$\cos A = 1$
$\cos B = 1$
$\cos C = -1$
area of triangle $ABC = 0$

Chapter 7 Test, p. 422

1. **a.** $(-4, -1, -3)$
b. $(-4, -1, -3)$
c. 0
d. 0

2. **a.** scalar projection: $\frac{1}{3}$,
vector projection: $\frac{1}{9}(2, -1, -2)$.
b. x-axis: 48.2°; y-axis: 109.5°; z-axis: 131.8°
c. $\sqrt{26}$ or 5.10

3. Both forces have a magnitude of 78.10 N. The resultant makes an angle 33.7° to the 40 N force and 26.3° to the 50 N force. The equilibrant makes an angle 146.3° to the 40 N force and 153.7° to the 50 N force.

4. 1004.99 km/h, N 5.7° W

5. **a.** 96 m downstream
b. 28.7° upstream

6. 3.50 square units.

7. cord at 45°: about 254.0 N;
cord at 70°: about 191.1 N

8. **a.** 0
33
$\frac{1}{4}|\vec{x}+\vec{y}|^2 - \frac{1}{4}|\vec{x}-\vec{y}|^2$
$= \frac{1}{4}(33) - \frac{1}{4}(33) = 0$
So, the equation holds for these vectors.

b.
$$\begin{aligned}|\vec{x}+\vec{y}|^2 &= (\vec{x}+\vec{y})(\vec{x}+\vec{y})\\ &= (\vec{x}\cdot\vec{x}) + (\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{x}) + (\vec{y}\cdot\vec{y})\\ &= (\vec{x}\cdot\vec{x}) + 2(\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{y})\end{aligned}$$
$$\begin{aligned}|\vec{x}-\vec{y}|^2 &= (\vec{x}-\vec{y})(\vec{x}-\vec{y})\\ &= (\vec{x}\cdot\vec{x}) + (\vec{x}\cdot-\vec{y}) + (-\vec{y}\cdot\vec{x}) + (-\vec{y}\cdot-\vec{y})\\ &= (\vec{x}\cdot\vec{x}) - 2(\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{y})\end{aligned}$$
So, the right side of the equation is
$$\begin{aligned}&\frac{1}{4}|\vec{x}+\vec{y}|^2 - \frac{1}{4}|\vec{x}-\vec{y}|^2\\ &= \frac{1}{4}(4(\vec{x}\cdot\vec{y}))\\ &= \vec{x}\cdot\vec{y}\end{aligned}$$

Chapter 8

Review of Prerequisite Skills, pp. 424–425

1. **a.** $(2, -9, 6)$
b. $(13, -12, -41)$

2. **a.** yes **c.** yes
b. yes **d.** no

3. yes

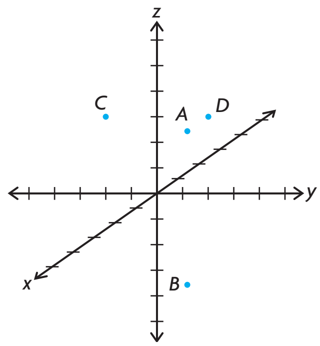
4. $t = 18$

5. **a.** $(3, 1)$
b. $(5, 6)$
c. $(-4, 7, 0)$

6. $\sqrt{2802}$

7. **a.** $(-22, -8, -13)$
b. $(0, 0, -3)$

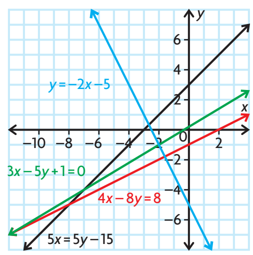
8.



9. **a.** $(-7, -3)$
b. $(10, 14)$
c. $(2, -8, 5)$
d. $(-4, 5, 4)$

10. **a.** $(7, 3)$
b. $(-10, -14)$
c. $(-2, 8, -5)$
d. $(4, -5, -4)$

11. **a.** slope: -2; y-intercept: -5
b. slope: $\frac{1}{2}$; y-intercept: -1
c. slope: $\frac{3}{5}$; y-intercept: $\frac{1}{5}$
d. slope: 1; y-intercept: 3



12. Answers may vary. For example:
a. $(8, 14)$
b. $(-15, 12, 9)$
c. $\vec{i} + 3\vec{j} - 2\vec{k}$
d. $-20\vec{i} + 32\vec{j} + 8\vec{k}$

13. **a.** 33
b. -33
c. 77
d. $(-11, -8, 28)$
e. $(11, 8, -28)$
f. $(55, 40, -140)$

14. The dot product of two vectors yields a real number, while the cross product of two vectors gives another vector.

Section 8.1, pp. 433–434

1. Direction vectors for a line are unique only up to scalar multiplication. So, since each of the given vectors is just a scalar multiple of $\left(\frac{1}{3}, \frac{1}{6}\right)$, each is an acceptable direction vector for the line.

2. **a.** Answers may vary. For example, $(-2, 7)$, $(1, 5)$, and $(4, 3)$.
b. $t = -5$
If $t = -5$, then $x = -14$ and $y = 15$. So $P(-14, 15)$ is a point on the line.

3. Answers may vary. For example:
a. direction vector: $(2, 1)$; point: $(3, 4)$
b. direction vector: $(2, -7)$; point: $(1, 3)$
c. direction vector: $(0, 2)$; point: $(4, 1)$
d. direction vector: $(-5, 0)$; point: $(0, 6)$

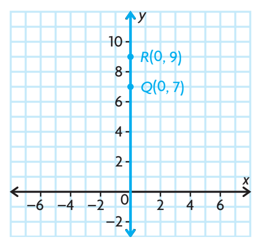
4. Answers may vary. For example:
$\vec{r} = (2, 1) + t(-5, 4), t \in \mathbf{R}$
$\vec{q} = (-3, 5) + s(5, -4), s \in \mathbf{R}$

5. **a.** $R(-9, 18)$ is a point on the line. When $t = 7$, $x = -9$ and $y = 18$.
b. Answers may vary. For example:
$\vec{r} = (-9, 18) + t(-1, 2), t \in \mathbf{R}$
c. Answers may vary. For example:
$\vec{r} = (-2, 4) + t(-1, 2), t \in \mathbf{R}$

6. Answers may vary. For example:
a. $(-3, -4)$, $(0, 0)$, and $(3, 4)$
b. $\vec{r} = t(1, 1), t \in \mathbf{R}$
c. This describes the same line as part a.

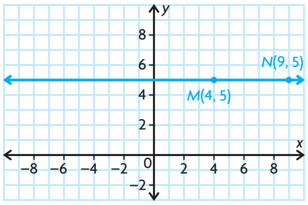
7. One can multiply a direction vector by a constant to keep the same line, but multiplying the point yields a different line.

8. **a.**



b. $\vec{r} = (0, 7) + t(0, 2), t \in \mathbf{R}$; $x = 0$, $y = 7 + 2t, t \in \mathbf{R}$

9. **a.**



b. $\vec{r} = (4, 5) + t(5, 0), t \in \mathbf{R}$;
$x = 4 + 5t, y = 5, t \in \mathbf{R}$

10. **a.** $\vec{r} = (2, 0) + t(5, -3), t \in \mathbf{R}$
b. $(0, -1.2)$

11. 9

12. First, all the relevant vectors are found.
$\overrightarrow{AB} = (-2, 3)$
$\overrightarrow{AC} = (-4, 6)$
$\overrightarrow{AD} = (-6, 9)$
a. $\overrightarrow{AC} = (-4, 6) = 2(-2, 3) = 2\overrightarrow{AB}$
b. $\overrightarrow{AD} = (-6, 9) = 3(-2, 3) = 3\overrightarrow{AB}$
c. $\overrightarrow{AC} = (-4, 6) = \frac{2}{3}(-6, 9) = \frac{2}{3}\overrightarrow{AD}$

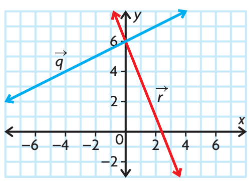
13. **a.** $A(5, 12)$; $B(-12, -5)$
b. $\sqrt{578}$ or about 24.04

14. In the parametric form, the second equation becomes $x = 1 + 6t$, $y = 6 + 4t, t \in \mathbf{R}$. If t is solved for in this equation, we obtain $t = \frac{x - 1}{6}$ and $t = \frac{y - 6}{4}$. Setting these two expressions equal to each other, the line is described by $\frac{x - 1}{6} = \frac{y - 6}{4}$, or by simplifying, $y - 6 = \frac{2}{3}x - \frac{2}{3}$. So, the second equation describes a line with a slope of $\frac{2}{3}$. If y is solved for in the first expression, we see that $y = \frac{2}{3}x + 5$. (1, 6) is on the second line but not the first. Hence, both equations are lines with slope of $\frac{2}{3}$ and must be parallel.

Section 8.2, pp. 443–444

1. **a.** $\vec{m} = (6, -5)$
b. $\vec{n} = (5, 6)$
c. $(0, 9)$
d. $\vec{r} = (7, 9) + t(6, -5), t \in \mathbf{R}$;
$x = 7 + 6t, y = 9 - 5t, t \in \mathbf{R}$
e. $\vec{r} = (-2, 1) + t(5, 6), t \in \mathbf{R}$;
$x = -2 + 5t, y = 1 + 6t, t \in \mathbf{R}$

2. **a.–b.**



c. It produces a different line.

3. **a.** $\vec{r} = (0, -6) + t(8, 7), t \in \mathbf{R}$;
$x = 8t, y = -6 + 7t, t \in \mathbf{R}$
b. $\vec{r} = (0, 5) + t(2, 3), t \in \mathbf{R}$;
$x = 2t, y = 5 + 3t, t \in \mathbf{R}$
c. $\vec{r} = (0, -1) + t(1, 0), t \in \mathbf{R}$;
$x = t, y = -1, t \in \mathbf{R}$
d. $\vec{r} = (4, 0) + t(0, 1), t \in \mathbf{R}$;
$x = 4, y = t, t \in \mathbf{R}$

4. If the two lines have direction vectors that are collinear and share a point in common, then the two lines are coincident. In this example, both have (3, 2) as a parallel direction vector and both have $(-4, 0)$ as a point on the line. Hence, the two lines are coincident.

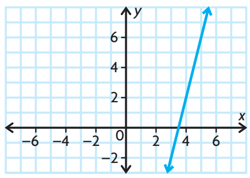
5. **a.** The normal vectors for the lines are $(2, -3)$ and $(4, -6)$, which are collinear. Since in two dimensions, any two direction vectors perpendicular to $(2, -3)$ are collinear, the lines have collinear direction vectors. Hence, the lines are parallel.
b. $k = 12$

6. $4x + 5y - 21 = 0$

7. $x + y - 2 = 0$

8. $2x + y - 16 = 0$

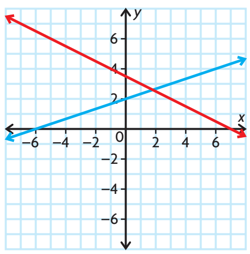
9. **a.**



b. $4x - y - 14 = 0$

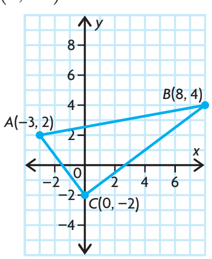
10. **a.** 82° **c.** 63° **e.** 54°
b. 42° **d.** 37° **f.** 63°

11. **a.**



b. acute: 45°, obtuse: 135°

12. **a.** $(0, -2)$
b.



c. $\overrightarrow{CA} = (-3 - 0, 2 - (-2))$
$= (-3, 4)$
$\overrightarrow{CB} = (8 - 0, 4 - (-2))$
$= (8, 6)$
$\overrightarrow{CA} \cdot \overrightarrow{CB} = (-3)(8) + (4)(6)$
$= -24 + 24$
$= 0$
Since the dot product of the vectors is 0, the vectors are perpendicular, and $\angle ACB = 90°$.

13. The sum of the interior angles of a quadrilateral is 360°. The normals make 90° angles with their respective lines at A and C. The angle of the quadrilateral at B is $180° - \theta$. Let x represent the measure of the interior angle of the quadrilateral at O.
$$90° + 90° + 180° - \theta + x = 360°$$
$$360° - \theta + x = 360°$$
$$x = \theta$$
Therefore, the angle between the normals is the same as the angle between the lines.

14. $2 \pm \sqrt{3}$

Section 8.3, pp. 449–450

1. **a.** $(-3, 1, 8)$
b. $(1, -1, 3)$
c. $(-2, 1, 3)$
d. $(-2, -3, 1)$
e. $(3, -2, -1)$
f. $\left(\frac{1}{3}, -\frac{3}{4}, \frac{2}{5}\right)$

2. **a.** $(-1, 1, 9)$
b. $(2, 1, -1)$
c. $(3, -4, -1)$
d. $(-1, 0, 2)$
e. $(0, 0, 2)$
f. $(2, -1, 2)$

3. **a.** $\vec{r} = (-1, 2, 4) + t(4, -5, 1), t \in \mathbf{R}$;
$\vec{q} = (3, -3, 5) + s(-4, 5, -1), s \in \mathbf{R}$
b. $x = -1 + 4t, y = 2 - 5t,$
$z = 4 + t, t \in \mathbf{R}$; $x = 3 - 4s,$
$y = -3 + 5s, z = 5 - s, s \in \mathbf{R}$

4. **a.** $\vec{r} = (-1, 5, -4) + t(1, 0, 0), t \in \mathbf{R}$
b. $x = -1 + t, y = 5, z = -4, t \in \mathbf{R}$
c. Since two of the coordinates in the direction vector are zero, a symmetric equation cannot exist.

5. **a.** $\vec{r} = (-1, 2, 1) + t(3, -2, 1), t \in \mathbf{R}$;
$x = -1 + 3t, y = 2 - 2t,$
$z = 1 + t, t \in \mathbf{R}$;
$\frac{x + 1}{3} = \frac{y - 2}{-2} = \frac{z - 1}{1}$

b. $\vec{r} = (-1, 1, 0) + t(0, 1, 1), t \in \mathbf{R}$; $x = -1, y = 1 + t, z = t, t \in \mathbf{R}$; $\frac{y-1}{1} = \frac{z}{1}, x = -1$

c. $\vec{r} = (-2, 3, 0) + t(0, 1, 1), t \in \mathbf{R}$; $x = -2, y = 3 + t, z = t, t \in \mathbf{R}$; $\frac{y-3}{1} = \frac{z}{1}, x = -2$

d. $\vec{r} = (-1, 0, 0) + t(0, 1, 0), t \in \mathbf{R}$; $x = -1, y = t, z = 0, t \in \mathbf{R}$; Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

e. $\vec{r} = t(-4, 3, 0), t \in \mathbf{R}$; $x = -4t, y = 3t, z = 0, t \in \mathbf{R}$; $\frac{x}{-4} = \frac{y}{3}, z = 0$

f. $\vec{r} = (1, 2, 4) + t(0, 0, 1), t \in \mathbf{R}$; $x = 1, y = 2, z = 4 + t, t \in \mathbf{R}$; Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

6. **a.** $x = -6 + t, y = 10 - t, z = 7 + t, t \in \mathbf{R}$; $x = -7 + s, y = 11 - s, z = 5, s \in \mathbf{R}$

b. about 35.3°

7. The directional vector of the first line is $(8, 2, -2) = -2(-4, -1, 1)$. So, $(-4, -1, 1)$ is a directional vector for the first line as well. Since $(-4, -1, 1)$ is also the directional vector of the second line, the lines are the same if the lines share a point. (1, 1, 3) is a point on the second line. Since $1 = \frac{1+7}{8} = \frac{1+1}{2} = \frac{3-5}{-2}$, (1, 1, 3) is a point on the first line as well. Hence, the lines are the same.

8. **a.** The line that passes through (0, 0, 3) with a directional vector of $(-3, 1, -6)$ is given by the parametric equation is $x = 3t, y = t, z = 3 - 6t, t \in \mathbf{R}$. So, the y-coordinate is equal to -2 only when $t = -2$. At $t = -2, x = -3(-2) = 6$ and $z = 3 - 6(-2) = 15$. So, $A(6, -2, 15)$ is a point on the line. So, the y-coordinate is equal to 5 only when $t = 5$. At $t = 5$, $x = -3(5) = -15$ and $z = 3 - 6(5) = -27$. So, $B(-15, 5, -27)$ is a point on the line.

b. $x = -3t, y = t, z = 3 - 6t, -2 \leq t \leq 5$

9. -1

10. **a.** $(8, 4, -3), (0, -8, 13), (4, -2, 5)$

b. $(-9, 3, 15), (1, 1, 3), (-4, 2, 9)$

c. $(-4, 3, -4), (2, 1, 4), (-1, 2, 0)$

d. $(-4, -1, -2), (-4, 5, 8)\ (-4, 2, 3)$

11. **a.** $x = 4 - 4t, y = -2 - 6t, z = 5 + 8t, t \in \mathbf{R}$; $\frac{x-4}{-4} = \frac{y+2}{-6} = \frac{z-5}{8}$

b. $\vec{r} = (-4, 2, 9) + s(5, -1, -6), s \in \mathbf{R}$; $\frac{x+4}{5} = \frac{y-2}{-1} = \frac{z-9}{-6}$

c. $\vec{r} = (-1, 2, 0) + t(3, -1, 4), t \in \mathbf{R}$; $x = -1 + 3t, y = 2 - t, z = 4t, t \in \mathbf{R}$

d. $\vec{r} = (-4, 2, 3) + t(0, 3, 5), t \in \mathbf{R}$; $x = -4, y = 2 + 3t, z = 3 + 5t, t \in \mathbf{R}$

12. $x = 2 - 34t, y = -5 + 25t, z = 13t, t \in \mathbf{R}$

13. $(-2, -1, 2), (2, 1, 2)$.

14. $P_1(2, 3, -2)$ and $P_2(4, -3, -4)$

15. about 17°

Mid-Chapter Review, pp. 451–452

1. **a.** $(-7, -2), (-5, 1), (-3, 4)$

b. $(-1, 5), (2, 3), (5, 1)$

c. $\left(-1, \frac{11}{5}\right), \left(0, \frac{8}{5}\right), (1, 1)$

d. $(-2, -4, 4), (4, 0, 6), (1, -2, 5)$

2. **a.** $\left(\frac{18}{5}, 0\right)$; $(0, 6)$

b. $\left(-\frac{14}{3}, 0\right)$; $(0, -3)$

3. approximately 86.8°

4. x-axis: about 51°; y-axis: about 39°

5. $5x - 7y - 41 = 0$

6. $\frac{x}{3} = \frac{y}{-4} = \frac{z-2}{4}$

7. $x = 1 + t, y = 2 - 9t, z = 5 + t, t \in \mathbf{R}$

8. approximately 79.3°, 137.7°, and 49.7°

9. $y = -4, \frac{x-3}{1} = \frac{z-6}{\sqrt{3}}$

10. x-axis: $x = t, y = 0, z = 0, t \in \mathbf{R}$; y-axis: $x = 0, y = t, z = 0, t \in \mathbf{R}$; z-axis: $x = 0, y = 0, z = t, t \in \mathbf{R}$

11. **a.** -7

b. $\frac{1}{19}$

12. 17.2 units, 12

13. **a.** $\vec{r} = (0, 6) + t(4, -3), t \in \mathbf{R}$

b. $x = 4t, y = 6 - 3t, t \in \mathbf{R}$

c. about 36.9°

d. $\vec{r} = t(3, 4), t \in \mathbf{R}$

14. $x + 6y - 32 = 0$; $\vec{r} = (-4, 6) + t(12, -2), t \in \mathbf{R}$; $x = -4 + 12t, y = 6 - 2t, t \in \mathbf{R}$

15. $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

16. **a.** $x = -5 + 3t, y = 10 - 2t, t \in \mathbf{R}$

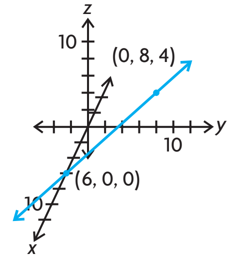
b. $x = 1 + t, y = -1 + t, t \in \mathbf{R}$

c. $x = 0, y = t, t \in \mathbf{R}$

17. **a.** yz-plane at (0, 8, 4); xz-plane at (6, 0, 0); xy-plane at (6, 0, 0)

b. x-axis at (6, 0, 0)

c.



18. **a.** $\vec{r} = (1, -2, 8) + t(-5, -2, 1), t \in \mathbf{R}$; $x = 1 - 5t, y = -2 - 2t, z = 8 + t, t \in \mathbf{R}$; $\frac{x-1}{-5} = \frac{y+2}{-2} = \frac{z-8}{1}$

b. $\vec{r} = (3, 6, 9) + t(2, 4, 6), t \in \mathbf{R}$; $x = 3 + 2t, y = 6 + 4t, z = 9 + 6t, t \in \mathbf{R}$; $\frac{x-3}{2} = \frac{y-6}{4} = \frac{z-9}{6}$

c. $\vec{r} = (0, 0, 6) + t(-1, 5, 1), t \in \mathbf{R}$; $x = -t, y = 5t, z = 6 + t, t \in \mathbf{R}$; $\frac{x}{-1} = \frac{y}{5} = \frac{z-6}{1}$

d. $\vec{r} = (2, 0, 0) + t(0, 0, -2), t \in \mathbf{R}$; $x = 2, y = 0, z = -2t, t \in \mathbf{R}$; There is no symmetric equation for this line.

19. $\vec{r} = t(5, -5, -1), t \in \mathbf{R}$

20. $x = t, y = -8 - 13t, z = 1, t \in \mathbf{R}$

21. $(1, 3, -5), -3(1, 3, -5)$.

22. Since $\frac{7-4}{3} = \frac{-1+2}{1} = \frac{8-6}{2} = 1$, the point $(7, -1, 8)$ lies on the line.

Section 8.4 pp. 459–460

1. **a.** plane; **b.** line; **c.** line; **d.** plane;

2. **a.** $(4, -24, 9)$

b. $(1, -2, 5)$

c. $\vec{r} = (2, 1, 3) + s(4, -24, 9) + t(1, -2, 5), t, s \in \mathbf{R}$

3. **a.** $(0, 0, -1)$

b. $(2, -3, -3)$ and $(0, 5, -2)$

c. $(-2, -17, 10)$

d. $m = 0$ and $n = 3$

e. For the point $B(0, 15, -8)$, the first two parametric equations are the same, yielding $m = 0$ and $n = 3$; however, the third equation would then give:
$-8 = -1 - 3m - 2n$
$-8 = -1 - 3(0) - 2(3)$
$-8 = -7$
which is not true. So, there can be no solution.

4. a. $\vec{r} = (-2, 3, 1) + t(0, 0, 1) + s(3, -3, 0), t, s \in \mathbf{R}$
b. $\vec{r} = (-2, 3, -2) + t(0, 0, 1) + s(3, -3, -1), t, s \in \mathbf{R}$

5. a. $\vec{r} = (1, 0, -1) + s(2, 3, -4) + t(4, 6, -8), t, s \in \mathbf{R}$, does not represent a plane because the direction vectors are the same. We can rewrite the second direction vector as $(2)(2, 3, -4)$. And so we can rewrite the equation as:
$\vec{r} = (1, 0, -1) + s(2, 3, -4) + 2t(2, 3, -4)$
$= (1, 0, -1) + (s + 2t)(2, 3, -4)$
$= (1, 0, -1) + n(2, 3, 4), n \in \mathbf{R}$
This is an equation of a line in $\mathbf{R}^3$.

6. a. $\vec{r} = (-1, 2, 7) + t(4, 1, 0) + s(3, 4, -1), t, s \in \mathbf{R}$;
$x = -1 + 4t + 3s,$
$y = 2 + t + 4s,$
$z = 7 - s, t, s \in \mathbf{R}$
b. $\vec{r} = (1, 0, 0) + t(-1, 1, 0) + s(-1, 0, 1), t, s \in \mathbf{R}$;
$x = 1 - t - s,$
$y = t,$
$z = s, t, s \in \mathbf{R}$
c. $\vec{r} = (1, 1, 0) + t(3, 4, -6) + s(7, 1, 2), t, s \in \mathbf{R}$;
$x = 1 + 3t + 7s,$
$y = 1 + 4t + s,$
$z = -6t + 2s, t, s \in \mathbf{R}$

7. a. $s = 1$ and $t = 1$
b. $(0, 5, -4) = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$ gives the following parametric equations:
$0 = 2 + 4s + t \Rightarrow t = 2 + 4s$
$5 = 2s + t$
$5 = 2s + (2 + 4s)$
$3 = 6s$
$\frac{1}{2} = s$
$t = 2 + 4\left(\frac{1}{4}\right)$
$t = 2 + 2 = 4$
The third equation then says:
$-4 = 1 - s + 2t$
$-4 = 1 - \frac{1}{2} + 2(4)$
$-4 = \frac{17}{2}$, which is a false statement. So, the point $A(0, 5, -4)$ is not on the plane.

8. a. $\vec{l} = (-3, 5, 6) + s(-1, 1, 2), s \in \mathbf{R}$;
$\vec{p} = (-3, 5, 6) + t(2, 1, -3), t \in \mathbf{R}$
b. $(-3, 5, 6)$

9. $(0, 0, 5)$

10. $\vec{r} = (2, 1, 3) + s(4, 1, 5) + t(3, -1, 2), t, s \in \mathbf{R}$

11. $\vec{r} = m(2, -1, 7) + n(-2, 2, 3), m, n \in \mathbf{R}$

12. a. $(1, 0, 0), (0, 1, 0)$ and $(1, 1, 0), (-1, 1, 0)$
b. $\vec{r} = s(1, 0, 0) + t(0, 1, 0), t, s \in \mathbf{R}$;
$x = s,$
$y = t,$
$z = 0, t, s \in \mathbf{R}$

13. a. $\vec{r} = s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
b. $\vec{r} = (-2, 2, 3) + s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
c. The planes are parallel since they have the same direction vectors.

14. $(-4, 7, 1) - (-3, 2, 4) = (-1, 5, -3),$
$\frac{27}{13}(-3, 2, 4) - \frac{17}{13}(-4, 7, 1)$
$= (-1, -5, 7)$

15. $\vec{r} = (0, 3, 0) + t(0, 3, 2), t \in \mathbf{R}$

16. The fact that the plane $\vec{r} = \overrightarrow{OP_0} + s\vec{a} + t\vec{b}$ contains both of the given lines is easily seen when letting $s = 0$ and $t = 0$, respectively.

Section 8.5 pp. 468–469

1. a. $\vec{n} = (A, B, C) = (1, -7, -18)$
b. In the Cartesian equation:
$Ax + By + Cz + D = 0$
If $D = 0$, the plane passes through the origin.
c. $(0, 0, 0), (11, -1, 1), (-11, 1, -1)$

2. a. $\vec{n} = (A, B, C) = (2, -5, 0)$
b. In the Cartesian equation: $D = 0$. So, the plane passes through the origin.
c. $(0, 0, 0), (5, 2, 0)\ (5, 2, 1)$

3. a. $\vec{n} = (A, B, C) = (1, 0, 0)$
b. In the Cartesian equation: $D = 0$. So, the plane passes through the origin.
c. $(0, 0, 0), (0, 1, 0)\ (0, 0, 1)$

4. a. $x + 5y - 7z = 0$
b. $-8x + 12y + 7z = 0$

5. *Method 1:* Let $A(x, y, z)$ be a point on the plane. Then,
$\overrightarrow{PA} = (x + 3, y - 3, z - 5)$ is a vector on the plane.
$\vec{n} \cdot \overrightarrow{PA} = 0$
$(x + 3) + 7(y - 3) + 5(z - 5) = 0$
$x + 7y + 5z - 43 = 0.$
Method 2: $\vec{n} = (1, 7, 5)$ so the Cartesian equation is
$x + 7y + 5z + D = 0$
We know the point $(-3, 3, 5)$ is on the plane and must satisfy the equation, so
$(-3) + 7(3) + 5(5) + D = 0$
$43 + D = 0$
$D = -43$
This also gives the equation:
$x + 7y + 5z - 43 = 0.$

6. a. $7x + 19y - 3z - 28 = 0$
b. $7x + 19y - 3z - 28 = 0$
c. There is only one simplified Cartesian equation that satisfies the given information, so the equations must be the same.

7. $7x + 17y - 13z - 24 = 0$

8. $20x + 9y + 7z - 47 = 0$

9. a. $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$
b. $\left(\frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$
c. $\left(\frac{3}{13}, -\frac{4}{13}, \frac{12}{13}\right)$

10. $21x - 15y - z - 1 = 0$

11. $2x - 4y - z + 6 = 0$

12. a. First determine their normal vectors, $\vec{n_1}$ and $\vec{n_2}$. Then the angle between the two planes can be determined from the formula:
$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$
b. 30°

13. a. 53.3°
b. $2x - 3y - z + 5 = 0$

14. a. 8
b. $-\frac{5}{2}$
c. No, the planes cannot ever be coincident. If they were, then they would also be parallel, so $k = 8$, and we would have the two equations:
$4x + 8y - 2z + 1 = 0.$
$2x + 4y - z + 4 = 0 \Rightarrow$
$4x + 8y - 2z + 8 = 0$. Here all of the coefficients are equal except for the D-values, which means that they don't coincide.

15. $3x + 5y - z - 18 = 0$

16. $-\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y + \sqrt{3}z = 0$

17. $8x - 2y - 16z - 5 = 0$

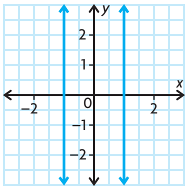
Section 8.6, pp. 476–477

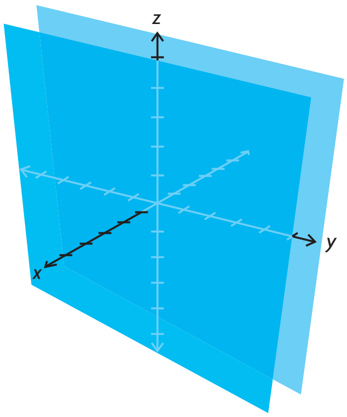
1. a. A plane parallel to the yz-axis, but two units away, in the negative x direction.
b. A plane parallel to the xz-axis, but three units away, in the positive y direction.
c. A plane parallel to the xy-axis, but 4 units away, in the positive z direction.

2. $(-2, 3, 4)$

3. P must lie on plane π_1 since the point has an x-coordinate of 5, and doesn't have a y-coordinate of 6.

4.





5. **a.** **i.** x-intercept $= 9$, y-intercept $= 6$, no z-intercept

ii. x-intercept $= 40$, y-intercept $= -30$, z-intercept $= 24$

iii. no x-intercept, y-intercept $= 3$, z-intercept $= -39$

b. **i.** $(0, 0, 1)$, $(3, -2, 0)$

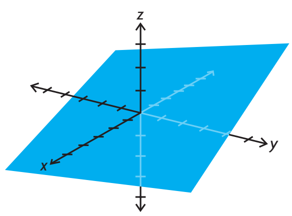
ii. $(4, 3, 0)$, $(5, 0, -3)$

iii. $(1, 0, 0)$, $(0, 1, 13)$

6. **a.** **i.** $(0, 0, 0)$, $(1, 2, 0)$, $(0, 5, 1)$

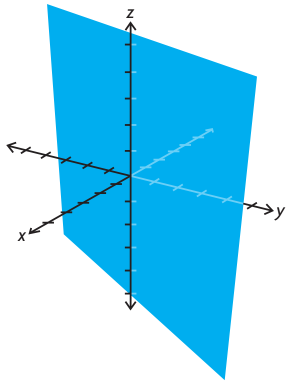
ii. $2x - y = 0$

b.

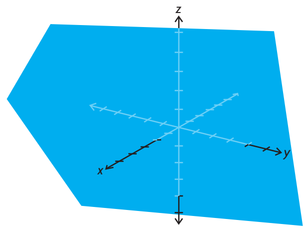


7. yz-plane, xz-plane, xy-plane

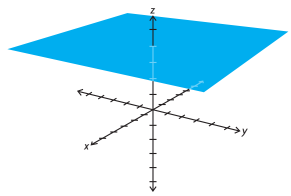
8. **a.**



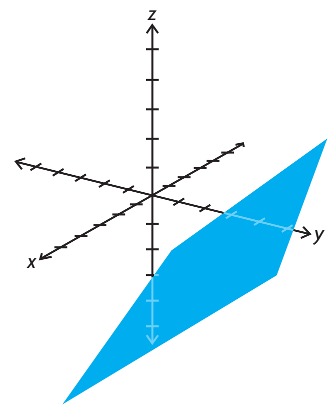
b.



c.

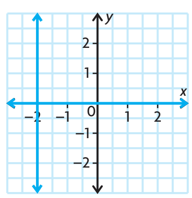


d.



9. **a.** $y(x + 2) = 0$

b.



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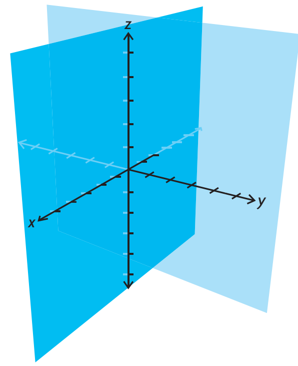
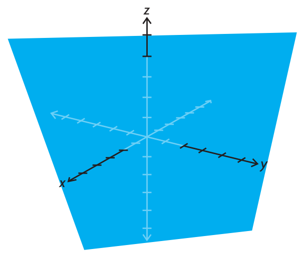
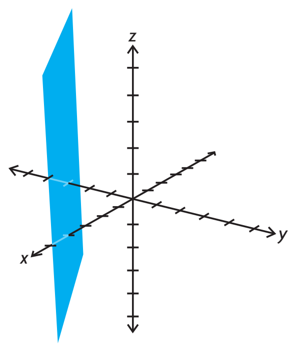


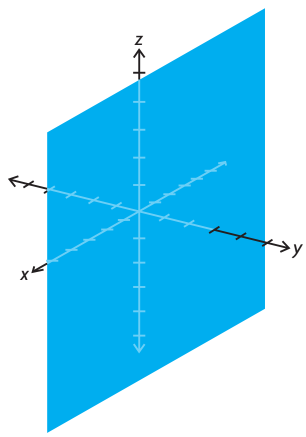
10. **a.**



b.



c.



11. **a.** $\frac{x}{3} + \frac{y}{4} + \frac{z}{6} = 1$

b. $\frac{x}{5} - \frac{z}{7} = 1$

c. $\frac{z}{8} = 1$

Review Exercise, pp. 480–483

1. Answers may vary. For example,
$x = 1 + s + t,$
$y = 2 - s,$
$z = -1 + 2s + 3t$

2. $3x + y - z - 6 = 0$
$\overrightarrow{AC} = (2, -1, 5) = \vec{c}$
$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$
$\vec{r} = (1, 2, -1) + s(2, -1, 5) + t(1, 0, 3),\ s, t \in \mathbf{R}$
$\vec{b} \times \vec{c} = (1, 0, 3) \times (2, -1, 5) = (3, 1, -1)$
$Ax + By + Cz + D = 0$
$(3)x + (1)y + (-1)z + D = 0$
$3(1) + (2) - 1(-1) + D = 0$
$D = -6$
$3x + y - z - 6 = 0$
Both Cartesian equations are the same regardless of which vectors are used.

3. **a.** Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1),\ t \in \mathbf{R};$
$x = 4 + 7t,\ y = 3 + t,\ z = 9 + t,\ t \in \mathbf{R};$
$\frac{x-4}{7} = \frac{y-3}{1} = \frac{z-9}{1}$
b. Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1) + s(3, 2, 3),\ t, s \in \mathbf{R};$
$x = 4 + 7t + 3s,\ y = 3 + t + 2s,\ z = 9 + t + 3s,\ t, s \in \mathbf{R}$
c. There are two parameters.

4. $\vec{r} = (7, 1, -2) + t(2, -3, 1),\ t \in \mathbf{R};$
$x = 7 + 2t,\ y = 1 - 3t,\ z = -2 + t;$
$\frac{x-7}{2} = \frac{y-1}{-3} = \frac{z+2}{1}$

5. **a.** $x - 3y - 3z - 3 = 0$
b. $3x + 5y - 2z - 7 = 0$
c. $3y + z - 7 = 0$

6. $19x - 7y - 8z = 0$

7. $\vec{r} = (-1, 2, 1) + t(0, 1, 0) + s(0, 0, 1)\ t, s \in \mathbf{R};$
$x = -1,\ y = 2 + t,\ z = 1 + s$

8. $3x + y - z - 7 = 0$

9. $34x + 32y - 7z - 229 = 0$

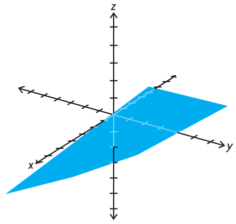
10. Answers may vary. For example,
$\vec{r} = (2, 3, -3) + s(3, -2, 1),\ s \in \mathbf{R};$
$x = 2 + 3s,\ y = 3 - 2s,\ z = -3 + s;$
$\frac{x-2}{3} = \frac{y-3}{-2} = \frac{z+3}{1}$

11. Answers may vary. For example,
$\vec{r} = (0, 0, 6) + s(1, 0, 3) + t(3, -5, -1),\ s, t \in \mathbf{R};$
$x = s + 3t,\ y = -5t,\ z = 6 + 3s - t$

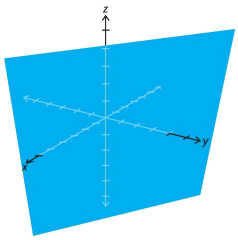
12. Answers may vary. For example,
$\vec{r} = (0, 0, 7) + t(1, 0, 2),\ t \in \mathbf{R};$
$x = t,\ y = 0,\ z = 7 + 2t;$

13. $\vec{r} = (3, -4, 1) + s(1, -3, -5) + t(4, 3, -1),\ s, t \in \mathbf{R};$
$x = 3 + s + 4t,$
$y = -4 - 3s + 3t$
$z = 1 - 5s - t,\ s, t \in \mathbf{R};$
$18x - 19y + 15z - 145 = 0$

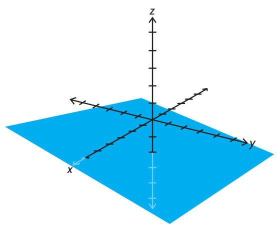
14. **a.**



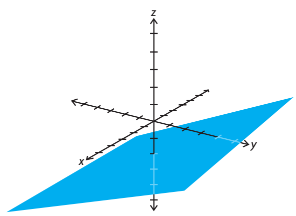
b.



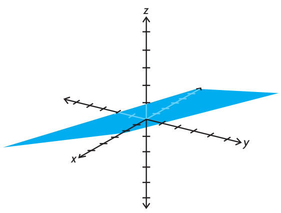
c.



d.



e.



15. **a.** Answers may vary. For example,
$\vec{r} = (3, 1, 2) + t(2, 4, 1) + s(2, 3, -3),\ t, s \in \mathbf{R};$
$x = 3 + 2t + 2s,$
$y = 1 + 4t + 3s,\ z = 2 + t - 3s;$
$15x - 8y + 2z - 41 = 0$
b. Answers may vary. For example,
$\overrightarrow{BC} = (-4, 0, 11)$
$D = -18$
$-4x + 11z - 18 = 0$
c. Answers may vary. For example,
$\vec{r} = (4, 1, -1) + t(1, -3, 5) + s(0, 0, 1),\ t, s \in \mathbf{R};$
$x = 4 + t,\ y = 1 - 3t,$
$z = -1 + 5t + s;$
$3x + y - 13 = 0$
d. Answers may vary. For example,
$\vec{r} = (1, 3, -5) + t(1, 3, 9) + s(1, -9, -1),\ t, s \in \mathbf{R};$
$x = 1 + t + s,\ y = 3 + 3t - 9s,$
$z = -5 + 9t - s;$
$78x + 10y - 12z - 168 = 0$

16. They are in the same plane because both planes have the same normal vectors and Cartesian equations.
$L_1: \vec{r} = (1, 2, 3) + s(-3, 5, 21) + t(0, 1, 3),\ s, t \in \mathbf{R}$
$L_2: \vec{r} = (1, -1, -6) + u(1, 1, 1) + v(2, 5, 11),\ u, v \in \mathbf{R}$
$(-3, 5, 21) \times (0, 1, 3) = (-6, 9, -3) = (2, -3, 1)$
$(1, 1, 1) \times (2, 5, 11) = (6, -9, 3) = (2, -3, 1)$
$Ax + By + Cz + D = 0$
$2x - 3y + z + D = 0$
$2(1) - 3(2) + (3) + D = 0$
$D = 1$
$2(1) - 3(-1) + (-6) + D = 0$
$D = 1$
$2x - 3y + z + 1 = 0$

17. $\left(\frac{20}{3}, \frac{10}{3}, -\frac{1}{3}\right)$

18. **a.** The plane is parallel to the z-axis through the points $(3, 0, 0)$ and $(0, -2, 0)$.
b. The plane is parallel to the y-axis through the points $(6, 0, 0)$ and $(0, 0, -2)$.
c. The plane is parallel to the x-axis through the points $(0, 3, 0)$ and $(0, 0, -6)$.

19. **a.** A
b. $a = -8,\ b = -1$

20. **a.** 45.0° **b.** 59.0° **c.** 37.4° **d.** 90°

21. **a.** 44.2°
b. 90°

22. **a.** **i.** no **ii.** yes **iii.** no
b. **i.** yes **ii.** no **iii.** no

23. $(x, y, z) = (4, 1, 6) + p(3, -2, 1) + q(-6, 6, -1)$
$(x, y, z) = (4, 1, 6) + 4(3, -2, 1) + 2(-6, 6, -1)$
$(x, y, z) = (4, 5, 8) \neq (4, 5, 6)$

24. $x = 1 + s + 3t,\ y = 4 - t,$
$z = 4 - 3s + t,\ s, t \in \mathbf{R}$

25. A plane has two parameters, because a plane goes in two different directions, unlike a line that goes only in one direction.

26. This equation will always pass through the origin, because you can always set $s = 0$ and $t = -1$ to obtain $(0, 0, 0)$.

27. **a.** They do not form a plane, because these three points are collinear.
$\vec{r} = (-1, 2, 1) + t(3, 1, -2)$

b. They do not form a plane, because the point lies on the line.
$\vec{r} = (4, 9, -3) + t(1, -4, 2)$
$\vec{r} = (4, 9, -3) + 4(1, -4, 2)$
$= (8, -7, 5)$

28. $bcx + acy + abz - abc = 0$

29. $6x - 5y + 12z + 46 = 0$

30. **a., b.** $\vec{r} = (1, -3, 2) + t(-3, 7, -4) + s(5, -2, 3)\ t, s \in \mathbf{R}$;
$x = 1 - 3t + 5s$,
$y = -3 + 7t - 2s$,
$z = 2 - 4t + 3s$

c. $13x - 11y - 29z + 12 = 0$

d. no

31. **a.** $4x - 2y + 5z = 0$

b. $4x - 2y + 5z + 19 = 0$

c. $4x - 2y + 5z - 22 = 0$

32. **a.** These lines are coincident. The angle between them is 0°.

b. $\left(\frac{3}{2}, 5\right)$, 86.82°

33. **a.** $\vec{r} = (1, 3, 5) + t(-2, -4, -10)$, $t \in \mathbf{R}$;
$x = 1 - 2t, y = 3 - 4t$,
$z = 5 - 10t$;
$\frac{x - 1}{-2} = \frac{y - 3}{-4} = \frac{z - 5}{-10}$

b. $\vec{r} = (1, 3, 5) + t(-8, 6, -2), t \in \mathbf{R}$;
$x = 1 - 8t, y = 3 + 6t$,
$z = 5 - 2t$;
$\frac{x - 1}{-8} = \frac{x - 3}{6} = \frac{x - 5}{-2}$

c. $\vec{r} = (1, 3, 5) + t(-6, -13, 14)$, $t \in \mathbf{R}$;
$x = 1 - 6t, y = 3 - 13t$,
$z = 5 + 14t$;
$\frac{x - 1}{-6} = \frac{x - 3}{-13} = \frac{x - 5}{14}$

d. $\vec{r} = (1, 3, 5) + t(1, 0, 0), t \in \mathbf{R}$;
$x = 1 + t, y = 3, z = 5$

e. $a = 0, b = 6, c = 4$;
$\vec{r} = (1, 3, 5) + t(0, 6, 4), t \in \mathbf{R}$

f. $\vec{r} = (1, 3, 5) + t(0, 1, 6)$;
$x = 1, y = 3 + t, z = 5 + 6t$

34. **a.** $2x - 4y + 5z + 23 = 0$

b. $29x + 27y + 24z - 86 = 0$

c. $z - 3 = 0$

d. $3x + y - 4z + 26 = 0$

e. $y - 2z - 4 = 0$

f. $-5x + y + 7z + 18 = 0$

Chapter 8 Test, p. 484

1. **a.** **i.** $\vec{r} = (1, 2, 4) + s(1, -2, -1) + t(3, 2, 0), s, t \in \mathbf{R}$;
$x = 1 + s + 3t$,
$y = 2 - 2s + 2t, z = 4 - s$,
$s, t \in \mathbf{R}$

ii. $2x - 3y + 8z - 28 = 0$

b. no

2. **a.** $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

b. $(6, 4, 3)$

3. **a.** $\vec{r} = s(2, 1, 3) + t(1, 2, 5), s, t \in \mathbf{R}$

b. $-x - 7y + 3z = 0$

4. **a.** $\vec{r} = (4, -3, 5) + s(2, 0, -3) + t(5, 1, -1), s, t \in \mathbf{R}$

b. $3x - 13y + 2z - 61 = 0$

5. **a.** $\left(0, 5, -\frac{1}{2}\right)$

b. $\frac{x}{4} = \frac{y - 5}{-2} = \frac{1}{2}$

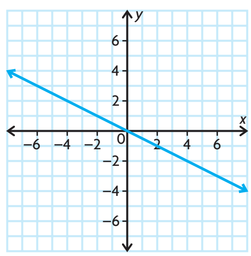
6. **a.** about 70.5°

b. **i.** 4

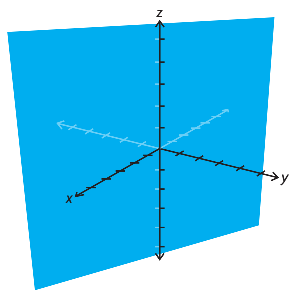
ii. $-\frac{1}{5}$

c. The y-intercepts are different and the planes are parallel.

7. **a.**



b.



c. The equation for the plane can be written as $Ax + By + 0z = 0$. For any real number t,
$A(0) + B(0) + 0(t) = 0$, so
$(0, 0, t)$ is on the plane. Since this is true for all real numbers, the z-axis is on the plane.

Chapter 9

Review of Prerequisite Skills, p. 487

1. **a.** yes **c.** yes
b. no **d.** no

2. Answers may vary. For example:

a. $\vec{r} = (2, 5) + t(5, -2), t \in \mathbf{R}$;
$x = 2 + 5t, y = 5 - 2t, t \in \mathbf{R}$

b. $\vec{r} = (-3, 7) + t(7, -14), t \in \mathbf{R}$;
$x = -3 + 7t, y = 7 - 14t, t \in \mathbf{R}$

c. $\vec{r} = (-1, 0) + t(-2, -11), t \in \mathbf{R}$;
$x = -1 + -2t, y = -11t, t \in \mathbf{R}$

d. $\vec{r} = (1, 3, 5) + t(5, -10, -5), t \in \mathbf{R}$;
$x = 1 + 5t, y = 3 - 10t, z = 5 - 5t$,
$t \in \mathbf{R}$

e. $\vec{r} = (2, 0, -1) + t(-3, 5, 3), t \in \mathbf{R}$;
$x = 2 - 3t, y = 5t, z = -1 + 3t$,
$t \in \mathbf{R}$

f. $\vec{r} = (2, 5, -1) + t(10, -10, -6)$,
$t \in \mathbf{R}$;
$x = 2 + 10t, y = 5 - 10t, z = -1 - 6t, t \in \mathbf{R}$

3. **a.** $2x + 6y - z - 17 = 0$

b. $y = 0$

c. $4x - 3y - 15 = 0$

d. $6x - 5y + 3z = 0$

e. $11x - 6y - 38 = 0$

f. $x + y - z - 6 = 0$

4. $5x + 11y + 2z - 21 = 0$

5. L_1 is not parallel to the plane. L_1 is on the plane.
L_2 is parallel to the plane.
L_3 is not parallel to the plane.

6. **a.** $x - y - z - 2 = 0$

b. $x + 6y - 10z - 30 = 0$

7. $\vec{r} = (1, -4, 3) + t(1, 3, 3) + s(0, 1, 0), s, t \in \mathbf{R}$

8. $3y + z = 13$

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1. **a.** $\pi: x - 2y - 3z = 6$,
$\vec{r} = (1, 2, -3) + s(5, 1, 1)\ s \in \mathbf{R}$

b. This line lies on the plane.

2. **a.** A line and a plane can intersect in three ways: (1) The line and the plane have zero points of intersection. This occurs when the lines are not incidental, meaning they do not intersect.
(2) The line and the plane have only one point of intersection. This occurs when the line crosses the plane at a single point.
(3) The line and the plane have an infinite number of intersections. This occurs when the line is