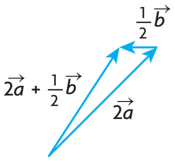
b.



23. **a.** $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

b. $\left(-\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$

24. **a.** $\overrightarrow{OC} = (8, 9)$, $\overrightarrow{BD} = (10, -5)$

b. about 74.9°

c. about 85.6°

25. **a.** $x = t, y = -1 + t, z = 1, t \in \mathbf{R}$

b. $(1, 2, -3)$

c. $x = 1, y = t, z = -3 + t, t \in \mathbf{R}$

d. $x = 1 + 3s + t, y = t, z = s$, $s, t \in \mathbf{R}$

26. **a.** yes; $x = 0, y = -1 + t, z = t, t \in \mathbf{R}$

b. no

c. yes; $x = 2 - 2t, y = t, z = 3t, t \in \mathbf{R}$

27. 30°

28. **a.** $-\frac{3}{2}$

b. 84

29. $\vec{r} = t(-1, 3, 1), t \in \mathbf{R}$, $-x + 3y + z - 11 = 0$

30. $(-1, 1, 0)$

31. **a.** 0.8 km

b. 12 min

32. **a.** Answers may vary. $\vec{r} = (6, 3, 4) + t(4, 4, 1), t \in \mathbf{R}$

b. The line found in part a will lie in the plane $x - 2y + 4z - 16 = 0$ if and only if both points $A(2, -1, 3)$ and $B(6, 3, 4)$ lie in this plane. We verify this by substituting these points into the equation of the plane, and checking for consistency.
For A:
$2 - 2(-1) + 4(3) - 16 = 0$
For B:
$6 - 2(3) + 4(4) - 16 = 0$
Since both points lie on the plane, so does the line found in part a.

33. 20 km/h at N 53.1° E

34. parallel: 1960 N, perpendicular: about 3394.82 N

35. **a.** True; all non-parallel pairs of lines intersect in exactly one point in $\mathbf{R}^2$. However, this is not the case for lines in $\mathbf{R}^3$ (skew lines provide a counterexample).

b. True; all non-parallel pairs of planes intersect in a line in $\mathbf{R}^3$.

c. True; the line $x = y = z$ has direction vector $(1, 1, 1)$, which is not perpendicular to the normal vector $(1, -2, 2)$ to the plane $x - 2y + 2z = k$, k is any constant. Since these vectors are not perpendicular, the line is not parallel to the plane, and so they will intersect in exactly one point.

d. False; a direction vector for the line $\frac{x}{2} = y - 1 = \frac{z+1}{2}$ is $(2, 1, 2)$. A direction vector for the line $\frac{x-1}{-4} = \frac{y-1}{-2} = \frac{z+1}{-2}$ is $(-4, -2, -2)$, or $(2, 1, 1)$ (which is parallel to $(-4, -2, -2)$). Since $(2, 1, 2)$ and $(2, 1, 1)$ are obviously not parallel, these two lines are not parallel.

36. **a.** A direction vector for L_1: $x = 2, \frac{y-2}{3} = z$ is $(0, 3, 1)$, and a direction vector for L_2: $x = y + k = \frac{z+14}{k}$ is $(1, 1, k)$. But $(0, 3, 1)$ is not a nonzero scalar multiple of $(1, 1, k)$ for any k, since the first component of $(0, 3, 1)$ is 0. This means that the direction vectors for L_1 and L_2 are never parallel, which means that these lines are never parallel for any k.

b. 6; $(2, -4, -2)$

Calculus Appendix

Implicit Differentiation, p. 564

1. The chain rule states that if y is a composite function, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$. To differentiate an equation implicitly, first differentiate both sides of the equation with respect to x, using the chain rule for terms involving y, then solve for $\frac{dy}{dx}$.

2. **a.** $-\frac{x}{y}$

b. $\frac{x^2}{5y}$

c. $\frac{-y^2}{2xy + y^2}$

d. $\frac{9x}{16y}$

e. $-\frac{13x}{48y}$

f. $-\frac{2x}{2y + 5}$

3. **a.** $y = \frac{2}{3}x - \frac{13}{3}$

b. $y = \frac{2}{3}(x + 8) + 3$

c. $y = -\frac{3\sqrt{3}}{5}x - 3$

d. $y = \frac{11}{10}(x + 11) - 4$

4. $(0, 1)$

5. **a.** 1

b. $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(-\frac{3}{\sqrt{5}}, -\sqrt{5}\right)$

6. -10

7. $7x - y - 11 = 0$

8. $y = \frac{1}{2}x - \frac{3}{2}$

9. **a.** $\frac{4}{(x + y)^2} - 1$

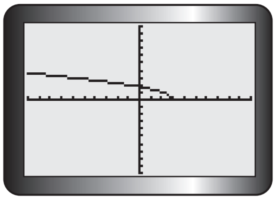
b. $4\sqrt{x + y} - 1$

10. **a.** $\frac{3x^2 - 8xy}{4x^2 - 3}$

b. $y = \frac{x^3}{4x^2 - 3}; \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

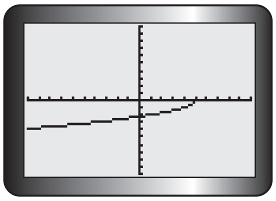
c.
$$\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$
$$y = \frac{x^3}{4x^2 - 3}$$
$$\frac{dy}{dx} = \frac{3x^2 - 8x\left(\frac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$$
$$= \frac{3x^2 - (4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

11. **a.**



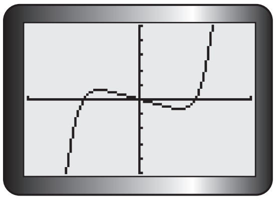
one tangent

b.



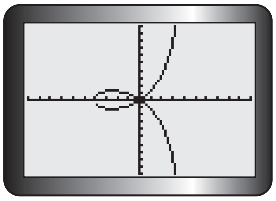
one tangent

c.



one tangent

d.



two tangents

12. $\frac{1}{2}\left(\frac{x}{y}\right)^{-\frac{1}{2}} \frac{1y - \frac{dy}{dx}x}{y^2}$

$+ \frac{1}{2}\left(\frac{y}{x}\right)^{-\frac{1}{2}} \frac{\frac{dy}{dx}x - y}{x^2} = 0$

$\frac{y^{\frac{1}{2}}}{2x^{\frac{1}{2}}} \frac{1y - \frac{dy}{dx}x}{y^2} + \frac{x^{\frac{1}{2}}}{2y^{\frac{1}{2}}} \frac{\frac{dy}{dx}x - y}{x^2} = 0$

Multiply by $2x^2y^2$:

$x^{\frac{3}{2}}y^{\frac{1}{2}}\left(y - x\frac{dy}{dx}\right) + x^{\frac{1}{2}}y^{\frac{3}{2}}\left(\frac{dy}{dx}x - y\right) = 0$

$x^{\frac{3}{2}}y^{\frac{3}{2}} - x^{\frac{5}{2}}y^{\frac{1}{2}}\frac{dy}{dx} + x^{\frac{3}{2}}y^{\frac{3}{2}}\frac{dy}{dx} - x^{\frac{1}{2}}y^{\frac{5}{2}} = 0$

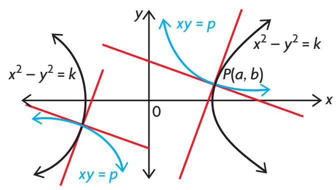
$\frac{dy}{dx}\left(x^{\frac{3}{2}}y^{\frac{3}{2}} - x^{\frac{5}{2}}y^{\frac{1}{2}}\right) = x^{\frac{1}{2}}y^{\frac{5}{2}} - x^{\frac{3}{2}}y^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{x^{\frac{1}{2}}y^{\frac{3}{2}}(y - x)}{x^{\frac{3}{2}}y^{\frac{1}{2}}(y - x)}$

$\frac{dy}{dx} = \frac{y}{x}$, as required.

13. $2x - 3y + 10 = 0$ and $x = 4$

14.



Let $P(a, b)$ be the point of intersection where $a \neq 0$ and $b \neq 0$.

For $x^2 - y^2 = k$,

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

At $P(a, b)$,

$\frac{dy}{dx} = \frac{a}{b}$

For $xy = P$,

$1 \cdot y + \frac{dy}{dx}x = P$

$\frac{dy}{dx} = -\frac{y}{x}$

At $P(a, b)$,

$\frac{dy}{dx} = -\frac{b}{a}$

At point $P(a, b)$, the slope of the tangent line of $xy = P$ is the negative reciprocal of the slope of the tangent line of $x^2 - y^2 = k$. Therefore, the tangent lines intersect at right angles, and thus, the two curves intersect orthogonally for all values of the constants k and P.

15. $\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}y^{\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Let $P(a, b)$ be the point of tangency.

$\frac{dy}{dx} - \frac{\sqrt{b}}{\sqrt{a}}$

Equation on tangent line l and P is

$\frac{y - b}{x - a} = -\frac{\sqrt{b}}{\sqrt{a}}$.

x-intercept is found when $y = 0$.

$\frac{-b}{x - a} = -\frac{\sqrt{b}}{\sqrt{a}}$

$-b\sqrt{a} = -\sqrt{b}x + a\sqrt{b}$

$x = \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}}$

Therefore, the x-intercept is $\frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}}$.

For the y-intercept, let $x = 0$,

$\frac{y - b}{-a} = -\frac{\sqrt{b}}{\sqrt{a}}$.

y-intercept is $\frac{a\sqrt{b}}{\sqrt{a}} + b$.

The sum of the intercepts is

$\frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}} + \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{a}}$

$= \frac{a^{\frac{3}{2}}b^{\frac{1}{2}} + 2ab + b^{\frac{3}{2}}a^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$

$= \frac{a^{\frac{1}{2}}b^{\frac{1}{2}}(a + 2\sqrt{a}\sqrt{b} + b)}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$

$= a + 2\sqrt{a}\sqrt{b} + b$

$= (a^{\frac{1}{2}} + b^{\frac{1}{2}})^2$

Since $P(a, b)$ is on the curve, then $\sqrt{a} + \sqrt{b} = \sqrt{k}$, or $a^{\frac{1}{2}} + b^{\frac{1}{2}} = k^{\frac{1}{2}}$.

Therefore, the sum of the intercepts is $(k^{\frac{1}{2}})^2 = k$, as required.

16. $(x + 2)^2 + (y - 5)^5 = 18$ and $(x - 4)^2 + (y + 1)^2 = 18$

Related Rates, pp. 569–570

1. **a.** $\frac{dA}{dt} = 4\ \text{m/s}^2$

b. $\frac{dS}{dt} = -3\ \text{m}^2\text{/min}$

c. $\frac{ds}{dt} = 70$ km/h, when $t = 0.25$

d. $\frac{dx}{dt} = \frac{dy}{dt}$

e. $\frac{d\theta}{dt} = \frac{\pi}{10}$ rad/s

2. **a.** decreasing at 5.9 °C/s

b. about 0.58 m

c. Solve $T''(x) = 0$.

3. area increasing at 100 cm^2/s; perimeter increasing at 20 cm/s

4. **a.** increasing at 300 cm^3/s

b. increasing at 336 cm^2/s

5. increasing at 40 cm^2/s

6. **a.** $\frac{5}{6\pi}$ km/h

b. $\frac{5}{3\pi}$ m/s

7. $\frac{1}{\pi}$ km/h

8. 4 m/s

9. 8 m/min

10. 214 m/s

11. $5\sqrt{13}$ km/h

12. **a.** $\frac{1}{72\pi}$ cm/s

b. $\frac{2}{49\pi}$ cm/s or about 0.01 cm/s

c. $\frac{1}{8\pi}$ cm/s or about 0.04 cm/s

13. $\frac{50}{\pi}$ cm/min; 94.25 min (or about 1.5 h)

14. Answers may vary. For example:

a. The diameter of a right-circular cone is expanding at a rate of 4 cm/min. Its height remains constant at 10 cm. Find its radius when the volume is increasing at a rate of 80π cm^3/min.

b. Water is being poured into a right-circular tank at the rate of 12π m^3/min. Its height is 4 m and its radius is 1 m. At what rate is the water level rising?

c. The volume of a right-circular cone is expanding because its radius is increasing at 12 cm/min and its height is increasing at 6 cm/min. Find the rate at which its volume is changing when its radius is 20 cm and its height is 40 cm.

15. 0.145π m^3/year

16. $\frac{2}{\pi}$ cm/min

17. $\frac{\sqrt{3}}{4}$ m/min

18. 144 m/min

19. 62.8 km/h

20. a. $\frac{4}{5\pi}$ cm/s
 b. $\frac{8}{25\pi}$ cm/s

21. a. $x^2 + y^2 = \left(\frac{l}{2}\right)^2$
 b. $\frac{y^2}{k^2} + \frac{y^2}{(l-k)^2} = 1$

The Natural Logarithm and its Derivative, p. 575

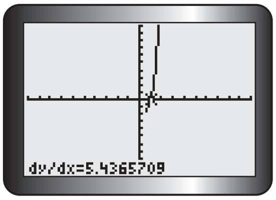
1. A natural logarithm has base e; a common logarithm has base 10.

2. Since $e = \lim_{h\to 0}(1+h)^{\frac{1}{h}}$, let $h = \frac{1}{n}$.
 Therefore,
 $e = \lim_{\frac{1}{n}\to 0}\left(1+\frac{1}{n}\right)^n.$
 But as $\frac{1}{n}\to 0$, $n\to\infty$.
 Therefore, $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$.
 If $n = 100$, $e \doteq \left(1+\frac{1}{100}\right)^{100}$
 $= 1.01^{100}$
 $\doteq 2.704\ 81$
 Try $n = 100\ 000$, etc.

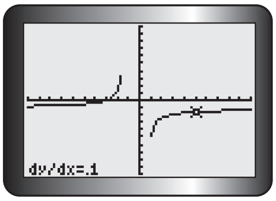
3. a. $\frac{5}{5x+8}$
 b. $\frac{2x}{x^2+1}$
 c. $\frac{15}{t}$
 d. $\frac{1}{2(x+1)}$
 e. $\frac{3t^2-4t}{t^3-2t^2+5}$
 f. $\frac{2z+3}{2(z^2+3z)}$

4. a. $\ln x + 1$
 b. 1
 c. $e^t \ln t + \frac{e^t}{t}$
 d. $\frac{-ze^{-z}}{e^{-z}+ze^{-z}}$
 e. $\frac{te^t\ln t - e^t}{t(\ln t)^2}$
 f. $\frac{1}{2}e^{\sqrt{u}}\left(\frac{1}{2}e^{\sqrt{u}}\ln u + \frac{1}{u}\right)$

5. a. $2e$
 b. 0.1
 c.



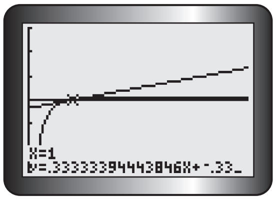
The value shown is approximately $2e$, which matches the calculation in part a.



This value matches the calculation in part b.

6. a. $x = 0$
 b. no solution
 c. $x = 0, \pm\sqrt{e-1}$

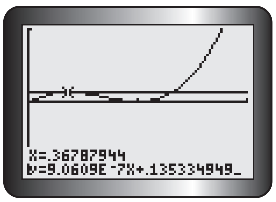
7. a. $x - 3y - 1 = 0$
 b.

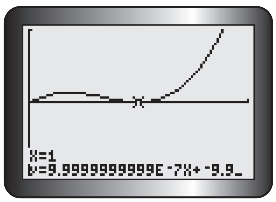


 c. The equation on the calculator is in a different form, but is equivalent to the equation in part a.

8. $x - 2y + (2\ln 2 - 4) = 0$

9. a. $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ and $(1, 0)$
 b.





 c. The solution in part a is more precise and efficient.

10. $y = -\frac{1}{2}x + \ln 2$

11. a. 90 km/h
 b. $\frac{-90}{3t+1}$
 c. about -12.8 km/h/s
 d. 6.36 s

12. $\frac{1}{2}$

13. a. $\frac{1}{x\ln x}$
 b. The function's domain is $\{x\in\mathbf{R} \mid x > 1\}$.
 The domain of the derivative is $\{x\in\mathbf{R} \mid x > 0 \text{ and } x \neq 1\}$.

The Derivatives of General Logarithmic Functions, p. 578

1. a. $\frac{1}{x\ln 5}$
 b. $\frac{1}{x\ln 3}$
 c. $\frac{2}{x\ln 4}$
 d. $\frac{-3}{x\ln 7}$
 e. $\frac{-1}{x\ln 10}$
 f. $\frac{3}{x\ln 6}$

2. a. $\frac{1}{(x+2)\ln 3}$
 b. $\frac{1}{x\ln 8}$
 c. $\frac{-6}{(2x+3)\ln 3}$
 d. $\frac{-2}{(5-2x)\ln 10}$
 e. $\frac{2}{(2x+6)\ln 8} = \frac{1}{(x+3)\ln 8}$
 f. $\frac{2x+1}{(x^2+x+1)\ln 7}$

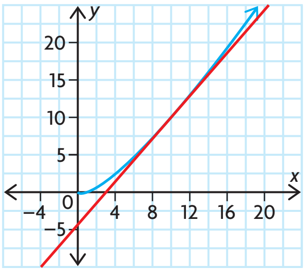
3. a. $\frac{5}{52\ln 2}$
 b. $\frac{1}{8\log_2(8)(\ln 3)(\ln 2)}$

4. a. $\frac{2}{(1-x^2)\ln 10}$
 b. $\frac{2x+3}{2(x^2+3x)\ln(2)}$
 c. $\frac{2\ln 5 - \ln 4}{\ln 3}$
 d. $\frac{x\ln 3(3^x)(\ln x) + 3^x}{x\ln 3}$

e. $\dfrac{\ln x + 1}{\ln 2}$

f. $\dfrac{4x + 1 - x\ln(3x^2)}{2x\ln 5(x+1)^{\frac{3}{2}}}$

5. $y = 1.434x - 4.343$



6. $y = \log_a kx$

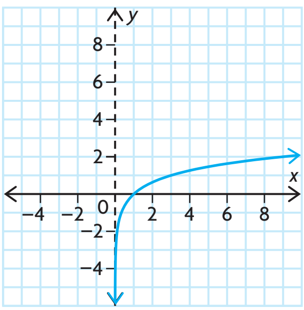
$$\frac{dy}{dx} = \frac{f'(x)}{f(x)\ln(a)} = \frac{k}{kx\ln(a)} = \frac{1}{x\ln(a)}$$

7. $y = 49.1x - 235.5$

8. Since the derivative is positive at $t = 15$, the distance is increasing at that point.

9. a. $y = 0.1x + 1.1$

b.



vertical asymptote at $x = 0$

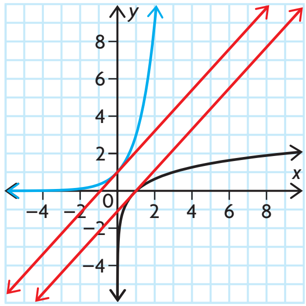
c. The tangent line will intersect this asymptote because it is defined for $x = 0$.

10. $D = \{x \in \mathbf{R} | x < -2 \text{ or } x > 2\}$; critical number at $x = 0$, $x = 2$, and $x = -2$; function is decreasing for $x < -2$ and increasing for $x > 2$

11. a. point of inflection at $x = 0$

b. $x = 0$ is a possible point of inflection. Since the graph is always concave up, there is no point of inflection.

12. The slope of $y = \log_3 x$ at $(1, 0)$ is $\frac{1}{\ln 3}$. Since $\ln 3 > 1$, the slope of $y = 3^x$ at $(0, 1)$ is greater than the slope of $y = \log_3 x$ at $(1, 0)$.



Logarithmic Differentiation, p. 582

1. a. $\sqrt{10}x^{\sqrt{10}-1}$

b. $15\sqrt{2}\,x^{3\sqrt{2}-1}$

c. $\pi t^{\pi - 1}$

d. $ex^{e-1} + e^x$

2. a. $\dfrac{2x^{\ln x}\ln x}{x}$

b. $\dfrac{(x+1)(x-3)^3}{(x+2)^3} \times \left(\dfrac{1}{x+1} + \dfrac{2}{x-3} - \dfrac{3}{x+2}\right)$

c. $\left(x^{\sqrt{x}}\right)\dfrac{\ln x + 2}{2\sqrt{x}}$

d. $\left(\dfrac{1}{t}\right)^t\left(\ln\dfrac{1}{t} - 1\right)$

3. a. $2e^e$

b. $e^2 + e \cdot 2^{e-1}$

c. $-\dfrac{4}{27}$

4. $y = 32(2\ln 2 + 1)(x - 128\ln 2 - 48)$

5. $-\dfrac{11}{36}$

6. $(e, e^{\frac{1}{e}})$

7. $(1, 1)$ and $(2, 4 + 4\ln 2)$

8. $\dfrac{32(\ln 4 + 1)^2}{\ln 4 + 2}$

9. $\dfrac{1}{8}$

10. $\left(\dfrac{x\sin x}{x^2 - 1}\right)^2 \times \left(\dfrac{2(\sin x + x\cos x)}{x\sin x} - \dfrac{4x}{x^2 - 1}\right)$

11. $x^{\cos x}\left(\sin x \ln x + \dfrac{\cos x}{x}\right)$

12. $y = x$

13. a. $v(t) = t^{\frac{1}{t}}\left(\dfrac{1 - \ln t}{t^2}\right)$,

$a(t) = \dfrac{t^{\frac{1}{t}}}{t^4}[1 - 2\ln t + (\ln t)^2 + 2t\ln t - 3t]$

b. $t = e$; $a(e) = -e^{\frac{1}{e} - 3}$

14. Using a calculator, $e^\pi \doteq 23.14$ and $\pi^e \doteq 22.46$. So, $e^\pi > \pi^e$.

Vector Appendix

Gaussian Elimination, pp. 588–590

1. a. $\left[\begin{array}{rrr|r} 1 & 2 & -1 & -1 \\ -1 & 3 & -2 & -1 \\ 0 & 3 & -2 & -3 \end{array}\right]$

b. $\left[\begin{array}{rrr|r} 2 & 0 & -1 & 1 \\ 0 & 2 & -1 & 16 \\ -3 & 1 & 0 & 10 \end{array}\right]$

c. $\left[\begin{array}{rrr|r} 2 & -1 & -1 & -2 \\ 1 & -1 & 4 & -1 \\ -1 & -1 & 0 & 13 \end{array}\right]$

2. Answers may vary. For example:

$\left[\begin{array}{rr|r} 1 & 1.5 & 0 \\ 0 & -5.5 & 1 \end{array}\right]$

$\left[\begin{array}{rr|r} 2 & 3 & 0 \\ 0 & -5.5 & 1 \end{array}\right]$

3. Answers may vary. For example:

$\left[\begin{array}{rrr|r} 2 & 1 & 6 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -37 & 4 \end{array}\right]$

4. a. Answers may vary. For example:

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -36 & 16 \end{array}\right]$

b. $x = -\dfrac{22}{9}, y = -\dfrac{8}{9}, z = -\dfrac{4}{9}$

5. a. $x - 2y = -1$
$2x - 3y = 1$
$2x - y = 0$

b. $-2x - z = 0$
$x - 2y = 4$
$y + 2z = -3$

c. $-z = 0$
$x = -2$
$y + z = 0$

6. a. $x = -\dfrac{9}{2}, y = -3$

b. $x = 13, y = 9, z = -6$

c. no solution

d. $x = -\dfrac{9}{4}, y = -4, z = -5$