{|\vec{a}|}\right) = -\vec{a}.$

11. **a.** scalar projection of $\overrightarrow{AB}$ on the x-axis is -2; vector projection of $\overrightarrow{AB}$ on the x-axis is $-2\vec{i}$; scalar projection of $\overrightarrow{AB}$ on the y-axis is 1; vector projection of $\overrightarrow{AB}$ on the y-axis is $\vec{j}$; scalar projection of $\overrightarrow{AB}$ on the z-axis is 2; vector projection of $\overrightarrow{AB}$ on the z-axis is $2\vec{k}$.
b. 70.5°

12. **a.** $|\overrightarrow{BD}|$

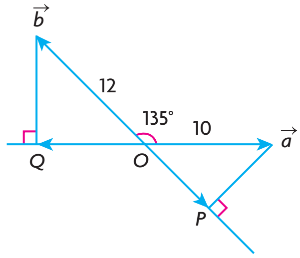


b. $|\overrightarrow{BD}|$



c. In an isosceles triangle, CD is a median and a right bisector of BA.
d. Yes

13. **a.** -7.07, -8.49
b.



$\overrightarrow{OQ}$ is the vector projection of $\vec{b}$ on $\vec{a}$
$\overrightarrow{OP}$ is the vector projection of $\vec{a}$ on $\vec{b}$

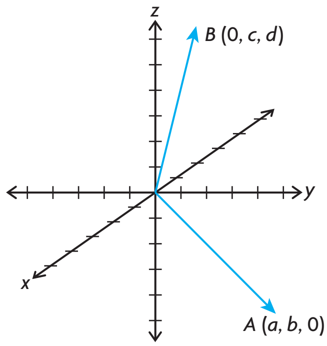
14. **a.** $-\frac{1}{3}$
b. The scalar projection of $\overrightarrow{BC}$ on $\overrightarrow{OD}$ is $\frac{19}{3}\cdot -\frac{1}{3} + \frac{19}{13} = 6$
The scalar projection of $\overrightarrow{AC}$ on $\overrightarrow{OD}$ is 6.
c. Same lengths and both are in the direction of $\overrightarrow{OD}$. Add to get one vector.

15. **a.**
$$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$
$$= \left(\frac{a}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right)^2$$
$$= \frac{a^2}{a^2+b^2+c^2} + \frac{b^2}{a^2+b^2+c^2} + \frac{c^2}{a^2+b^2+c^2}$$
$$= \frac{a^2+b^2+c^2}{a^2+b^2+c^2}$$
$$= 1$$
b. Answers may vary. For example: $\left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $(0, \sqrt{3}, 1)$
c. If two angles add to 90°, then all three will add to 180°.

16. **a.** about 54.7°
b. about 125.3°

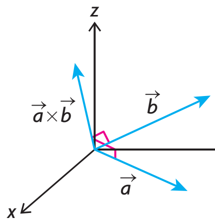
17.
$\cos^2 x + \sin^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$
$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$
$1 = (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma)$
$1 = 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

18. Answers may vary. For example:

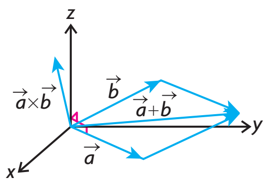


Section 7.6, pp. 407–408

1. **a.**



$\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. Thus, their dot product must equal 0. The same applies to the second case.



b. $\vec{a} + \vec{b}$ is still in the same plane formed by $\vec{a}$ and $\vec{b}$, thus $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} \times \vec{b}$ making the dot product 0 again.
c. Once again, $\vec{a} - \vec{b}$ is still in the same plane formed by $\vec{a}$ and $\vec{b}$, thus $\vec{a} - \vec{b}$ is perpendicular to $\vec{a} \times \vec{b}$ making the dot product 0 again.

2. $\vec{a} \times \vec{b}$ produces a vector, not a scalar. Thus, the equality is meaningless.

3. a. It's possible because there is a vector crossed with a vector, then dotted with another vector, producing a scalar.
b. This is meaningless because $\vec{a} \cdot \vec{b}$ produces a scalar. This results in a scalar crossed with a vector, which is meaningless.
c. This is possible. $\vec{a} \times \vec{b}$ produces a vector, and $\vec{c} + \vec{d}$ also produces a vector. The result is a vector dotted with a vector producing a scalar.
d. This is possible. $\vec{a} \times \vec{b}$ produces a scalar, and $\vec{c} \times \vec{d}$ produces a vector. The product of a scalar and vector produces a vector.
e. This is possible. $\vec{a} \times \vec{b}$ produces a vector, and $\vec{c} \times \vec{d}$ produces a vector. The cross product of a vector and vector produces a vector.
f. This is possible. $\vec{a} \times \vec{b}$ produces a vector. When added to another vector, it produces another vector.

4. a. $(-7, -8, -2)$
b. $(1, 5, 1)$
c. $(-11, -33, 22)$
d. $(-19, -22, 7)$
e. $(3, 3, -1)$
f. $(-8, -26, 11)$

5. 1

6. a. $(-4, 0, 0)$
b. Vectors of the form $(0, b, c)$ are in the yz-plane. Thus, the only vectors perpendicular to the yz-plane are those of the form $(a, 0, 0)$ because they are parallel to the x-axis.

7. a. $(1, 2, 1) \times (2, 4, 2)$
$= (2(2) - 1(4), 1(2) - 1(2), 1(4) - 2(2))$
$= (0, 0, 0)$
b. $(a, b, c) \times (ka, kb, kc)$
$= (b(kc) - c(kb), c(ka) - a(kc), a(kb) - b(ka))$
Using the associative law of multiplication, we can rearrange this:
$= (bck - bck, ack - ack, abk - abk)$
$= (0, 0, 0)$

8. a. $\vec{p} \times (\vec{q} + \vec{r}) = (-26, -7, 3)$
$\vec{p} \times \vec{q} + \vec{p} \times \vec{r} = (-26, -7, 3)$
b. $\vec{p} \times (\vec{q} + \vec{r}) = (-3, 2, 5)$
$\vec{p} \times \vec{q} + \vec{p} \times \vec{r} = (-3, 2, 5)$

9. a. $\vec{i} \times \vec{j} = (1, 0, 0) \times (0, 1, 0) = \vec{k}$
$-\vec{j} \times \vec{i} = (0, -1, 0) \times (1, 0, 0) = \vec{k}$
b. $\vec{j} \times \vec{k} = (0, 1, 0) \times (0, 0, 1) = \vec{i}$
$-\vec{k} \times \vec{j} = (0, 0, -1) \times (0, 1, 0) = \vec{i}$
c. $\vec{k} \times \vec{i} = (0, 0, 1) \times (1, 0, 0) = \vec{j}$
$-\vec{i} \times \vec{k} = (-1, 0, 0) \times (0, 0, 1) = \vec{j}$

10. $k(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)(a_1, a_2, a_3)$
$= k(a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_2a_1b_3 + a_3a_1b_2 - a_3a_2b_1)$
$= k(0)$
$= 0$
$\vec{a}$ is perpendicular to $k(\vec{a} \times \vec{b})$.

11. a. $(0, 0, 6), (0, 0, -6)$
b. $(0, 0, 0)$
c. All the vectors are in the xy-plane. Thus, their cross product in part b. is between vectors parallel to the z-axis and so parallel to each other. The cross product of parallel vectors is $\vec{0}$.

12. Let $\vec{x} = (1, 0, 1), \vec{y} = (1, 1, 1)$, and $\vec{z} = (1, 2, 3)$
Then, $\vec{x} \times \vec{y} = (0 - 1, 1 - 1, 1 - 0)$
$= (-1, 0, 1)$
$(\vec{x} \times \vec{y}) \times \vec{z}$
$= (0 - 2, 1 - (-3), -3 - 0)$
$= (-2, 4, -3)$
$\vec{y} \times \vec{z} = (3 - 2, 1 - 3, 2 - 1)$
$= (1, -2, 1)$
$\vec{x} \times (\vec{y} \times \vec{z}) = (0 + 2, 1 - 1, -2 - 0)$
$= (2, 0, -2)$
Thus, $(\vec{x} \times \vec{y}) \times \vec{z} \neq \vec{x} \times (\vec{y} \times \vec{z})$.

13. By the distributive property of cross product:
$= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$
By the distributive property again:
$= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$
A vector crossed with itself equals $\rightarrow 0$, thus:
$= -\vec{b} \times \vec{a} + \vec{a} \times \vec{b}$
$= \vec{a} \times \vec{b} - \vec{b} \times \vec{a}$
$= \vec{a} \times \vec{b} - (-\vec{a} \times \vec{b})$
$= 2\vec{a} \times \vec{b}$

Section 7.7, pp. 414–415

1. By pushing as far away from the hinge as possible, $|\vec{r}|$ is increased, making the cross product bigger. By pushing at right angles, sine is its largest value, 1, making the cross product larger.

2. a. 0
b. This makes sense because the vectors lie on the same line. Thus, the parallelogram would just be a line making its area 0.

3. a. 450 J
b. about 10 078.91 J
c. about 32 889.24 J
d. 35 355.34 J

4. a. $\vec{k}$
b. $-\vec{k}$
c. $-\vec{j}$
d. $\vec{j}$

5. a. $\sqrt{3}$ square units
b. $\sqrt{213}$ square units

6. $2, \frac{-12}{5}$

7. a. $\frac{5\sqrt{6}}{2}$ square units
b. $\frac{5\sqrt{6}}{2}$ square units.
c. Any two sides of a triangle can be used to calculate its area.

8. about 0.99 J

9. $\frac{6}{\sqrt{26}}$ or about 1.18

10. a. $\vec{p} \times \vec{q} = (-6 - 3, 6 - 3, 1 + 4)$
$= (-9, 3, 5)$
$(\vec{p} \times \vec{q}) \times \vec{r}$
$= (0 - 5, 5 + 0, -9 - 3)$
$= (-5, 5, -12)$
$a(1, -2, 3) + b(2, 1, 3)$
$= (-5, 5, -12)$
Looking at x-components:
$a + 2b = -5$
$a = -5 - 2b$
y-components:
$-2a + b = 5$
Substitute a:
$10 + 4b + b = 5$
$5b = -5$
$b = -1$
Substitute b back into the x-components:
$a = -5 + 2$
$a = -3$
Check in z-components:
$3a + 3b = -12$
$-9 - 3 = -12$
b. $\vec{p} \cdot \vec{r} = 1 - 2 + 0 = -1$
$\vec{q} \cdot \vec{r} = 2 + 1 + 0 = 3$
$(\vec{p} \cdot \vec{r})\vec{q} - (\vec{q} \cdot \vec{r})\vec{p}$
$= -1(2, 1, 3) - 3(1, -2, 3)$
$= (-2, -1, -3) - (3, -6, 9)$
$= (-2 - 3, -1 + 6, -3 - 9)$
$= (-5, 5, -12)$

Review Exercise, pp. 418–421

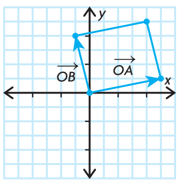
1. a. $(2, 0, 2)$
b. $(-4, 0, -4)$
c. 16
d. The cross products are parallel, so the original vectors are in the same plane.

2. **a.** 3
b. 7
c. $4\sqrt{3}$
d. $2\sqrt{17}$
e. 5
f. -1

3. **a.** 6
b. $-\frac{54}{5}$

4. about 18.52°

5. **a.**

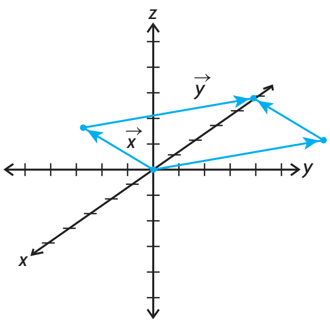


b. about 77.9°

6. rope at 45°: about 87.86 N,
rope at 30°: about 71.74 N

7. 304.14 km/h, W 9.46° N

8. **a.**



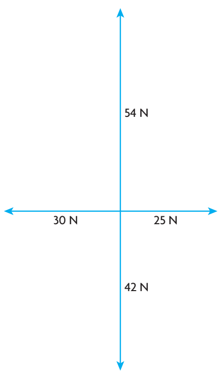
b. approximately 56.78

9. $\left(\frac{9}{\sqrt{115}}, -\frac{5}{\sqrt{115}}, -\frac{3}{\sqrt{115}}\right)$

10. **a.** about 77.64° is the largest angle
b. 36.50

11. 30 cm string: 78.4 N;
40 cm string: 58.8 N

12. **a.**



b. The resultant is 13 N in a direction N22.6°W. The equilibrant is 13 N in a direction S22.6°E.

13. **a.** Let D be the origin, then:
$A = (2, 0, 0)$, $B = (2, 4, 0)$,
$C = (0, 4, 0)$, $D = (0, 0, 0)$,
$E = (2, 0, 3)$, $F = (2, 4, 3)$,
$G = (0, 4, 3)$, $H = (0, 0, 3)$
b. about 44.31°
c. about 3.58

14. 7.5

15. **a.** about 48.2°
b. about 8 min 3 s
c. Such a situation would have resulted in a right triangle where one of the legs is longer than the hypotenuse, which is impossible.

16. **a.** $\overrightarrow{OA} + \overrightarrow{OB} = (-3, 8, -8)$,
$\overrightarrow{OA} - \overrightarrow{OB} = (9, -4, -4)$
b. about 84.36°

17. **a.** $a = 4$ and $b = -4$
b. $\vec{p}\cdot\vec{q} = 2a - 2b - 18 = 0$
c. $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$

18. **a.** about 74.62°
b. about 0.75
c. $(0.1875)(\sqrt{3}, -2, -3)$
d. about 138.59°

19. **a.** special
b. not special

20. **a.** $(-1, 1, 3)$
b. $(-2, 2, 6)$
c. 0
d. $(-1, 1, 3)$

21. about 11.55 N

22. $(2, -8, -10)$

23. -141

24. 5 or -7

25. about 103.34°

26. **a.** $C = (3, 0, 5)$, $F = (0, 4, 0)$
b. $(-3, 4, -5)$
c. about 111.1°

27. **a.** about 7.30
b. about 3.84
c. about 3.84

28. **a.** scalar: 1,
vector: $\vec{i}$
b. scalar: 1,
vector: $\vec{j}$
c. scalar: $\frac{1}{\sqrt{2}}$,
vector: $\frac{1}{2}(\vec{k} + \vec{j})$

29. **a.** $|\vec{b}|$, $|\vec{c}|$
b. $\vec{a}$; When dotted with $\vec{d}$, it equals 0.

30. 7.50 J

31. **a.** $\vec{a}\cdot\vec{b} = 6 - 5 - 1 = 0$
b. $\vec{a}$ with the x-axis:
$|\vec{a}| = \sqrt{4 + 25 + 1} = \sqrt{30}$
$\cos(\alpha) = \frac{2}{\sqrt{30}}$
$\vec{a}$ with the y-axis:
$\cos(\beta) = \frac{5}{\sqrt{30}}$
$\vec{a}$ with the z-axis:
$\cos(\gamma) = \frac{-1}{\sqrt{30}}$
$|\vec{b}| = \sqrt{9 + 1 + 1} = \sqrt{11}$
$\vec{b}$ with the x-axis:
$\cos(\alpha) = \frac{3}{\sqrt{11}}$
$\vec{b}$ with the y-axis:
$\cos(\beta) = \frac{-1}{\sqrt{11}}$
$\vec{b}$ with the z-axis:
$\cos(\gamma) = \frac{1}{\sqrt{11}}$
c. $\vec{m}_1 \times \vec{m}_2 = \frac{6}{\sqrt{330}} - \frac{5}{\sqrt{330}} - \frac{1}{\sqrt{330}} = 0$

32. $|3\vec{i} + 3\vec{j} + 10\vec{k}| = \sqrt{118}$
$|-\vec{i} + 9\vec{j} - 6\vec{k}| = \sqrt{118}$

33. **a.** $\cos\alpha = \frac{\sqrt{3}}{2}$,
$\cos\beta = \cos\gamma = \pm\frac{1}{2\sqrt{2}}$
b. acute case: 69.3°,
obtuse case: 110.7°

34. -5

35. $|\vec{a} + \vec{b}| = \sqrt{1 + 1 + 64} = \sqrt{66}$
$|\vec{a} - \vec{b}| = \sqrt{1 + 81 + 16} = \sqrt{98}$
$\frac{1}{4}|\vec{a} + \vec{b}|^2 - \frac{1}{4}|\vec{a} - \vec{b}|^2$
$= \frac{66}{4} - \frac{98}{4} = -8$

36. $\vec{c} = \vec{b} - \vec{a}$
$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2$
$= (\vec{b} - \vec{a})(\vec{b} - \vec{a})$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

37. $\overrightarrow{AB} = (2, 0, 4)$
$|\overrightarrow{AB}| = 2\sqrt{5}$
$\overrightarrow{AC} = (1, 0, 2)$
$|\overrightarrow{AC}| = \sqrt{5}$
$\overrightarrow{BC} = (-1, 0, -2)$
$|\overrightarrow{BC}| = \sqrt{5}$

$\cos A = 1$
$\cos B = 1$
$\cos C = -1$
area of triangle $ABC = 0$

Chapter 7 Test, p. 422

1. **a.** $(-4, -1, -3)$
b. $(-4, -1, -3)$
c. 0
d. 0

2. **a.** scalar projection: $\frac{1}{3}$,
vector projection: $\frac{1}{9}(2, -1, -2)$.
b. x-axis: 48.2°; y-axis: 109.5°; z-axis: 131.8°
c. $\sqrt{26}$ or 5.10

3. Both forces have a magnitude of 78.10 N. The resultant makes an angle 33.7° to the 40 N force and 26.3° to the 50 N force. The equilibrant makes an angle 146.3° to the 40 N force and 153.7° to the 50 N force.

4. 1004.99 km/h, N 5.7° W

5. **a.** 96 m downstream
b. 28.7° upstream

6. 3.50 square units.

7. cord at 45°: about 254.0 N; cord at 70°: about 191.1 N

8. **a.** 0
33
$\frac{1}{4}|\vec{x} + \vec{y}|^2 - \frac{1}{4}|\vec{x} - \vec{y}|^2$
$= \frac{1}{4}(33) - \frac{1}{4}(33) = 0$
So, the equation holds for these vectors.

b.
$$|\vec{x} + \vec{y}|^2 = (\vec{x} + \vec{y})(\vec{x} + \vec{y}) = (\vec{x}\cdot\vec{x}) + (\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{x}) + (\vec{y}\cdot\vec{y}) = (\vec{x}\cdot\vec{x}) + 2(\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{y})$$
$$|\vec{x} - \vec{y}|^2 = (\vec{x} - \vec{y})(\vec{x} - \vec{y}) = (\vec{x}\cdot\vec{x}) + (\vec{x}\cdot -\vec{y}) + (-\vec{y}\cdot\vec{x}) + (-\vec{y}\cdot -\vec{y}) = (\vec{x}\cdot\vec{x}) - 2(\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{y})$$
So, the right side of the equation is
$$\frac{1}{4}|\vec{x} + \vec{y}|^2 - \frac{1}{4}|\vec{x} - \vec{y}|^2 = \frac{1}{4}(4(\vec{x}\cdot\vec{y})) = \vec{x}\cdot\vec{y}$$

Chapter 8

Review of Prerequisite Skills, pp. 424–425

1. **a.** $(2, -9, 6)$
b. $(13, -12, -41)$

2. **a.** yes **b.** yes **c.** yes **d.** no

3. yes

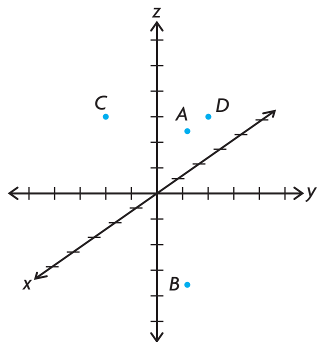
4. $t = 18$

5. **a.** $(3, 1)$
b. $(5, 6)$
c. $(-4, 7, 0)$

6. $\sqrt{2802}$

7. **a.** $(-22, -8, -13)$
b. $(0, 0, -3)$

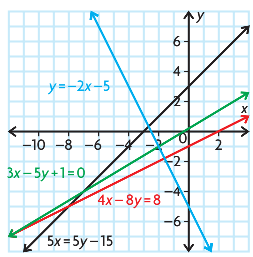
8.



9. **a.** $(-7, -3)$
b. $(10, 14)$
c. $(2, -8, 5)$
d. $(-4, 5, 4)$

10. **a.** $(7, 3)$
b. $(-10, -14)$
c. $(-2, 8, -5)$
d. $(4, -5, -4)$

11. **a.** slope: -2; y-intercept: -5
b. slope: $\frac{1}{2}$; y-intercept: -1
c. slope: $\frac{3}{5}$; y-intercept: $\frac{1}{5}$
d. slope: 1; y-intercept: 3



12. Answers may vary. For example:
a. $(8, 14)$
b. $(-15, 12, 9)$
c. $\vec{i} + 3\vec{j} - 2\vec{k}$
d. $-20\vec{i} + 32\vec{j} + 8\vec{k}$

13. **a.** 33
b. -33
c. 77
d. $(-11, -8, 28)$
e. $(11, 8, -28)$
f. $(55, 40, -140)$

14. The dot product of two vectors yields a real number, while the cross product of two vectors gives another vector.

Section 8.1, pp. 433–434

1. Direction vectors for a line are unique only up to scalar multiplication. So, since each of the given vectors is just a scalar multiple of $\left(\frac{1}{3}, \frac{1}{6}\right)$, each is an acceptable direction vector for the line.

2. **a.** Answers may vary. For example, $(-2, 7)$, $(1, 5)$, and $(4, 3)$.
b. $t = -5$
If $t = -5$, then $x = -14$ and $y = 15$. So $P(-14, 15)$ is a point on the line.

3. Answers may vary. For example:
a. direction vector: $(2, 1)$; point: $(3, 4)$
b. direction vector: $(2, -7)$; point: $(1, 3)$
c. direction vector: $(0, 2)$; point: $(4, 1)$
d. direction vector: $(-5, 0)$; point: $(0, 6)$

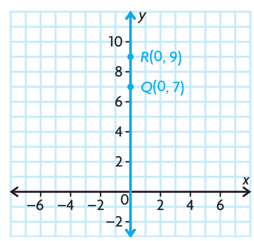
4. Answers may vary. For example:
$\vec{r} = (2, 1) + t(-5, 4), t \in \mathbf{R}$
$\vec{q} = (-3, 5) + s(5, -4), s \in \mathbf{R}$

5. **a.** $R(-9, 18)$ is a point on the line. When $t = 7$, $x = -9$ and $y = 18$.
b. Answers may vary. For example:
$\vec{r} = (-9, 18) + t(-1, 2), t \in \mathbf{R}$
c. Answers may vary. For example:
$\vec{r} = (-2, 4) + t(-1, 2), t \in \mathbf{R}$

6. Answers may vary. For example:
a. $(-3, -4)$, $(0, 0)$, and $(3, 4)$
b. $\vec{r} = t(1, 1), t \in \mathbf{R}$
c. This describes the same line as part a.

7. One can multiply a direction vector by a constant to keep the same line, but multiplying the point yields a different line.

8. **a.**



b. $\vec{r} = (0, 7) + t(0, 2), t \in \mathbf{R}$; $x = 0$, $y = 7 + 2t, t \in \mathbf{R}$