25. A plane has two parameters, because a plane goes in two different directions, unlike a line that goes only in one direction.

26. This equation will always pass through the origin, because you can always set $s = 0$ and $t = -1$ to obtain (0, 0, 0).

27. **a.** They do not form a plane, because these three points are collinear.
$\vec{r} = (-1, 2, 1) + t(3, 1, -2)$

b. They do not form a plane, because the point lies on the line.
$\vec{r} = (4, 9, -3) + t(1, -4, 2)$
$\vec{r} = (4, 9, -3) + 4(1, -4, 2)$
$= (8, -7, 5)$

28. $bcx + acy + abz - abc = 0$

29. $6x - 5y + 12z + 46 = 0$

30. **a., b.** $\vec{r} = (1, -3, 2) + t(-3, 7, -4) + s(5, -2, 3)\ t, s \in \mathbf{R}$;
$x = 1 - 3t + 5s$,
$y = -3 + 7t - 2s$,
$z = 2 - 4t + 3s$

c. $13x - 11y - 29z + 12 = 0$

d. no

31. **a.** $4x - 2y + 5z = 0$

b. $4x - 2y + 5z + 19 = 0$

c. $4x - 2y + 5z - 22 = 0$

32. **a.** These lines are coincident. The angle between them is 0°.

b. $\left(\frac{3}{2}, 5\right)$, 86.82°

33. **a.** $\vec{r} = (1, 3, 5) + t(-2, -4, -10)$, $t \in \mathbf{R}$;
$x = 1 - 2t$, $y = 3 - 4t$,
$z = 5 - 10t$;
$\frac{x - 1}{-2} = \frac{y - 3}{-4} = \frac{z - 5}{-10}$

b. $\vec{r} = (1, 3, 5) + t(-8, 6, -2)$, $t \in \mathbf{R}$;
$x = 1 - 8t$, $y = 3 + 6t$,
$z = 5 - 2t$;
$\frac{x - 1}{-8} = \frac{x - 3}{6} = \frac{x - 5}{-2}$

c. $\vec{r} = (1, 3, 5) + t(-6, -13, 14)$, $t \in \mathbf{R}$;
$x = 1 - 6t$, $y = 3 - 13t$,
$z = 5 + 14t$;
$\frac{x - 1}{-6} = \frac{x - 3}{-13} = \frac{x - 5}{14}$

d. $\vec{r} = (1, 3, 5) + t(1, 0, 0)$, $t \in \mathbf{R}$;
$x = 1 + t$, $y = 3$, $z = 5$

e. $a = 0$, $b = 6$, $c = 4$;
$\vec{r} = (1, 3, 5) + t(0, 6, 4)$, $t \in \mathbf{R}$

f. $\vec{r} = (1, 3, 5) + t(0, 1, 6)$;
$x = 1$, $y = 3 + t$, $z = 5 + 6t$

34. **a.** $2x - 4y + 5z + 23 = 0$

b. $29x + 27y + 24z - 86 = 0$

c. $z - 3 = 0$

d. $3x + y - 4z + 26 = 0$

e. $y - 2z - 4 = 0$

f. $-5x + y + 7z + 18 = 0$

Chapter 8 Test, p. 484

1. **a.** **i.** $\vec{r} = (1, 2, 4) + s(1, -2, -1) + t(3, 2, 0)$, $s, t \in \mathbf{R}$;
$x = 1 + s + 3t$,
$y = 2 - 2s + 2t$, $z = 4 - s$,
$s, t \in \mathbf{R}$

ii. $2x - 3y + 8z - 28 = 0$

b. no

2. **a.** $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

b. (6, 4, 3)

3. **a.** $\vec{r} = s(2, 1, 3) + t(1, 2, 5)$, $s, t \in \mathbf{R}$

b. $-x - 7y + 3z = 0$

4. **a.** $\vec{r} = (4, -3, 5) + s(2, 0, -3) + t(5, 1, -1)$, $s, t \in \mathbf{R}$

b. $3x - 13y + 2z - 61 = 0$

5. **a.** $\left(0, 5, -\frac{1}{2}\right)$

b. $\frac{x}{4} = \frac{y - 5}{-2} = \frac{1}{2}$

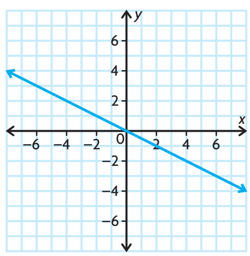
6. **a.** about 70.5°

b. **i.** 4

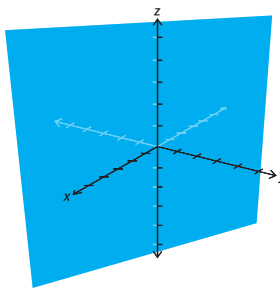
ii. $-\frac{1}{5}$

c. The y-intercepts are different and the planes are parallel.

7. **a.**



b.



c. The equation for the plane can be written as $Ax + By + 0z = 0$. For any real number t,
$A(0) + B(0) + 0(t) = 0$, so
$(0, 0, t)$ is on the plane. Since this is true for all real numbers, the z-axis is on the plane.

Chapter 9

Review of Prerequisite Skills, p. 487

1. **a.** yes　**c.** yes
b. no　**d.** no

2. Answers may vary. For example:

a. $\vec{r} = (2, 5) + t(5, -2)$, $t \in \mathbf{R}$;
$x = 2 + 5t$, $y = 5 - 2t$, $t \in \mathbf{R}$

b. $\vec{r} = (-3, 7) + t(7, -14)$, $t \in \mathbf{R}$;
$x = -3 + 7t$, $y = 7 - 14t$, $t \in \mathbf{R}$

c. $\vec{r} = (-1, 0) + t(-2, -11)$, $t \in \mathbf{R}$;
$x = -1 + -2t$, $y = -11t$, $t \in \mathbf{R}$

d. $\vec{r} = (1, 3, 5) + t(5, -10, -5)$, $t \in \mathbf{R}$;
$x = 1 + 5t$, $y = 3 - 10t$, $z = 5 - 5t$,
$t \in \mathbf{R}$

e. $\vec{r} = (2, 0, -1) + t(-3, 5, 3)$, $t \in \mathbf{R}$;
$x = 2 - 3t$, $y = 5t$, $z = -1 + 3t$,
$t \in \mathbf{R}$

f. $\vec{r} = (2, 5, -1) + t(10, -10, -6)$,
$t \in \mathbf{R}$;
$x = 2 + 10t$, $y = 5 - 10t$, $z = -1 - 6t$, $t \in \mathbf{R}$

3. **a.** $2x + 6y - z - 17 = 0$

b. $y = 0$

c. $4x - 3y - 15 = 0$

d. $6x - 5y + 3z = 0$

e. $11x - 6y - 38 = 0$

f. $x + y - z - 6 = 0$

4. $5x + 11y + 2z - 21 = 0$

5. L_1 is not parallel to the plane. L_1 is on the plane.
L_2 is parallel to the plane.
L_3 is not parallel to the plane.

6. **a.** $x - y - z - 2 = 0$

b. $x + 6y - 10z - 30 = 0$

7. $\vec{r} = (1, -4, 3) + t(1, 3, 3) + s(0, 1, 0)$, $s, t \in \mathbf{R}$

8. $3y + z = 13$

Section 9.1, pp. 496–498

1. **a.** $\pi: x - 2y - 3z = 6$,
$\vec{r} = (1, 2, -3) + s(5, 1, 1)\ s \in \mathbf{R}$

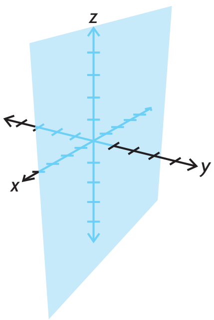
b. This line lies on the plane.

2. **a.** A line and a plane can intersect in three ways: (1) The line and the plane have zero points of intersection. This occurs when the lines are not incidental, meaning they do not intersect.
(2) The line and the plane have only one point of intersection. This occurs when the line crosses the plane at a single point.
(3) The line and the plane have an infinite number of intersections. This occurs when the line is

coincident with the plane, meaning the line lies on the plane.

b. Assume that the line and the plane have more than one intersection, but not an infinite number. For simplicity, assume two intersections. At the first intersection, the line crosses the plane. In order for the line to continue on, it must have the same direction vector. If the line has already crossed the plane, then it continues to move away from the plane, and can not intersect again. So, the line and the plane can only intersect zero, one, or infinitely many times.

3. **a.** The line $\vec{r} = s(1, 0, 0)$ is the x-axis.
b. The plane is parallel to the xz-plane, but just one unit away to the right.
c.



d. There are no intersections between the line and the plane.

4. **a.** For $x + 4y + z - 4 = 0$, if we substitute our parametric equations, we have $(-2 + t) + 4(1 - t) + (2 + 3t) + 4 = 0$
All values of t give a solution to the equation, so all points on the line are also on the plane.
b. For the plane $2x - 3y + 2x - 3y + 4z - 11 = 0$, we can substitute the parametric equations derived from $\vec{r} = (1, 5, 6) + t(1, -2, -2)$:
$2(1 + t) - 3(5 - 2t) + 4(6 - 2t) - 11 = 0$
All values of t give a solution to this equation, so all points on the line are also on the plane.

5. **a.** $2(-1 - s) - 2(1 + 2s) + 3(2s) - 1 = -5$
Since there are no values of s such that $-5 = 0$, this line and plane do not intersect.
b. $2(1 + 2t) - 4(-2 + 5t) + 4(1 + 4t) - 13 = 1$
Since there are no values of t such that $1 = 0$, there are no solutions, and the plane and the line do not intersect.

6. **a.** The direction vector is $\vec{m} = (-1, 2, 2)$ and the normal is $\vec{n} = (2, -2, 3)$, $\vec{m} \cdot \vec{n} = 0$. So the line is parallel to the plane, but $2(-1) - 2(1) + 3(0) - 1 = -5 \neq 0$. So, the point on the line is not on the plane.
b. The direction vector is $\vec{m} = (2, 5, 4)$ and the normal is $\vec{n} = (2, -4, 4)$, $\vec{m} \cdot \vec{n}$, $= 0$, so the line is parallel to the plane. and
$2(1) - 4(-2) + 4(1) - 13 = 1 \neq 0$
So, the point on the line is not on the plane.

7. **a.** $(-19, 0, 10)$
b. $(-11, 1, 0)$

8. **a.** There is no intersection and the lines are skew.
b. $(4, 1, 2)$

9. **a.** not skew
b. not skew
c. not skew
d. skew

10. 8

11. **a.** Comparing components results in the equation $s - t = -4$ for each component.
b. From L_1, we see that at $(-2, 3, 4)$, $s = 0$. When this occurs, $t = 4$. Substituting this into L_2, we get $(-30, 11, -4) + 4(7, -2, 2) = (-2, 3, 4)$. Since both of these lines have the same direction vector and a common point, the lines are coincidental.

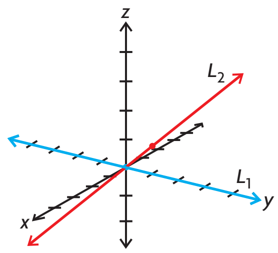
12. **a.** 3
b. $\left(\frac{2}{11}, \frac{53}{11}, \frac{46}{11}\right)$

13. 3

14. **a.** $(-6, 1, 3)$
b. 3

15. **a.** $(4, 1, 12)$
b. $\vec{r} = (4, 1, 12) + t(42, 55, -10)$, $t \in \mathbf{R}$

16. **a.**



b. (0, 0, 0)
c. If $p = 0$ and $q = 0$, the intersection occurs at (0, 0, 0).

17. **a.** Represent the lines parametrically, and then substitute into the equation for the plane. For the first equation, $x = t, y = 7 - 8t, z = 1 + 2t$.
Substituting into the plane equation, $2t + 7 - 8t + 3 + 6t - 10 = 0$.
Simplifying, $0t = 0$. So, the line lies on the plane.
For the second line, $x = 4 + 3s, y = -1, z = 1 - 2s$.
Substituting into the plane equation, $8 + 6s - 1 + 3 - 6s - 10 = 0$.
Simplifying, $0s = 0$. This line also lies on the plane.
b. $(1, -1, 3)$

18. Answers may vary. For example, $\vec{r} = (2, 0, 0) + p(2, 0, 1)$, $p \in \mathbf{R}$.

Section 9.2, pp. 507–509

1. **a.** linear
b. not linear
c. linear
d. not linear

2. Answers may vary. For example:
a. $x + y + 2z = -15$
$x + 2y + z = -3$
$2x + y + z = -10$
b. $(-3, 4, -8)$

3. **a.** yes
b. no

4. **a.** $(-2, -3)$
b. $(-2, -3)$
The two systems are equivalent because they have the same solution.

5. **a.** $(6, 1)$
b. $(-3, 5)$
c. $(-4, 3)$

6. **a.** These two lines are parallel, and therefore cannot have an intersection.
b. The second equation is five times the first; therefore, the lines are coincident.

7. **a.** $x = t, y = 2t - 3, t \in \mathbf{R}$
b. $x = t, y = s, z = 2s - t, t \in \mathbf{R}$

8. **a.** $2x + y = -11$
b. $2x + y = -11$
$2(3t + 3) + (-6t - 17) =$
$6t - 6t + 6 - 17 = -11$

9. **a.** $k \neq 12$
b. not possible
c. $k = 12$

10. **a** infinitely many

b. $x = t$,

$$y = \frac{11}{4} - \frac{1}{2}t, t \in \mathbf{R}$$

c. This equation will not have any integer solutions because the left side is an even function and the right side is an odd function.

11. **a.** $x = -a + b, y = -\frac{1}{3}b + \frac{2}{3}a$

b. Since they have different direction vectors, these two equations are not parallel or coincident and will intersect somewhere.

12. **a.** $(-1, -2, 3)$

b. $(3, 4, 12)$

c. $(4, 6, -8)$

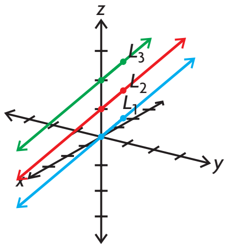
d. $(60, 120, -180)$

e. $(2, 4, 1)$

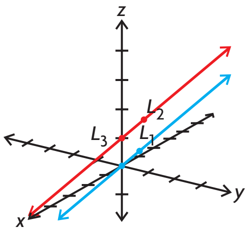
f. $(-2, 3, 6)$

13. Answers may vary. For example:

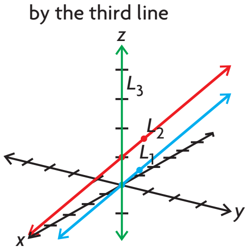
a. Three lines parallel



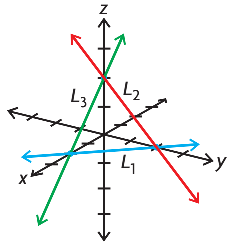
Two lines coincident and the third parallel



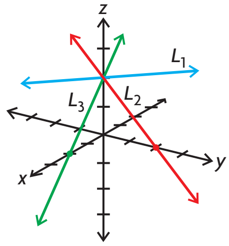
Two parallel lines cut by the third line



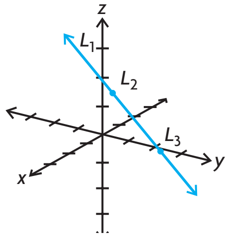
The lines form a triangle



b. Lines meet in a point



c. Three coincident lines



14. $(a - c, -a + b + c, a - b)$

15. **a.** $k = 2$

b. $k = -2$

c. $k \neq \pm 2$

Section 9.3, pp. 516–517

1. **a.** The two equations represent planes that are parallel and not coincident.

b. Answers may vary. For example: $x - y + z = 1, x - y + z = -2$

2. **a.** $x = \frac{1}{2} + \frac{1}{2}s - t, y = s, z = t;$ $s, t \in \mathbf{R}$; the two planes are coincident.

b. Answers may vary. For example: $x - y + z = -1,$ $2x - 2y + 2z = -2$

3. **a.** $x = 1 + s, y = s, z = -2, s \in \mathbf{R}$; the two planes intersect in a line.

b. Answers may vary. For example: $x - y + z = -1, x - y - z = 3$

4. **a.** $m = \frac{1}{2}, p = 2q, q = 1$, and $p = 2$; The value for m is unique, but p just has to be twice q and arbitrary values can be chosen.

b. $m = \frac{1}{2}, q = 1$, and $p = 3$; The value for m is unique, but p and q can be arbitrarily chosen as long as $p \neq 2q$.

c. $m = -20$; This value is unique, since only one value was found to satisfy the given conditions.

d. $m = -20, p = 1, q = 1$; The value for m is unique from the solution to **c.**, but the values for p and q can be arbitrary since the only value which can change the angle between the planes is m.

5. **a.** $x = 9s, y = -3s, z = s, s \in \mathbf{R}$

b. $x = -3t, y = t, z = -\frac{1}{3}t, t \in \mathbf{R}$

c. Since t is an arbitrary real number, we can express t as part b. $t = -3s$, $s \in \mathbf{R}$.

6. **a.** yes; plane

b. no

c. yes; line

d. yes; line

e. yes; line

f. yes; line

7. **a.** $x = 1 - s - t, y = s, z = t, s, t \in \mathbf{R}$

b. no solution

c. $x = -2s, y = -2, z = s, s \in \mathbf{R}$

d. $x = -s + 5, y = -s - 1, z = s,$ $s \in \mathbf{R}$

e. $x = \frac{5}{4}s, y = s, z = 1 - \frac{3}{4}s, s \in \mathbf{R}$

f. $x = s - 8, y = s, z = 4, s \in \mathbf{R}$

8. **a.** The system will have an infinite number of solutions for any value of k.

b. No, there is no value of k for which the system will not have a solution.

9. $\vec{r}_2 = (-2, 3, 6) + s(-5, -8, 2),$ $s \in \mathbf{R}$

10. The line of intersection of the two planes,

$x = 1 - 2s, y = 2 - 2s, z = s; s \in \mathbf{R};$

$5x + 3y + 16z - 11 = 0$

$5(1 - 2s) + 3(2 - 2s) +$

$16(s) - 11 = 0$

$5 + 6 - 11 - 10s - 6s + 16s = 0$

$0 = 0$

Since this is true, the line is contained in the plane.

11. **a.** $x = 1 + s, y = 1 + s, z = s, s \in \mathbf{R}$

b. about 1.73

12. $8x + 14y - 3z - 8 = 0$

Mid-Chapter Review, pp. 518–519

1. **a.** $(-2, 6, 0)$
b. $(2, 0, 10)$
c. $(0, 3, 5)$

2. **a, c.** Answers may vary. For example:
$x = 2 + 3t, y = 1 + 3t,$
$z = 3 - 2t, t \in \mathbf{R};$
$x = 3 + t, y = -2, z = 5\ t \in \mathbf{R};$
$x = -8 + 7t, y = -5 + 3t,$
$z = 7 - 2t, t \in \mathbf{R}$
b. $(-1, -2, 5)$
d. $C\colon x = -8 + 7t, y = -5 + 3t,$
$z = 7 - 2t, t \in \mathbf{R}$
$t = 1$
$x = -8 + 7(1), y = -5 + 3(1),$
$z = 7 - 2(1)$
$x = -1, y = -2, z = 5$
$(-1, -2, 5)$
e. $(-1, -2, 5)$

3. **a.** $\vec{r} = (-7, 20, 0) + t(0, -2, 1),$
$t \in \mathbf{R}$
b. $\vec{r} = \left(-\frac{19}{7}, \frac{30}{7}, 0\right) + t(3, 3, -7),$
$t \in \mathbf{R}$
c. $(-7, 0, 10)$

4. **a.** $x = -\frac{11t}{5} - \frac{1}{40}, y = -\frac{2t}{5} - \frac{117}{40},$
$z = t, t \in \mathbf{R}$
b. $x = -\frac{11}{5}s + \frac{227}{5}, y = -\frac{2}{5}s + \frac{94}{5},$
$z = s, t \in \mathbf{R}$
c. The lines found in 4.a. and 4.b. do not intersect, because they are in parallel and distinct planes.

5. **a.** $a = 3$
b. $a = -3$
c. $a \neq \pm 3, a \in \mathbf{R}$

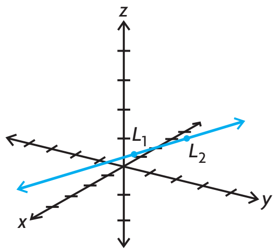
6. Since there is no t-value that satisfies the equations, there is no intersection, and these lines are skew.

7. **a.** no intersection
b. The lines are skew.

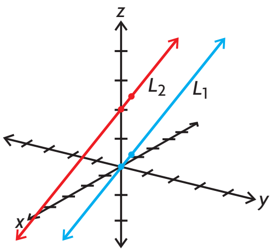
8. $(-3, 6, 6)$

9. **a.** $(3, 1, 2)$
b. $t \in \mathbf{R}$
These lines are the same, so either one of these lines can be used as their intersection.

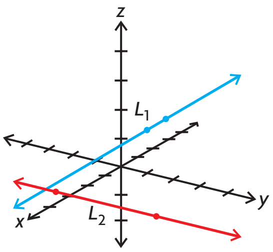
10. **a.** Answers may vary. For example:
i. coincident



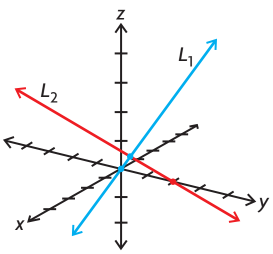
ii. parallel and distinct



iii. skew



iv. intersect in a point



b. **i.** When lines are the same, they are a multiple of each other.
ii. When lines are parallel, one equation is a multiple of the other equation, except for the constant term.
iii. When lines are skew, there are no common solutions to make each equation consistent.
iv. When the solution meets in a point, there is only one unique solution for the system.

11. **a.** when the line lies in the plane
b. Answers may vary. For example:
$\vec{r} = t(3, -5, -3), t \in \mathbf{R};$
$\vec{r} = t(3, -5, -3) + s(1, 1, 1),$
$t, s \in \mathbf{R}$

12. **a.** $(3, 8)$
b. no solution
c. $(2, 1, 4)$

13. **a.** The two lines intersect at a point.
b. The two planes are parallel and do not meet.
c. The three planes intersect at a point.

14. **a.** $\left(-\frac{1}{2}, -\frac{3}{2}, -\frac{3}{2}\right)$
b. $\theta = 90°$
c. $2x - y + z + 1 = 0$

Section 9.4, pp. 530–533

1. **a.** $(-9, -5, -4)$
b. This solution is the point at which all three planes meet.

2. **a.** Answers may vary. For example,
$3x - 3y + 3z = 12$ and
$2x - 2y + 2z = 8.$
b. These three planes are intersecting in one single plane because all three equations can be changed into one equivalent equation. They are coincident planes.
c. $x = t, y = s, z = s - t + 4, s, t \in \mathbf{R}$
d. $y = t, z = s, x = t - s + 4, s, t \in \mathbf{R}$

3. **a.** Answers may vary. For example,
$2x - y + 3z = -2, x - y + 4z = 3,$
and $3x - 2y + 7z = 2;$
$2x - y + 3z = -2, x - y + 4z = 3,$
and $2x - 2y + 8z = 5.$
b. no solutions

4. **a.** $\left(-3, \frac{11}{4}, -\frac{3}{2}\right)$
b. This solution is the point at which all three planes meet.

5. **a.** Since equation ③ = equation ②, equation ② and equation ③ are consistent or lie in the same plane. Equation ① meets this plane in a line.
b. $x = 0, y = t,$ and $z = 1 + t, t \in \mathbf{R}$

6. If you multiply equation ② by 5, you obtain a new equation, $5x - 5y + 15z = -1005$, which is inconsistent with equation ③.

7. **a.** Yes, when this equation is alone, this is true.
b. Answers may vary. For example:
$x + y + z = 2$
$2x + 2y + 2z = 4$
$3x + 3y + 3z = 12$

8. a. $(-1, -1, 0)$ is the point at which the three planes meet.
   b. $(-6, \frac{1}{2}, 3)$ is the point at which the three planes meet.
   c. $(-99, 100, -101)$ is the point at which the three planes meet.
   d. $(4, 2, 3)$ is the point at which the three planes meet.
9. a. $x = -\frac{1}{7}t - \frac{9}{7}, y = -\frac{15}{7} + \frac{3}{7}t$, and $z = t, t \in \mathbf{R}$; the planes intersect in a line.
   b. no solution
   c. $x = -t, y = 2$, and $z = t, t \in \mathbf{R}$; the planes intersect in a line.
10. a. $x = 0, y = t - 2$, and $z = t, t \in \mathbf{R}$
    b. $x = \frac{t - 3s}{2}, y = t$, and $z = s$, $s, t \in \mathbf{R}$
11. a. ① $x + y + z = 1$
    ② $x - 2y + z = 0$
    ③ $x - y + z = 0$
    Equation ① − equation ③ =
    Equation ④ $= 2y = 1$ or $y = \frac{1}{2}$
    Equation ② − equation ③ =
    Equation ⑤ $= -y = 0$ or $y = 0$
    Since the y-variable is different in Equation ④ and Equation ⑤, the system is inconsistent and has no solution.
    b. Answers may vary. For example:
    $\vec{n_1} = (1, 1, 1)$
    $\vec{n_2} = (1, -2, 1)$
    $\vec{n_3} = (1, -1, 1)$
    $m_1 = \vec{n_1} \times \vec{n_2} = (3, 0, -3)$
    $m_2 = \vec{n_1} \times \vec{n_3} = (2, 0, -2)$
    $m_3 = \vec{n_2} \times \vec{n_3} = (-1, 0, 1)$
    c. The three lines of intersection are parallel and coplanar, so they form a triangular prism.
    d. Since $(\vec{n_1} \times \vec{n_2}) \cdot \vec{n_3} = 0$, a triangular prism forms.
12. a. Equation ① and equation ② have the same set of coefficients and variables; however, equations ① equals 3, while equation ② equals 6, which means there is no possible solution.
    b. All three equations equal different numbers, so there is no possible solution.
    c. Equation ② equals 18, while equation ③ equals 17, which means there is no possible solution.
    d. The coefficients of equation ① are half the coefficients of equation ③, but the constant term is not half the other constant term.
13. a. $(4, 3, -5)$
    b. $x = \frac{t - 2}{3}, y = \frac{5t + 5}{3}, z = t, t \in \mathbf{R}$
    c. $x = 0, y = t, z = t, t \in \mathbf{R}$
    d. no solution
    e. $x = -t, y = 2, z = t, t \in \mathbf{R}$
    f. $(0, 0, 0)$
14. a. $p = q = 5$
    b. $x = -\frac{2}{3}t + 3, y = \frac{1}{3}t - 2, z = t$, $t \in \mathbf{R}$
15. a. $m = 2$
    b. $m \neq \pm 2, m \in \mathbf{R}$
    c. $m = -2$
16. $(3, 6, 2)$

Section 9.5, pp. 540–541

1. a. $\frac{3}{5}$
   b. $\frac{56}{13}$ or 4.31
   c. $\frac{236}{\sqrt{1681}}$ or 5.76
2. a. $\frac{5}{\sqrt{5}}$ or 2.24
   b. $\frac{504}{25}$ or 20.16
3. a. 1.4
   b. about 3.92
   c. about 2.88
4. a. $d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$
   If you substitute the coordinates $(0, 0)$, the formula changes to $d = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}}$, which reduces to $d = \frac{|C|}{\sqrt{A^2 + B^2}}$.
   b. $\frac{24}{5}$
   c. $\frac{24}{5}$; the answers are the same
5. a. 3
   b. $\frac{7}{5}$ or 1.4
   c. $\frac{4}{\sqrt{13}}$ or 1.11
   d. $\frac{240}{13}$ or 18.46
6. a. about 1.80
   b. about 2.83
   c. about 3.44
7. a. about 2.83
   b. about 3.28
8. a. $\left(\frac{17}{11}, \frac{7}{11}, \frac{16}{11}\right)$
   b. about 1.65
9. about 3.06; $\left(-\frac{11}{14}, \frac{5}{14}, \frac{22}{14}\right)$
10. $\left(\frac{38}{21}, -\frac{44}{21}, \frac{167}{21}\right)$
11. a. about 1.75
    b. D and G
    c. about 3.61 units^2

Section 9.6, pp. 549–550

1. a. Yes, the calculations are correct. Point A lies in the plane.
   b. The answer 0 means that the point lies in the plane.
2. a. 3 c. 2 e. $\frac{11}{27}$ or 0.41
   b. 3 d. $\frac{5}{13}$ or 0.38
3. a. 5
   b. $6x + 8y - 24z + 13 = 0$
   c. Answers may vary. For example: $\left(-\frac{1}{6}, 0, \frac{1}{2}\right)$
4. a. 4 b. 4 c. 2
5. $\frac{2}{3}$ or 0.67
6. 3
7. about 1.51
8. a. about 3.46
   b. $U(1, 1, 2)$ is the point on the first line that produces the minimal distance to the second line at point $V(-1, -1, 0)$.

Review Exercise, pp. 552–555

1. $-\frac{4}{99}$
2. no solution
3. a. no solution
   b. $(99, 100, 101)$
4. a. All four points lie on the plane $3x + 4y - 2z + 1 = 0$
   b. about 0.19
5. a. 3
   b. $\frac{1}{12}$ or 0.08
6. $\vec{r} = (3, 1, 1) + t(2, -1, 2), t \in \mathbf{R}$
7. a. no solution
   b. no solution
   c. no solution

8. a. $x = -\frac{5}{7}t, y = 1 + \frac{2}{7}t, z = t, t \in \mathbf{R}$
b. $x = 3, y = \frac{1}{4}, z = -\frac{1}{2}$
c. $x = 3t - 3s + 7, y = t, z = s,$ $s, t \in \mathbf{R}$

9. a. $x = \frac{1}{2} + \frac{1}{36}t, y = -\frac{1}{2} + \frac{5}{12}t,$ $z = t, t \in \mathbf{R}$
b. $x = \frac{9}{8} - \frac{31}{24}t, y = \frac{1}{4} + \frac{1}{12}t, z = t,$ $t \in \mathbf{R}$

10. a. These three planes meet at the point $(-1, 5, 3)$.
b. The planes do not intersect. Geometrically, the planes form a triangular prism.
c. The planes meet in a line through the origin, with equation $x = t,$ $y = -7t, z = -5t, t \in \mathbf{R}$

11. 4.90

12. a. $x - 2y + z + 4 = 0$
$\vec{r} = (3, 1, -5) + s(2, 1, 0), s \in \mathbf{R}$
$\vec{m} \times \vec{n} = (2, 1, 0)(1, -2, 1) = 0$
Since the line's direction vector is perpendicular to the normal of the plane and the point $(3, 1, -5)$ lies on both the line and the plane, the line is in the plane.
b. $(-1, -1, -5)$
c. $x - 2y + z + 4 = 0$
$-1 - 2(-1) + (-5) + 4 = 0$
The point $(-1, -1, -5)$ is on the plane since it satisfies the equation of the plane.
d. $7x - 2y - 11z - 50 = 0$

13. a. 5.48
b. $(3, 0, -1)$

14. a. $(-2, -3, 0)$.
b. $\vec{r} = (-2, -3, 0) + t(1, -2, 1),$ $t \in \mathbf{R}$

15. a. $-10x + 9y + 8z + 16 = 0$
b. about 0.45

16. a. 1
b. $\vec{r} = (0, 0, -1) + t(4, 3, 7), t \in \mathbf{R}$

17. a. $x = 2, y = -1, z = 1$
b. $x = 7 - 3t, y = 3 - t, z = t, t \in \mathbf{R}$

18. $a = \frac{2}{3}, b = \frac{3}{4}, c = \frac{1}{2}$

19. $\left(4, -\frac{7}{4}, \frac{7}{2}\right)$

20. $\left(-\frac{5}{3}, \frac{8}{3}, \frac{4}{3}\right)$

21. a. $\vec{r} = \left(\frac{45}{4}, 0, -\frac{21}{4}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = \left(-\frac{37}{2}, 0, \frac{15}{2}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = (7, 0, -1) + t(11, 2, -5),$ $t \in \mathbf{R}; z = -1 - 5t, t \in \mathbf{R}$
b. All three lines of intersection found in part a. have direction vector $(11, 2, -5)$, and so they are all parallel. Since no pair of normal vectors for these three planes is parallel, no pair of these planes is coincident.

22. $\left(\frac{1}{2}, 1, \frac{1}{3}\right), \left(\frac{1}{2}, 1, -\frac{1}{3}\right), \left(\frac{1}{2}, -1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right), \left(-\frac{1}{2}, 1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right) \left(-\frac{1}{2}, 1, -\frac{1}{3}\right)$, and $\left(-\frac{1}{2}, -1, \frac{1}{3}\right)$

23. $y = \frac{7}{6}x^2 - \frac{3}{2}x - \frac{2}{3}$

24. $\left(\frac{29}{7}, \frac{4}{7}, -\frac{33}{7}\right)$

25. $A = 5, B = 2, C = -4$

26. a. $\vec{r} = (-1, -4, -6) + t(-5, -4, -3), t \in \mathbf{R}$
b. $\left(\frac{13}{2}, 2, -\frac{3}{2}\right)$
c. about 33.26 units^2

27. $6x - 8y + 9z - 115 = 0$

Chapter 9 Test, p. 556

1. a. $(3, -1, -5)$
b. $3 - (-1) + (-5) + 1 = 0$
$3 + 1 - 5 + 1 = 0$
$0 = 0$

2. a. $\frac{13}{12}$ or 1.08
b. $\frac{40}{3}$ or 13.33

3. a. $x = \frac{4t}{5}, y = 1 - \frac{t}{5}, z = t, t \in \mathbf{R}$
b. $(4, 0, 5)$

4. a. $(1, -5, 4)$
b. The three planes intersect at the point $(1, -5, 4)$.

5. a. $x = -\frac{1}{2} - \frac{t}{4}, y = \frac{3t}{4} + \frac{1}{2}, z = t,$ $t \in \mathbf{R}$
b. The three planes intersect at this line.

6. a. $m = -1, n = -3$
b. $x = -1, y = 1 - t, z = t, t \in \mathbf{R}$

7. 10.20

Cumulative Review of Vectors, pp. 557–560

1. a. about 111.0°
b. scalar projection: $-\frac{14}{13}$, vector projection: $\left(-\frac{52}{169}, \frac{56}{169}, -\frac{168}{169}\right)$
c. scalar projection: $-\frac{14}{3}$, vector projection: $\left(-\frac{28}{9}, \frac{14}{9}, \frac{28}{9}\right)$

2. a. $x = 8 + 4t, y = t, z = -3 - 3t,$ $t \in \mathbf{R}$
b. about 51.9°

3. a. $\frac{1}{2}$
b. 3
c. $\frac{3}{2}$

4. a. $-7\vec{i} - 19\vec{j} - 14\vec{k}$
b. 18

5. x-axis: about 42.0°, y-axis: about 111.8°, z-axis: about 123.9°

6. a. $(-7, -5, -1)$
b. $(-42, -30, -6)$
c. about 8.66 square units
d. 0

7. $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

8. a. vector equation: Answers may vary. $\vec{r} = (2, -3, 1) + t(-1, 5, 2), t \in \mathbf{R};$ parametric equation: $x = 2 - t, y = -3 + 5t,$ $z = 1 + 2t, t \in \mathbf{R}$
b. If the x-coordinate of a point on the line is 4, then $2 - t = 4$, or $t = -2$. At $t = -2$, the point on the line is $(2, -3, 1) - 2(-1, 5, 2) = (4, -13, -3)$. Hence, $C(4, -13, -3)$ is a point on the line.

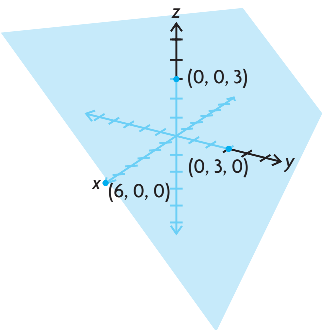
9. The direction vector of the first line is $(-1, 5, 2)$ and of the second line is $(1, -5, -2) = -(-1, 5, 2)$. So they are collinear and hence parallel.
The lines coincide if and only if for any point on the first line and second line, the vector connecting the two points is a multiple of the direction vector for the lines. $(2, 0, 9)$ is a point on the first line and $(3, -5, 10)$ is a point on the second line.
$(2, 0, 9) - (3, -5, 10) = (-1, 5, -1) \neq k(-1, 5, 2)$ for $k \in \mathbf{R}$. Hence, the lines are parallel and distinct.

10. vector equation:
$\vec{r} = (0, 0, 4) + t(0, 1, 1), t \in \mathbf{R}$;
parametric equation: $x = 0, y = t$,
$z = 4 + t, t \in \mathbf{R}$

11. -13

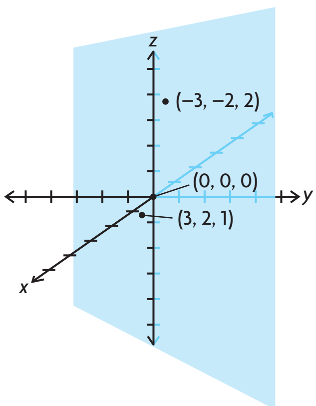
12. $\left(\frac{3}{2}, -\frac{31}{6}, \frac{13}{6}\right)$

13. **a.**



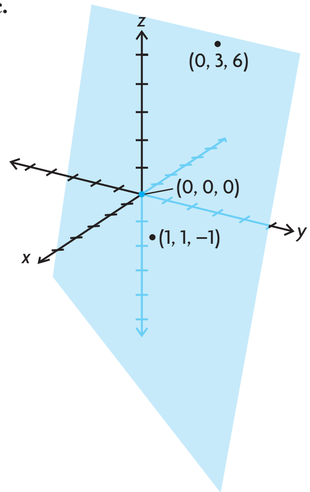
Answers may vary. For example, $(0, 3, -3)$ and $(6, 0, -3)$.

b.



Answers may vary. For example, $(-3, -2, 2)$ and $(3, 2, 1)$.

c.



Answers may vary. For example, $(0, 3, 6)$ and $(1, 1, -1)$.

14. $(-7, 10, 20)$

15. $\vec{q} = (1, 0, 2) + t(-11, 7, 2), t \in \mathbf{R}$

16. **a.** $12x - 9y - 6z + 24 = 0$
b. about 1.49 units

17. **a.** $3x - 5y + 4z - 7 = 0$
b. $x - y + 12z - 27 = 0$
c. $z - 3 = 0$
d. $x + 2z + 1 = 0$

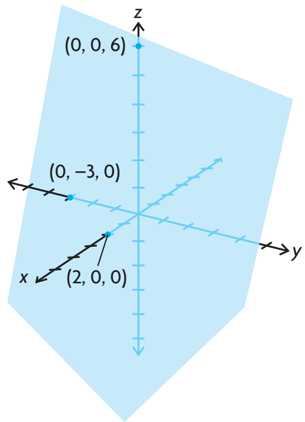
18. 336.80 km/h, N 12.1° W

19. **a.** $\vec{r} = (0, 0, 6) + s(1, 0, -3) + t(0, 1, 2), s, t \in \mathbf{R}$. To verify, find the Cartesian equation corresponding to the above vector equation and see if it is equivalent to the Cartesian equation given in the problem. A normal vector to this plane is the cross product of the two directional vectors.
$\vec{n} = (1, 0, -3) \times (0, 1, 2)$

$= (0(2) - (-3)(1), -3(0) - 1(2), 1(1) - 0(0))$
$= (3, -2, 1)$

So the plane has the form $3x + 2y + z + D = 0$, for some constant D. To find D, we know that $(0, 0, 6)$ is a point on the plane, so $3(0) - 2(0) + (6) + D = 0$. So, $6 + D = 0$, or $D = -6$. So, the Cartesian equation for the plane is $3x - 2y + z - 6 = 0$. Since this is the same as the initial Cartesian equation, the vector equation for the plane is correct.

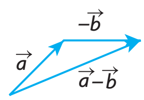
b.



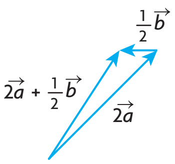
20. **a.** 16°
b. The two planes are perpendicular if and only if their normal vectors are also perpendicular. A normal vector for the first plane is $(2, -3, 1)$ and a normal vector for the second plane is $(4, -3, -17)$. The two vectors are perpendicular if and only if their dot product is zero.
$(2, -3, 1) \cdot (4, -3, -17)$
$= 2(4) - 3(-3) + 1(-17)$
$= 0$
Hence, the normal vectors are perpendicular. Thus, the planes are perpendicular.
c. The two planes are parallel if and only if their normal vectors are also parallel. A normal vector for the first plane is $(2, -3, 2)$ and a normal vector for the second plane is $(2, -3, 2)$. Since both normal vectors are the same, the planes are parallel. Since $2(0) - 3(-1) + 2(0) - 3 = 0$, the point $(0, -1, 0)$ is on the second plane. Yet since $2(0) - 3(-1) + 2(0) - 1 = 2 \neq 0$, $(0, -1, 0)$ is not on the first plane. Thus, the two planes are parallel but not coincident.

21. resultant: about 56.79 N, 37.6° from the 25 N force toward the 40 N force, equilibrant: about 56.79 N, 142.4° from the 25 N force away from the 40 N force

22. **a.**



b.



23. **a.** $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

b. $\left(-\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$

24. **a.** $\overrightarrow{OC} = (8, 9)$,
$\overrightarrow{BD} = (10, -5)$

b. about 74.9°

c. about 85.6°

25. **a.** $x = t, y = -1 + t, z = 1, t \in \mathbf{R}$

b. $(1, 2, -3)$

c. $x = 1, y = t, z = -3 + t, t \in \mathbf{R}$

d. $x = 1 + 3s + t, y = t, z = s,$
$s, t \in \mathbf{R}$

26. **a.** yes; $x = 0, y = -1 + t, z = t, t \in \mathbf{R}$

b. no

c. yes;
$x = 2 - 2t, y = t, z = 3t, t \in \mathbf{R}$

27. 30°

28. **a.** $-\frac{3}{2}$

b. 84

29. $\vec{r} = t(-1, 3, 1), t \in \mathbf{R}$,
$-x + 3y + z - 11 = 0$

30. $(-1, 1, 0)$

31. **a.** 0.8 km

b. 12 min

32. **a.** Answers may vary.
$\vec{r} = (6, 3, 4) + t(4, 4, 1), t \in \mathbf{R}$

b. The line found in part a will lie in the plane $x - 2y + 4z - 16 = 0$ if and only if both points $A(2, -1, 3)$ and $B(6, 3, 4)$ lie in this plane. We verify this by substituting these points into the equation of the plane, and checking for consistency.
For A:
$2 - 2(-1) + 4(3) - 16 = 0$
For B:
$6 - 2(3) + 4(4) - 16 = 0$
Since both points lie on the plane, so does the line found in part a.

33. 20 km/h at N 53.1° E

34. parallel: 1960 N,
perpendicular: about 3394.82 N

35. **a.** True; all non-parallel pairs of lines intersect in exactly one point in $\mathbf{R}^2$. However, this is not the case for lines in $\mathbf{R}^3$ (skew lines provide a counterexample).

b. True; all non-parallel pairs of planes intersect in a line in $\mathbf{R}^3$.

c. True; the line $x = y = z$ has direction vector $(1, 1, 1)$, which is not perpendicular to the normal vector $(1, -2, 2)$ to the plane $x - 2y + 2z = k$, k is any constant. Since these vectors are not perpendicular, the line is not parallel to the plane, and so they will intersect in exactly one point.

d. False; a direction vector for the line $\frac{x}{2} = y - 1 = \frac{z + 1}{2}$ is $(2, 1, 2)$. A direction vector for the line $\frac{x - 1}{-4} = \frac{y - 1}{-2} = \frac{z + 1}{-2}$ is $(-4, -2, -2)$, or $(2, 1, 1)$ (which is parallel to $(-4, -2, -2)$). Since $(2, 1, 2)$ and $(2, 1, 1)$ are obviously not parallel, these two lines are not parallel.

36. **a.** A direction vector for
L_1: $x = 2, \frac{y - 2}{3} = z$ is $(0, 3, 1)$,
and a direction vector for
L_2: $x = y + k = \frac{z + 14}{k}$ is $(1, 1, k)$.
But $(0, 3, 1)$ is not a nonzero scalar multiple of $(1, 1, k)$ for any k, since the first component of $(0, 3, 1)$ is 0. This means that the direction vectors for L_1 and L_2 are never parallel, which means that these lines are never parallel for any k.

b. 6; $(2, -4, -2)$

Calculus Appendix

Implicit Differentiation, p. 564

1. The chain rule states that if y is a composite function, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$. To differentiate an equation implicitly, first differentiate both sides of the equation with respect to x, using the chain rule for terms involving y, then solve for $\frac{dy}{dx}$.

2. **a.** $-\frac{x}{y}$

b. $\frac{x^2}{5y}$

c. $\frac{-y^2}{2xy + y^2}$

d. $\frac{9x}{16y}$

e. $-\frac{13x}{48y}$

f. $-\frac{2x}{2y + 5}$

3. **a.** $y = \frac{2}{3}x - \frac{13}{3}$

b. $y = \frac{2}{3}(x + 8) + 3$

c. $y = -\frac{3\sqrt{3}}{5}x - 3$

d. $y = \frac{11}{10}(x + 11) - 4$

4. $(0, 1)$

5. **a.** 1

b. $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(-\frac{3}{\sqrt{5}}, -\sqrt{5}\right)$

6. -10

7. $7x - y - 11 = 0$

8. $y = \frac{1}{2}x - \frac{3}{2}$

9. **a.** $\frac{4}{(x + y)^2} - 1$

b. $4\sqrt{x + y} - 1$

10. **a.** $\frac{3x^2 - 8xy}{4x^2 - 3}$

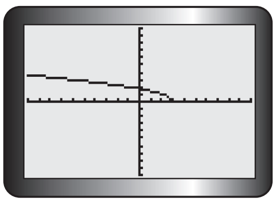
b. $y = \frac{x^3}{4x^2 - 3}; \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

c. $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$

$y = \frac{x^3}{4x^2 - 3}$

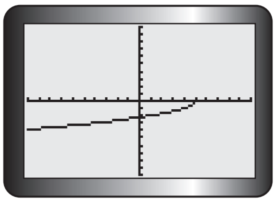
$$\frac{dy}{dx} = \frac{3x^2 - 8x\left(\frac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$$
$$= \frac{3x^2 - (4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

11. **a.**



one tangent

b.



one tangent