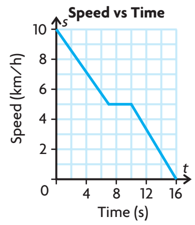
9\. a) Answers may vary. For example:

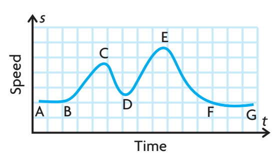



b) $-\frac{5}{7}$ km/h/s

c) From (7, 5) to (12, $\frac{10}{3}$), the rate of change of speed in $-\frac{1}{3}$ km/h/s

d) $-\frac{5}{6}$ km/h/s

10\. The roller coaster moves at a slow steady speed between A and B. At B, it begins to accelerate as it moves down to C. Going uphill from C to D it decelerates. At D, it starts to move down and accelerates to E, where the speed starts to decrease until F, where it maintains a slower speed to G, the end of the track.



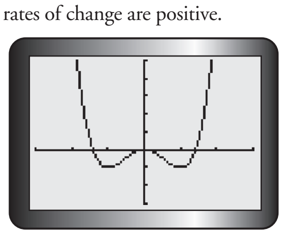

11\. a) minimum
b) maximum
c) maximum
d) minimum
e) minimum
f) maximum

12\. a) i) $m = h - 26$ ii) $m = -4h - 48$
b) i) $m = -26$ ii) $m = -48$

13\. a) To the left of a maximum, the instantaneous rates of change are positive. To the right, the instantaneous rates of change are negative.

b) To the left of a minimum, the instantaneous rates of change are negative. To the right, the instantaneous rates of change are positive.

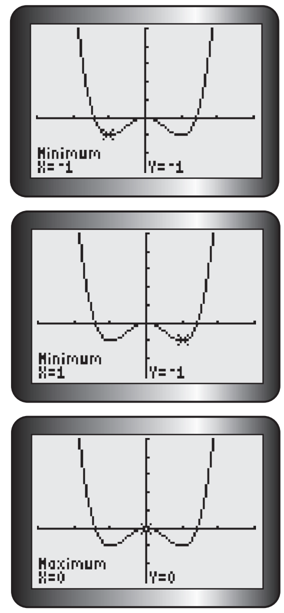
14\. a)



b) minimum: $x = -1$, $x = 1$
maximum: $x = 0$

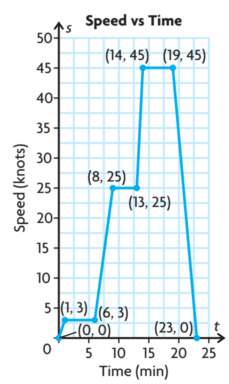
c) The slopes of tangent lines for points to the left of a minimum will be negative, while the slopes of tangent lines for points to the right of a minimum will be positive. The slopes of tangent lines for points to the left of a maximum will be positive, while the slopes of tangent lines for points to the right of a minimum will be negative.

d)




## Chapter Self-Test, p. 118

1\. a)




b) 11 kn/min; 0 kn/min; the two different average rates of change indicate that the boat was increasing its speed from $t = 6$ to $t = 8$ at a rate of 11knots/min and moving at a constant speed from $t = 8$ to $t = 13$.

c) 11 kn/min

2\. a) $-1$

b) The hot cocoa is cooling by 1 °C/min on average.

c) $-0.75$

d) The hot cocoa is cooling by 0.75 °C/min after 30 min.

e) The rate decreases over the interval, until it is nearly 0 and constant.

3\. a) \$310 per dollar spent

b) $-$\$100 per dollar spent

c) The positive sign for part a) means that the company is increasing its profit when it spends between \$8000 and \$10 000 on advertising. The negative sign means the company's profit is decreasing when it spends \$50 000 on advertising.

4\. a) $-1$; 0 (minimum); 7

b) 4.5; $-4.5$; 0 (maximum)

# Chapter 3

## Getting Started, p. 122

1\. a) $6x^3 - 22x^2$
b) $x^2 + 2x - 24$
c) $24x^3 - 44x^2 - 40x$
d) $5x^3 + 31x^2 - 68x + 32$

2\. a) $(x + 7)(x - 4)$
b) $2(x - 2)(x - 7)$

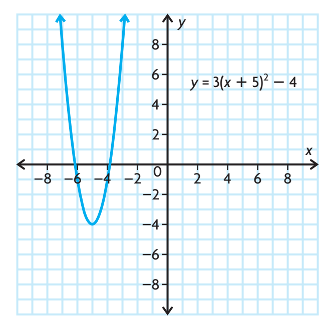
3\. a) $x = -6$
b) $x = -3, 4.5$
c) $x = -3, -8$
d) $x = \frac{1}{3}, -4$

4\. a) vertical compression by a factor of $\frac{1}{4}$; horizontal translation 3 units to the right; vertical translation 9 units up

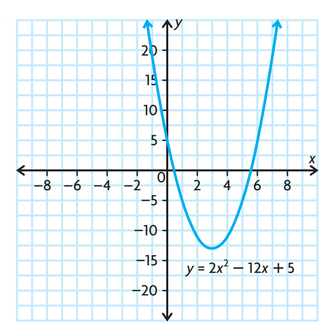
b) vertical compression by a factor of $\frac{1}{4}$; vertical translation 7 units down

5\. a) $y = 2(x - 5)^2 - 2$
b) $y = -2x^2 + 3$

6\. a)

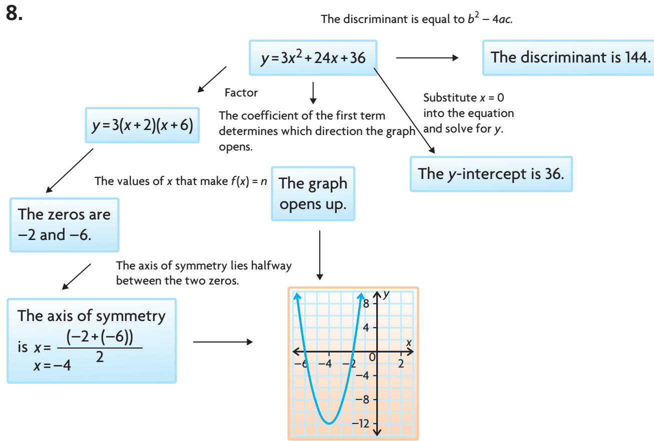



b)




7\. a) quadratic
b) other
c) other
d) linear

**8.**




## Lesson 3.1, pp. 127–128

**1.** **a)** This represents a polynomial function because the domain is the set of all real numbers, the range does not have a lower bound, and the graph does not have horizontal or vertical asymptotes.

**b)** This represents a polynomial function because the domain is the set of all real numbers, the range is the set of all real numbers, and the graph does not have horizontal or vertical asymptotes.

**c)** This is not a polynomial function because it has a horizontal asymptote.

**d)** This represents a polynomial function because the domain is the set of all real numbers, the range does not have an upper bound, and the graph does not have horizontal or vertical asymptotes.

**e)** This is not a polynomial function because its domain is not all real numbers.

**f)** This is not a polynomial function because it is a periodic function.

**2.** **a)** polynomial; the exponents of the variables are all natural numbers

**b)** polynomial; the exponents of the variables are all natural numbers

**c)** polynomial; the exponents of the variables are all natural numbers

**d)** other; the variable is under a radical sign

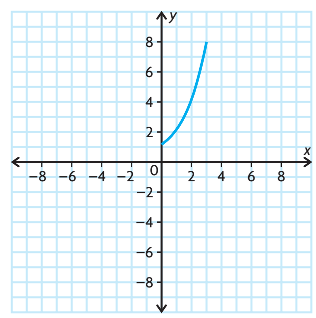
**e)** other; the function contains another function in the denominator

**f)** polynomial; the exponents of the variables are all natural numbers

**3.** **a)** linear **c)** linear

**b)** quadratic **d)** cubic

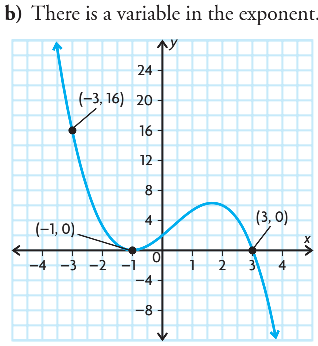
**4.**



**a)** The graph looks like one half of a parabola, which is the graph of a quadratic equation.

**b)** There is a variable in the exponent.

**5.**




**6.** Answers may vary. For example, any equation of the form $y = a\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ will have the same zeros, but have a different $y$-intercept and a different value for $f(-3)$. Any equation of the form $y = x\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ would have two of the same zeros, but a different value for $f(-3)$ and different positive/negative intervals.

**7.** $y = x + 5, y = x^2 + 5,$
$y = x^3 + 5, y = x^4 + 5$

**8.** Answers may vary. For example:

| Definition | Characteristics |
|---|---|
| A polynomial is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and $n$ is a whole number. | The domain of the function is all real numbers, but the range can have restrictions; except for polynomial functions of degree zero (whose graphs are horizontal lines), the graphs of polynomials do not have horizontal or vertical asymptotes. The shape of the graph depends on its degree. |
| **Polynomials** | |
| **Examples** | **Non-Examples** |
| $x^2 + 4x + 6$ | $\sqrt{x} + 1$ |

## Lesson 3.2, pp. 136–138

**1.** **a)** 4; −4; as $x \to +/-\infty, y \to -\infty$

**b)** 5; 2; as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$

**c)** 3; −3; as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$

**d)** 4; 24; as $x \to +/-\infty, y \to \infty$

**2.** **a)** Turning points

**a)** minimum 1, maximum 3

**b)** minimum 0, maximum 4

**c)** minimum 0, maximum 2

**d)** minimum 1, maximum 3

**b)** Zeros

**a)** minimum 0, maximum 4

**b)** minimum 1, maximum 5

**c)** minimum 1, maximum 3

**d)** minimum 0, maximum 4

**3.** **i)** **a)** The degree is even.

**b)** The leading coefficient is negative.

**ii)** **a)** The degree is even.

**b)** The leading coefficient is negative.

**iii)** **a)** The degree is odd.

**b)** The leading coefficient is negative.

**iv)** **a)** The degree is even.

**b)** The leading coefficient is positive.

**v)** **a)** The degree is odd.

**b)** The leading coefficient is negative.

**vi)** **a)** The degree is odd.

**b)** The leading coefficient is positive.

**4.** **a)** as $x \to +/-\infty, y \to \infty$

**b)** as $x \to -\infty, y \to \infty$ and as $x \to \infty$, $y \to -\infty$

**c)** as $x \to -\infty, y \to -\infty$ and as $x \to \infty$, $y \to \infty$

**d)** as $x \to +/-\infty, y \to -\infty$

e) as $x \rightarrow +/-\infty$, $y \rightarrow \infty$

f) as $x \rightarrow -\infty$, $y \rightarrow \infty$ and as $x \rightarrow \infty$, $y \rightarrow -\infty$

**5.** a) D: The graph extends from quadrant III to quadrant I and the $y$-intercept is 2.

b) A: The graph extends from quadrant III to quadrant IV.

c) E: The graph extends from quadrant II to quadrant I and the $y$-intercept is $-5$.

d) C: The graph extends from quadrant II to quadrant I and the $y$-intercept is 0.

e) F: The graph extends from quadrant II to quadrant IV.

f) B: The graph extends from quadrant III to quadrant I and the $y$-intercept is 1.

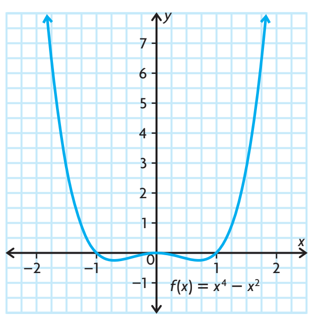
**6.** a) Answers may vary. For example, $f(x) = 2x^3 + 5$.

b) Answers may vary. For example, $f(x) = 6x^2 + x - 4$.

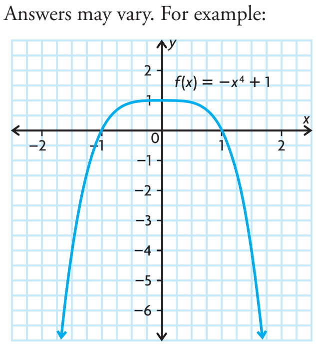
c) Answers may vary. For example, $f(x) = -x^4 - x^3 + 7$.

d) Answers may vary. For example, $f(x) = -9x^5 + x^4 - x^3 - 2$.

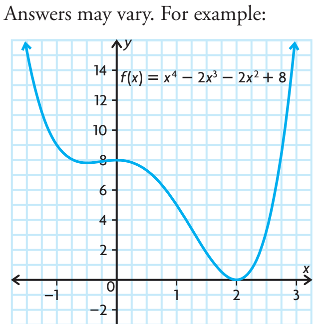
**7.** a) Answers may vary. For example:



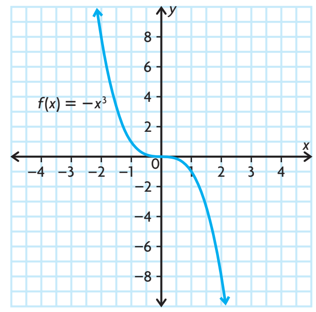

b) Answers may vary. For example:



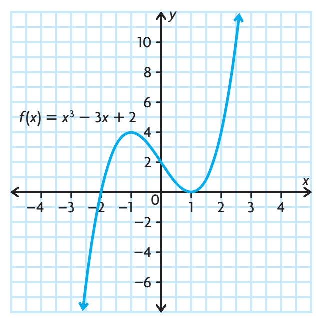

c) Answers may vary. For example:



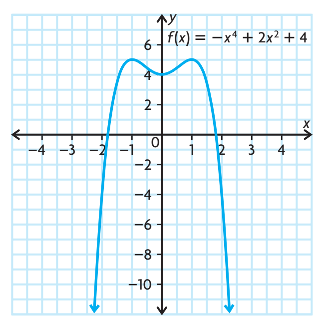

d) Answers may vary. For example:




e) Answers may vary. For example:



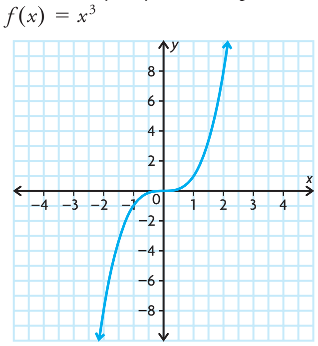

f) Answers may vary. For example:



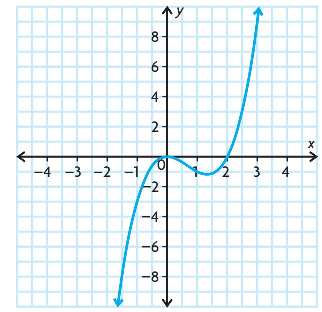

**8.** An odd-degree polynomial can have only local maximums and minimums because the $y$-value goes to $-\infty$ and $\infty$ at each end of the function. An even-degree polynomial can have absolute maximums and minimums because it will go to either $-\infty$ at both ends or $\infty$ at both ends of the function.

**9.** even number of turning points

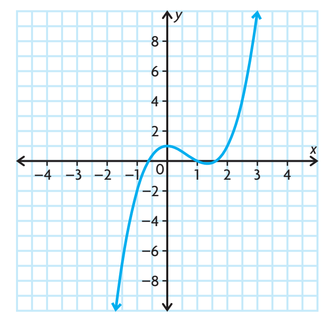
**10.** a) Answers may vary. For example: $f(x) = x^3$



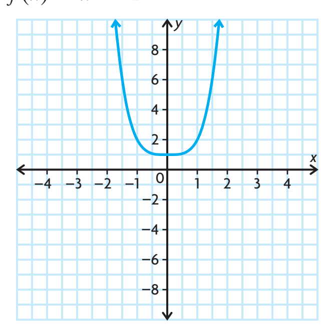
b) Answers may vary. For example: $f(x) = x^3 - 2x^2$



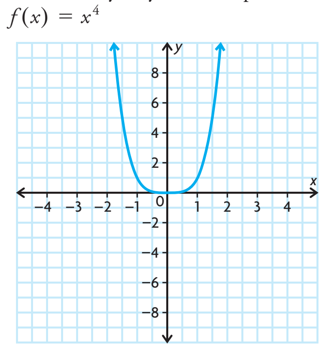
c) Answers may vary. For example: $f(x) = x^3 - 2x^2 + 1$



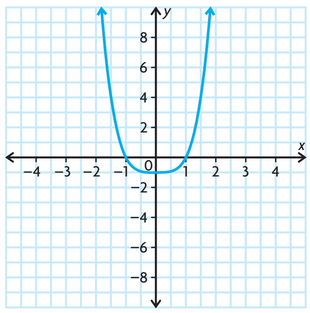
**11.** a) Answers may vary. For example: $f(x) = x^4 + 1$



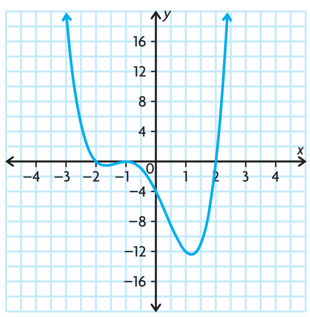
b) Answers may vary. For example: $f(x) = x^4$



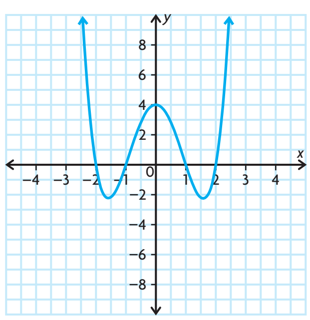
**c)** Answers may vary. For example:
$f(x) = x^4 - 1$



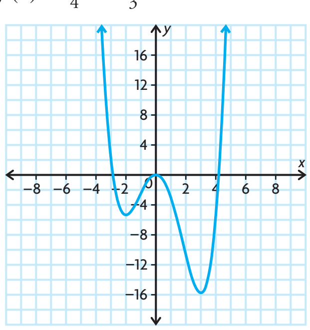
**d)** Answers may vary. For example:
$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$



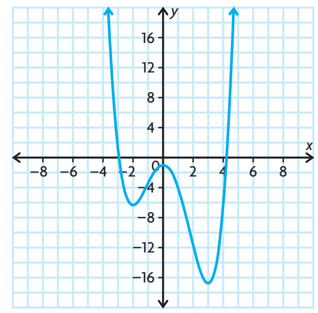
**e)** Answers may vary. For example:
$f(x) = x^4 - 5x^2 + 4$



**12. a)** Answers may vary. For example:
$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$



and $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 - 1$



**b)** zero and leading coefficient of the function

**13. a)** 700 people

**b)** The population will decrease because the leading coefficient is negative.

**14. a)** False; Answers may vary. For example, $f(x) = x^2 + x$ is not an even function.

**b)** True

**c)** False; Answers may vary. For example, $f(x) = x^2 + 1$ has no zeros.

**d)** False; Answers may vary. For example, $f(x) = -x^2$ has end behaviour opposite the behaviour stated.

**15.** Answers may vary. For example, "What are the turning points of the function?", "What is the leading coefficient of the function?", and "What are the zeros of the function?" If the function has 0 turning points or an even number of turning points, then it must extend to the opposite side of the $x$-axis. If it has an odd number of turning points, it must extend to the same side of the $x$-axis. If the leading coefficient is known, it can be determined exactly which quadrants the function extends to/from and if the function has been vertically stretched. If the zeros are known, it can be determined if the function has been vertically translated up or down.

**16. a)** $b = 0$

**b)** $b = 0, d = 0$

## Lesson 3.3, pp. 146–148

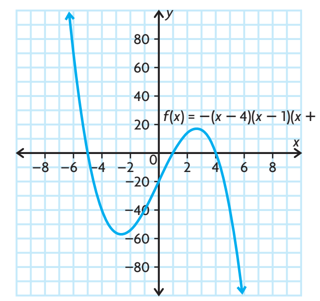
**1. a)** C: The graph has zeros of $-1$ and 3, and it extends from quadrant III to quadrant I.

**b)** A: The graph has zeros of $-1$ and 3, and it extends from quadrant II to quadrant III.

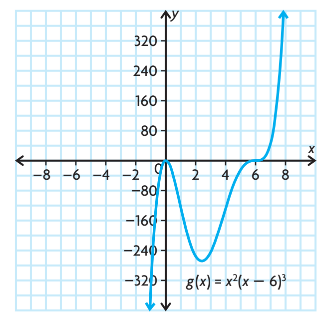
**c)** B: The graph has zeros of $-1$ and 3, and it extends from quadrant II to quadrant IV.

**d)** D: The graph has zeros of $-1$, 0, 3, and 5, and it extends from quadrant II to quadrant I.

**2. a)**

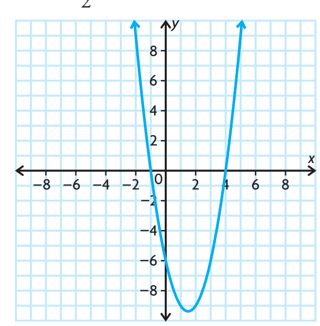



**b)**




**3. a)** $f(x) = k(x + 1)(x - 4)$;
$f(x) = 4(x + 1)(x - 4)$;
$f(x) = -2(x + 1)(x - 4)$

**b)** $f(x) = \frac{3}{2}(x + 1)(x - 4)$

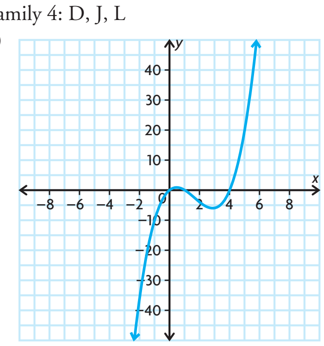


**4. a)** $y = 0.5(x + 3)(x - 2)(x - 5)$

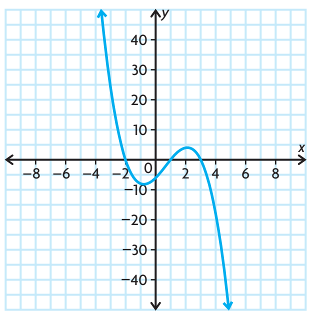
**b)** $y = -(x + 1)^2(x - 2)(x - 4)$

**5.** Family 1: A, G, I
Family 2: B, E
Family 3: C, F, H, K
Family 4: D, J, L

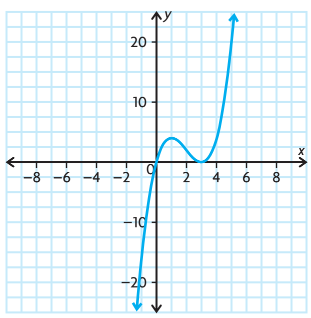
**6. a)**



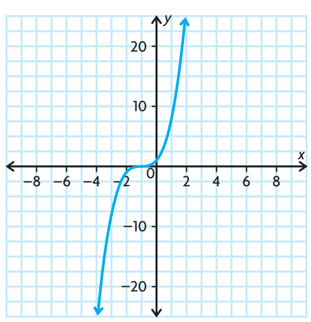
**b)**



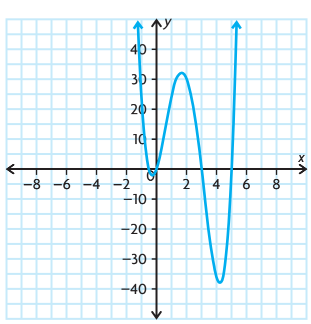

**c)**



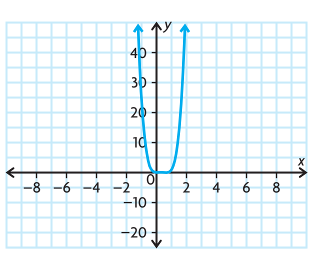

**d)**




**e)**

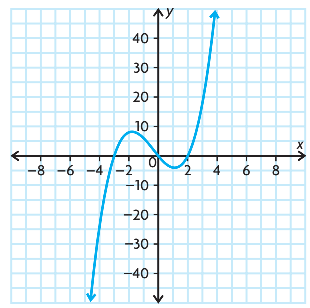



**f)**

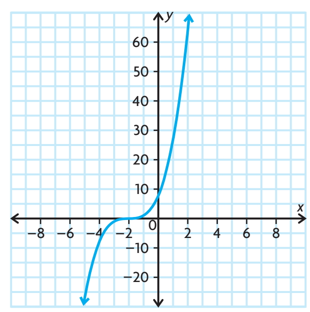



**7.** **a)** Answers may vary. For example:

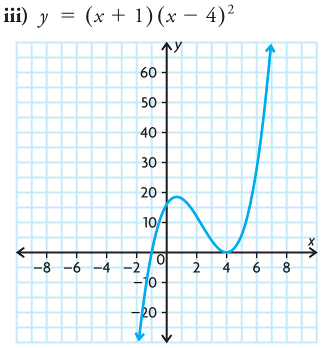
**i)** $y = x(x + 3)(x - 2)$



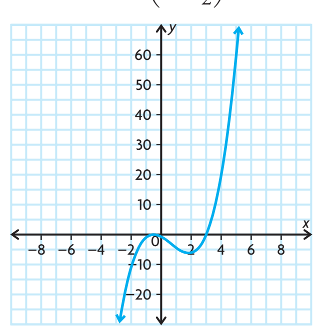

**ii)** $y = (x + 2)^3$




**iii)** $y = (x + 1)(x - 4)^2$



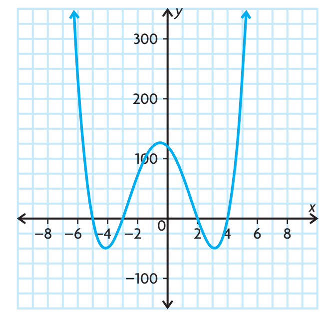

**iv)** $y = (x - 3)\left(x + \frac{1}{2}\right)^2$




**b)** No, as all the functions belong to a family of equations.

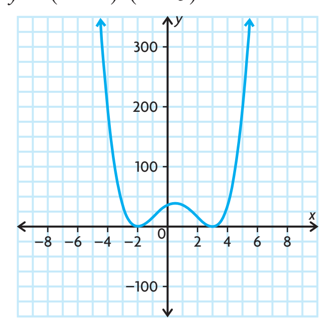
**8.** Answers may vary. For example:

**a)** $y = (x + 5)(x + 3)(x - 2)(x - 4)$



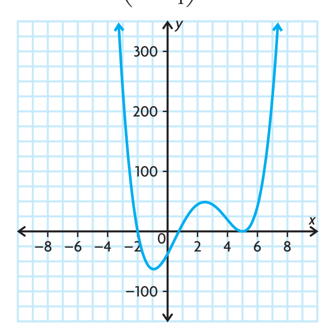

$y = 2(x + 5)(x + 3)(x - 2)(x - 4)$
$y = -5(x + 5)(x + 3)(x - 2)(x - 4)$

**b)** $y = (x + 2)^2(x - 3)^2$




$y = 10(x + 2)^2(x - 3)^2$
$y = 7(x + 2)^2(x - 3)^2$

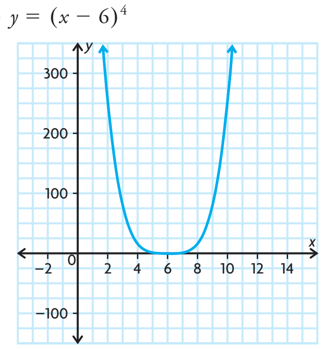
**c)** $y = (x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$




$y = -(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

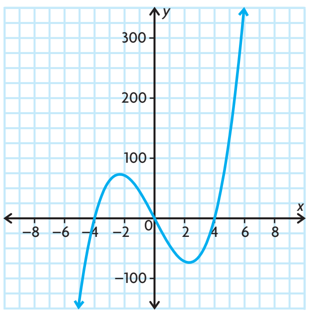
$y = \frac{2}{5}(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

**d)** $y = (x - 6)^4$

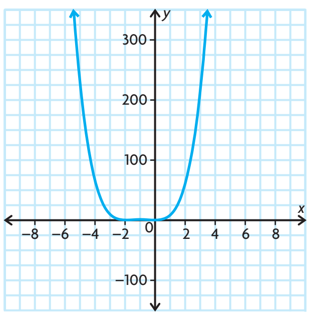



$y = 15(x - 6)^4$
$y = -3(x - 6)^4$

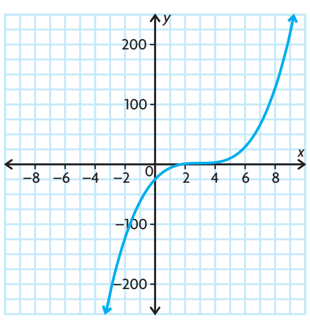
**9.** **a)**



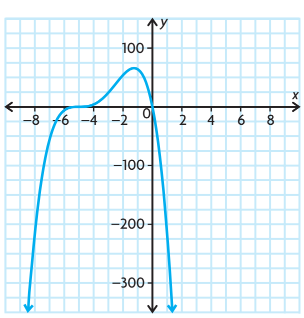
**b)**



**c)**

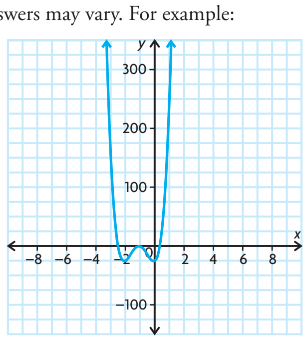


**d)**

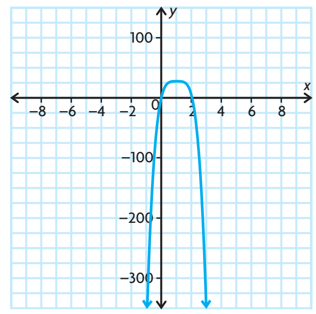


**10.** Answers may vary. For example:

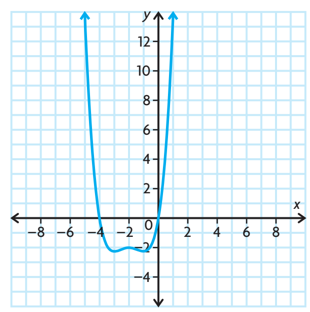
**a)**



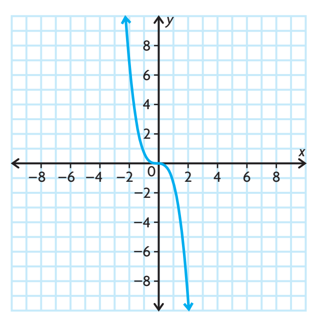
**b)**



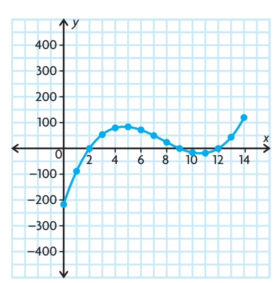
**c)**



**d)**



**11.** **a)**



**b)** $y = (x - 2)(x - 9)(x - 12)$

**c)** No; $\{x \in \mathbf{R} | 0 \leq x \leq 14\}$

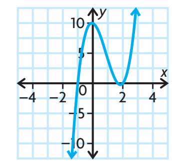
**12.** **a)** $y = x^3 + 2x^2 - x - 2$

**b)** $y = -\frac{2}{5}(x - 1)(x + 2)(x + 4)$

**13.** **a)** $f(x) = -6(x + 3)(x + 5)$

**b)** $f(x) = 2(x + 2)(x - 3)(x - 4)$

**14.** $k = 3$



The zeros are $\frac{5}{3}$, $-1$, and 2.

$f(x) = (3x - 5)(x + 1)(x - 2)$

**15.** **a)** It has zeros at 2 and 4, and it has turning points at 2, 3, and 4. It extends from quadrant II to quadrant I.

**b)** It has zeros at $-4$ and 3, and it has turning points at $-\frac{5}{3}$ and 3. It extends from quadrant III to quadrant I.

**16.** **a)** 832 $cm^3$

**b)** 2.93 cm by 24.14 cm by 14.14 cm or 5 cm by 20 cm by 10 cm

**c)** $0 < x < 10$; The values of $x$ are the side lengths of squares that can be cut from the sheet of cardboard to produce a box with positive volume. Since the sheet of cardboard is 30 cm by 20 cm, the side lengths of a square cut from each corner have to be less than 10 cm, or an entire edge would be cut away, leaving nothing to fold up.

**d)** The square that is cut from each corner must be larger than 0 cm by 0 cm but smaller than 10 cm by 10 cm.

## Lesson 3.4, pp. 155–158

**1.** **a)** B: $y = x^3$ has been vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 1 unit up.

**b)** C: $y = x^3$ has been reflected in the $x$-axis, vertically compressed by a factor of $\frac{1}{3}$, horizontally translated 1 unit to the left, and vertically translated 1 unit down.

**c)** A: $y = x^4$ has been vertically compressed by a factor of 0.2, horizontally translated 4 units to the right, and vertically translated 3 units down.

**d)** D: $y = x^4$ has been reflected in the $x$-axis, vertically stretched by a factor of 1.5, horizontally translated 3 units to the left, and vertically translated 4 units up.

**2.** **a)** $y = x^4$; vertical stretch by a factor of $\frac{5}{4}$ and vertical translation of 3 units up

**b)** $y = x$; vertical stretch by a factor of 3 and vertical translation of 4 units down

**c)** $y = x^3$; horizontal compression by a factor of $\frac{1}{3}$, horizontal translation of $\frac{4}{3}$ units to the left, and vertical translation of 7 units down

**d)** $y = x^4$; reflection in the $x$-axis and horizontal translation of 8 units to the left

**e)** $y = x^2$; reflection in the $x$-axis, vertical stretch by a factor of 4.8, and horizontal translation 3 units left

**f)** $y = x^3$; vertical stretch by a factor of 2, horizontal stretch by a factor of 5, horizontal translation of 7 units to the left, and vertical translation of 4 units down

**3.** **a)** $y = x^3$ has been translated 3 units to the left and 4 units down.
$y = (x + 3)^3 - 4$

**b)** $y = x^4$ has been reflected in the $x$-axis, vertically stretched by a factor of 2, horizontally translated 4 units to the left, and vertically translated 5 units up.
$y = -2(x + 4)^4 + 5$

**c)** $y = x^4$ has been vertically compressed by a factor of $\frac{1}{4}$, horizontally translated 1 unit to the right, and vertically translated 2 units down.
$y = \frac{1}{4}(x - 1)^4 - 2$

**d)** $y = x^3$ has been reflected in the $x$-axis, vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 4 units down.
$y = -2(x - 3)^3 - 4$

**4.** **a)** vertically stretched by a factor of 12, horizontally translated 9 units to the right, and vertically translated 7 units down

**b)** horizontally stretched by a factor of $\frac{8}{7}$, horizontally translated 1 unit to the left, and vertically translated 3 units up

**c)** vertically stretched by a factor of 2, reflected in the $x$-axis, horizontally translated 6 units to the right, and vertically translated 8 units down

**d)** horizontally translated 9 units to the left

**e)** reflected in the $x$-axis, vertically stretched by a factor of 2, reflected in the $y$-axis, horizontally compressed by a factor of $\frac{1}{3}$, horizontally translated 4 units to the right, and vertically translated 5 units down

**f)** horizontally stretched by a factor of $\frac{4}{3}$ and horizontally translated 10 units to the right

**5.** **a)** $y = 8x^2 - 11$
$y = x^2$ was vertically stretched by a factor of 8 and vertically translated 11 units down.

**b)** $y = -\frac{1}{4}x^2 + 1.25$
$y = x^2$ was reflected in the $x$-axis, vertically compressed by a factor of $\frac{1}{4}$, and vertically translated 1.25 units up.

**6.** **a)** $\left(-6\frac{1}{5}, -\frac{1}{2}\right), \left(-6, 0\right), \left(-5\frac{3}{5}, 4\right)$

**b)** $(2, 2), (0, 3), (-4, 11)$

**c)** $\left(3, 2\frac{1}{2}\right), \left(4, -\frac{1}{2}\right), \left(6, -24\frac{1}{2}\right)$

**d)** $\left(-7, -2\frac{1}{10}\right), \left(0, -2\right), \left(14, -1\frac{1}{5}\right)$

**e)** $\left(1, 1\frac{9}{10}\right), \left(0, \frac{9}{10}\right), \left(-2, -7\frac{1}{10}\right)$

**f)** $(-11, -8), (-4, -7), (10, 1)$

**7.** $y = -\frac{1}{4}(x - 1)^4 + 3$

**8.** $(-2, 8), (0, 0), (2, -8)$

**9.** **a)** $-2$ and $-4$

**b)** 4

**c)** $-3$ and 1

**d)** no $x$-intercepts

**e)** 6.68 and 9.32

**f)** $-3.86$

**10.** **a)** 1; $0 = 2(x - 4)^3 + 1$ has only one solution.

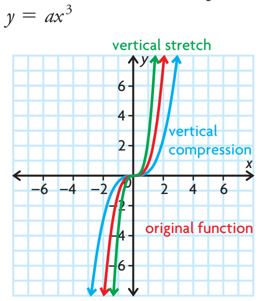
**b)** 0; $0 = 2(x - 4)^4 + 1$ has no solution.

**c)** 1 when $n$ is odd, since an odd root results in only one value; 0 when $n$ is even, since there is no value for an even root of a negative number.

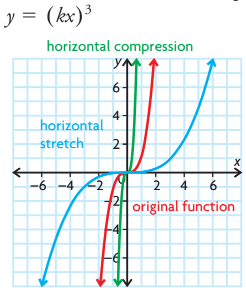
**11.** **a)** The reflection of the function $y = x^n$ in the $x$-axis will be the same as its reflection in the $y$-axis for odd values of $n$.

**b)** The reflections will be different for even values of $n$. The reflection in the $x$-axis will be $y = -x^n$, and the reflection in the $y$-axis will be $y = (-x)^n$. For odd values of $n$, $-x^n$ equals $(-x^n)$. For even values of $n$, $-x^n$ does not equal $(-x^n)$.

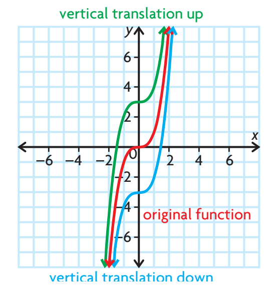
**12.** **a)** Vertical stretch and compression:
$y = ax^3$



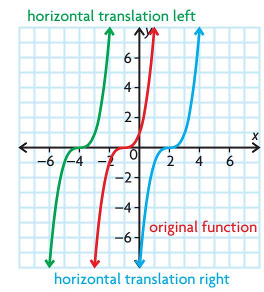

Horizontal stretch and compression:
$y = (kx)^3$



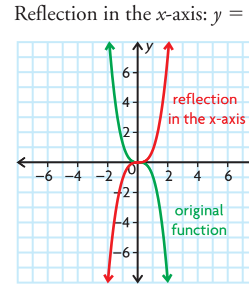

Vertical translation up or down:
$y = x^3 + c$



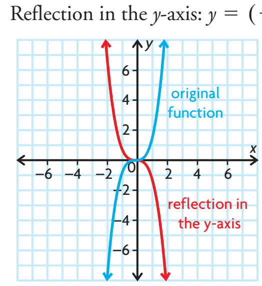

Horizontal translation left or right:
$y = (x - d)^3$




Reflection in the $x$-axis: $y = -x^3$




Reflection in the $y$-axis: $y = (-x)^3$




**b)** When using a table of values to sketch the graph of a function, you may not select a large enough range of values for the domain to produce an accurate representation of the function.

**13.** Yes, you can. The zeros of the first function have the same spacing between them as the zeros of the second function. Also, the ratio of the distances of the two curves above or below the $x$-axis at similar distances between the zeros is always the same. Therefore, the two curves have the same general shape, and one can be transformed into the other.

Answers

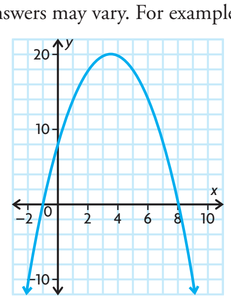
14. $y = (x - 1)^2 (x + 1)^2$ has zeroes at $x = \pm 1$ where the $x$-axis is tangent to these points. $y = 2(x - 1)^2(x + 1)^2 + 1$ is obtained by vertically stretching the original function by a factor of 2 and vertically translating up 1 unit. This results in a new graph that has no zeroes.

15. $f(x) = 5(2(x + 3))^2 + 1$

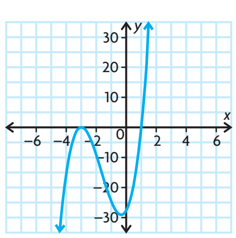
## Mid-Chapter Review, p. 161

1. a) Yes
   b) No; it contains a rational exponent.
   c) Yes
   d) No; it is a rational function.
2. a) Answers may vary. For example, $f(x) = x^3 + 2x^2 - 8x + 1$.
   b) Answers may vary. For example, $f(x) = 5x^4 - x^2 - 7$.
   c) Answers may vary. For example, $f(x) = 7x^6 + 3$.
   d) Answers may vary. For example, $f(x) = -2x^5 - 4x^4 + 3x^3 - 2x^2 + 9$.
3. a) As $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.
   b) As $x \to \pm\infty$, $y \to \infty$.
   c) As $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.
   d) As $x \to \pm\infty$, $y \to -\infty$.
4. a) even c) odd
   b) odd d) even
5. Answers may vary. For example:

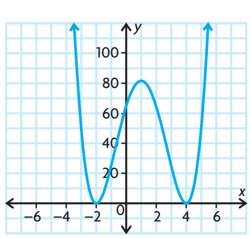
a)



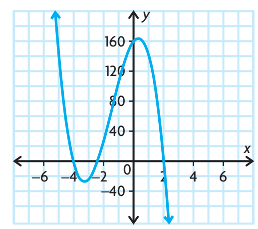

b)




c)




d)



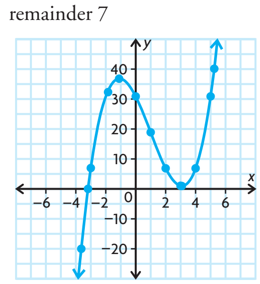

6. end behaviours
7. $y = 5(x - 2)(x + 3)^2(x - 5)$
8. a) reflection in the $x$-axis, vertical stretch by a factor of 25, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 4 units to the left, vertical translation 60 units down
   b) vertical stretch by a factor of 8, horizontal stretch by a factor of $\frac{4}{3}$, vertical translation 43 units up
   c) reflection in the $y$-axis, horizontal compression by a factor of $\frac{1}{13}$, horizontal translation 2 units to the right, vertical translation 13 units up
   d) vertical compression by a factor of $\frac{8}{11}$, reflection in the $y$-axis, vertical translation 1 unit down
9. vertically stretched by a factor of 5, horizontally translated 4 units to the left, and vertically translated 2 units down

## Lesson 3.5, pp. 168–170

1. a) i) $x^3 - 14x^2 - 24x - 38$ remainder $-87$
      ii) $x^3 - 20x^2 + 84x - 326$ remainder 1293
      iii) $x^3 - 15x^2 - 11x - 1$ remainder $-12$
   b) No; because for each division problem there is a remainder.
2. a) 2 b) 2 c) 1 d) not possible
3. a) $x^2 - 15x + 6$ remainder $-48x + 14$
   b) $5x^2 - 19x + 60$ remainder $-184$
   c) $x - 6$ remainder $-6x^2 + 22x + 6$
   d) Not possible
4.

| Dividend | Divisor | Quotient | Remainder |
|---|---|---|---|
| $2x^3 - 5x^2 + 8x + 4$ | $x + 3$ | $2x^2 - 11x + 41$ | $-119$ |
| $6x^4 + 12x^3 - 10x^2 - 4x + 29$ | $2x + 4$ | $3x^3 - 5x + 8$ | $-3$ |
| $6x^4 + 2x^3 + 3x^2 - 11x - 9$ | $3x + 1$ | $2x^3 + x - 4$ | $-5$ |
| $3x^3 + x^2 - 6x + 16$ | $x + 2$ | $3x^2 - 5x + 4$ | 8 |

5. a) $x^2 + 4x + 14$ remainder 57
   b) $x^2 - 6$ remainder 13
   c) $x^2 + 2x - 3$ remainder $-2$
   d) $x^2 + 3x - 9$ remainder $-16x + 62$
   e) $x + 1$ remainder $8x^2 - 8x + 11$
   f) $x + 3$ remainder $-4x^3 - 4x^2 + 8x + 14$
6. a) $x^2 + 3x + 2$ no remainder
   b) $2x^2 - 5x - 12$ remainder 7
   c) $6x^3 - 5x^2 - 19x + 10$ remainder $-2$
   d) $x^2 + 2x - 8$ remainder $-2$
   e) $6x^3 - 31x^2 + 45x - 18$ no remainder
   f) $3x^2 - 1$ no remainder
7. a) $x^3 + 4x^2 - 51x + 89$
   b) $3x^4 - 2x^3 + 3x^2 - 38x + 39$
   c) $5x^4 + 22x^3 - 17x^2 + 21x + 10$
   d) $x^6 + 8x^5 + 5x^4 - 13x^3 - 72x^2 + 49x - 3$
8. a) $r = 20$ c) $r = 0$
   b) $r = x - 22$ d) $r = 2x^2 + 2$
9. a) $x + 3$ c) $x + 4$
   b) $x + 10$ d) $x - 2$
10. a) $x + 5$ is a factor since there is no remainder.
    b) $x + 2$ is a factor since there is no remainder.
    c) $x - 2$ is not a factor since there is a remainder of 2.
    d) $x - 1$ is not a factor since there is a remainder of 1.
    e) $3x + 5$ is not a factor since there is a remainder of $-\frac{13}{3}$.
    f) $5x - 1$ is not a factor since there is a remainder of $-8$.
11. $(x + 1)$ cm
12. a) 7 b) 3
13. 2
14. Yes, $f(x)$ is always divisible by $x - 1$. Regardless of the value of $n$, $f(x) = x^n - 1$ can always be written as $f(x) = x^n + 0x^{n-1} + 0x^{n-2} + \ldots 0x - 1$. Therefore, the same pattern continues when dividing $x^n - 1$ by $x - 1$, regardless of how large $n$ is, and there is never a remainder.
15. a) $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x^2 + x - 6)$ remainder 7
    b) $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x + 3)(x - 2)$ remainder 7
    c)



**16.** Answers may vary. For example:

$$\begin{array}{rl} & 2x^3 + 9x^2 + 2x - 1 \\ x - 3 & \overline{)2x^4 + 3x^3 - 25x^2 - 7x - 14} \\ 2x^3(x-3) \rightarrow & \underline{2x^4 - 6x^3} \\ & 9x^3 - 25x^2 \\ 9x^2(x-3) \rightarrow & \underline{9x^3 - 27x^2} \\ & 2x^2 - 7x \\ 2x(x-3) \rightarrow & \underline{2x^2 - 6x} \\ & -1x - 14 \\ -1(x-3) \rightarrow & \underline{-1x + 3} \\ & -17 \end{array}$$

**17.** $r = 2x + 5$ cm

**18.** a) $x^2 + xy + y^2$
b) $x^2 - 2xy + y^2$

**19.** $x - y$ is a factor because there is no remainder.

**20.** $[q(x) + 1](x + 5)$

## Lesson 3.6, pp. 176–177

**1.** a) i) 64
ii) 22
iii) 12
b) No, according to the factor theorem, $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

**2.** a) not divisible by $x - 1$
b) divisible by $x - 1$
c) not divisible by $x - 1$
d) divisible $x - 1$

**3.** $(x + 1)(x + 3)(x - 2)$

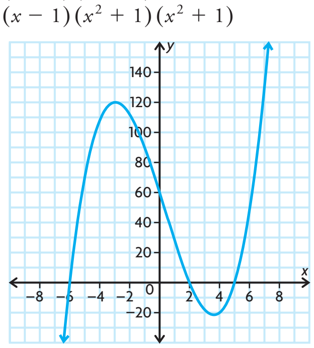
**4.** a) $-1$ b) $-5$ c) 0 d) $-34$ e) 30 f) 0

**5.** a) yes b) no c) yes d) no

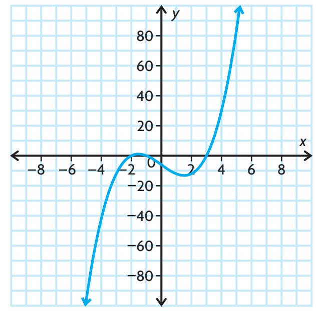
**6.** a) $(x - 2)(x - 4)(x + 3)$
b) $(x - 1)(2x + 3)(2x + 5)$
c) $x(x - 2)(x + 4)(x + 6)$
d) $(x + 2)(x + 5)(4x - 9)(x - 3)$
e) $x(x + 2)(x + 1)(x - 3)(x - 5)$
f) $(x - 3)(x - 3)(x + 4)(x + 4)$

**7.** a) $(x - 2)(x + 5)(x + 6)$
b) $(x + 1)(x - 3)(x + 2)$
c) $(x + 1)(x - 1)(x - 2)(x + 2)$
d) $(x - 2)(x + 1)(x + 8)(x - 4)$
e) $(x - 1)(x^2 + 1)$
f) $(x - 1)(x^2 + 1)(x^2 + 1)$

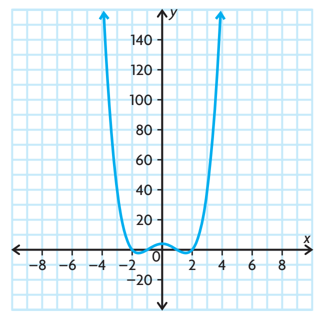
**8.** a)



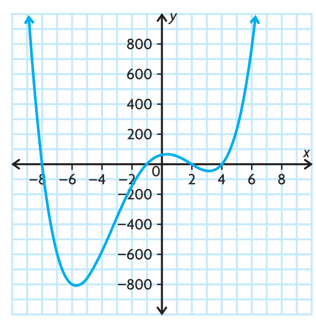

b)



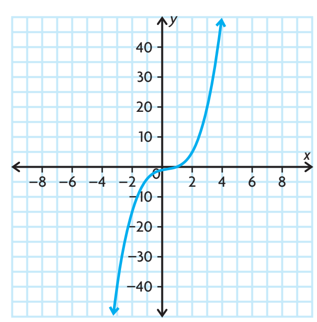

c)



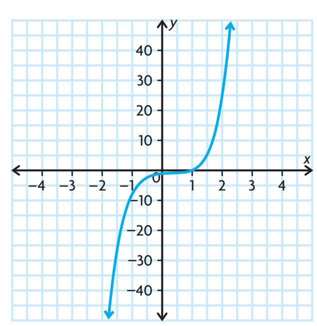

d)




e)




f)




**9.** 20

**10.** $a = 6, b = 3$

**11.** For $x^n - a^n$, if $n$ is even, they're both factors. If $n$ is odd, only $(x - a)$ is a factor. For $x^n + a^n$, if $n$ is even, neither is a factor. If $n$ is odd, only $(x + a)$ is a factor.

**12.** $a = -2, b = 22$;
The other factor is $-2x + 3$.

**13.** $-6$

**14.** $x^4 - a^4$
$= (x^2)^2 - (a^2)^2$
$= (x^2 + a^2)(x^2 - a^2)$
$= (x^2 + a^2)(x + a)(x - a)$

**15.** Answers may vary. For example: if $f(x) = k(x - a)$, then $f(a) = k(a - a) = k(0) = 0$.

**16.** $x^2 - x - 2 = (x - 2)(x + 1)$;
If $f(x) = x^3 - 6x^2 + 3x + 10$, then $f(2) = 0$ and $f(-1) = 0$.

**17.** If $f(x) = (x + a)^5 + (x + c)^5 + (a - c)^5$, then $f(-a) = 0$

## Lesson 3.7, p. 182

**1.** $(x + b)(x^2 - bx + b^2)$

**2.** a) $(x - 4)(x^2 + 4x + 16)$
b) $(x - 5)(x^2 + 5x + 25)$
c) $(x + 2)(x^2 - 2x + 4)$
d) $(2x - 3)(4x^2 + 6x + 9)$
e) $(4x - 5)(16x^2 + 20x + 25)$
f) $(x + 1)(x^2 - x + 1)$
g) $(3x + 2)(9x^2 - 6x + 4)$
h) $(10x + 9)(100x^2 - 90x + 81)$
i) $8(3x - 1)(9x^2 + 3x + 1)$

**3.** a) $(4x + 3y)(16x^2 - 12xy + 9y^2)$
b) $(-3x)(x - 2)(x^2 + 2x + 4)$
c) $(4 - x)(7x^2 + 25x + 31)$
d) $(x^2 + 4)(x^4 - 4x^2 + 16)$

**4.** a) $(x - 7)(x^2 + 7x + 49)$
b) $(6x - 1)(36x^2 + 6x + 1)$
c) $(x + 10)(x^2 - 10x + 100)$
d) $(5x - 8)(25x^2 + 40x + 64)$
e) $(4x - 11)(16x^2 + 44x + 121)$
f) $(7x + 3)(49x^2 - 21x + 9)$
g) $(8x + 1)(64x^2 - 8x + 1)$
h) $(11x + 12)(121x^2 - 132x + 144)$
i) $(8 - 11x)(64 + 88x + 121x^2)$

**5.** a) $\left(\frac{1}{3}x - \frac{2}{5}\right)\left(\frac{1}{9}x^2 + \frac{2}{15}x + \frac{4}{25}\right)$
b) $-16x^2(3x + 2)(9x^2 - 6x + 4)$
c) $7(4x - 5)(x^2 - x + 1)$
d) $\left(\frac{1}{2}x - 2\right)\left(\frac{1}{4}x^2 + x + 4\right)\left(\frac{1}{64}x^6 + x^3 + 64\right)$

**6.** Agree; by the formulas for factoring the sum and difference of cubes, the numerator of the fraction is equivalent to $(a^3 + b^3) + (a^3 - b^3)$. Since $(a^3 + b^3) + (a^3 - b^3) = 2a^3$, the entire fraction is equal to 1.

Answers

7. a) $1^3 + 12^3 = (1 + 12)(1^2 - (1)(12) + 12^2)$
$= (13)(133) = 1729$

b) $9^3 + 10^3 = (9 + 10)(9^2 - (9)(10) + 10^2)$
$= (19)(91) = 1729$

8. $x^9 + y^9$
$= x^{18} + 2x^9y^9 + y^{18}$
$= (x^{18} + y^{18}) + 2x^9y^9$
$= (x^6 + y^6)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$
$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$

9. Answers may vary. For example, this statement is true because $a^3 - b^3$ is the same as $a^3 + (-b)^3$.

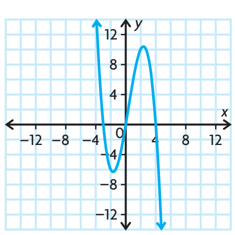
10. a) 1729 was the number of the taxicab that G. H. Hardy rode in when going to visit the mathematician Ramanujan. When Hardy told Ramanujan that the number of the taxicab he rode in was uninteresting, Ramanujan replied that the number was interesting because it was the smallest number that could be expressed as the sum of two cubes in two different ways. This is how such numbers came to be known as taxicab numbers.

b) Yes;

TN(1) = 2
TN(2) = 1729
TN(3) = 87 539 319
TN(4) = 6 963 472 309 248
TN(5) = 48 988 659 276 962 496
TN(6) = 24 153 319 581 254 312 065 344

## Chapter Review, pp. 184–185

1.




2. As $x \to -\infty$, $y \to +\infty$ and as $x \to \infty$, $y \to \infty$.

3. a) degree: 2 + 1; leading coefficient: positive; turning points: 2

b) degree: 3 + 1; leading coefficient: positive; turning points: 3

4. a) Answers may vary. For example,
$f(x) = (x + 3)(x - 6)(x - 4)$,
$f(x) = 10(x + 3)(x - 6)(x - 4)$,
$f(x) = -4(x + 3)(x - 6)(x - 4)$

b) Answers may vary. For example,
$f(x) = (x - 5)(x + 1)(x + 2)$,
$f(x) = -6(x - 5)(x + 1)(x + 2)$,
$f(x) = 9(x - 5)(x + 1)(x + 2)$

c) Answers may vary. For example,
$f(x) = (x + 7)(x - 2)(x - 3)$,
$f(x) = \frac{1}{4}(x + 7)(x - 2)(x - 3)$,
$f(x) = 3(x + 7)(x - 2)(x - 3)$

d) Answers may vary. For example,
$f(x) = (x - 9)(x + 5)(x + 4)$,
$f(x) = 7(x - 9)(x + 5)(x + 4)$,
$f(x) = -\frac{1}{3}(x - 9)(x + 5)(x + 4)$

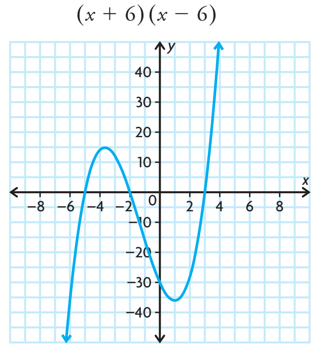
5. a) Answers may vary. For example,
$f(x) = (x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = 2(x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = -8(x + 6)(x - 2)(x - 5)(x - 8)$

b) Answers may vary. For example,
$f(x) = (x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = \frac{3}{4}(x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = -12(x - 4)(x + 8)(x - 1)(x - 2)$

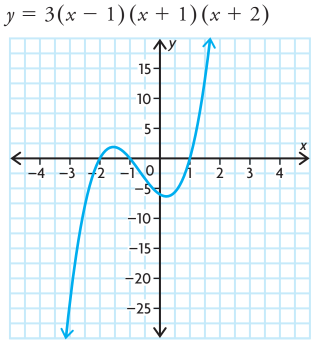
c) Answers may vary. For example,
$f(x) = x(x + 1)(x - 9)(x - 10)$,
$f(x) = 5x(x + 1)(x - 9)(x - 10)$,
$f(x) = -3x(x + 1)(x - 9)(x - 10)$

d) Answers may vary. For example,
$f(x) = (x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = \frac{2}{5}(x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = -10(x + 3)(x - 3)(x + 6)(x - 6)$

6.




7. $y = 3(x - 1)(x + 1)(x + 2)$




8. a) reflected in the $x$-axis, vertically stretched by a factor of 2, horizontally translated 1 unit to the right, and vertically translated 23 units up

b) horizontally stretched by a factor of $\frac{13}{12}$, horizontally translated 9 units to the left, and vertically translated 14 units down

c) horizontally translated 4 units to the right

d) horizontally translated $\frac{3}{7}$ units to the left

e) vertically stretched by a factor of 40, reflected in the $y$-axis, horizontally compressed by a factor of $\frac{1}{7}$, horizontally translated 10 units to the right, and vertically translated 9 units up

9. a) Answers will vary. For example, $(-2, -5400)$, $(3, 0)$, and $(8, 5400)$.

b) Answers will vary. For example, $(-7, -18)$, $(0, -19)$, and $(7, -20)$.

c) Answers will vary. For example, $\left(-6, \frac{182}{11}\right)$, $(-5, 16)$, and $\left(-4, \frac{170}{11}\right)$.

d) Answers will vary. For example, $(-2, -86)$, $(0, 14)$, and $(2, 114)$.

e) Answers will vary. For example, $(-1, -44)$, $(0, -45)$, and $(1, -46)$.

f) Answers will vary. For example, $(5, 1006)$, $(12, 6)$, and $(19, -994)$.

10. a) $2x^2 - 5x + 28$ remainder $-144$

b) $x^2 + 4x + 5$ remainder $26x + 33$

c) $2x - 6$ remainder $10x^2 + 27x - 34$

d) $x - 4$ remainder $4x^3 + 17x^2 - 8x - 18$

11. a) $(x + 2)(2x^2 + x - 3)$ remainder 1

b) $(x + 2)(3x^2 + 7x + 3)$ remainder $-3$

c) $(x + 2)(2x^3 + x^2 - 18x - 9)$ remainder 0

d) $(x + 2)(2x^2 - 5)$ remainder 6

12. a) $2x^3 - 7x^2 - 107x + 75$

b) $4x^4 + 3x^3 - 8x^2 + 22x + 17$

c) $3x^4 + 14x^3 - 42x^2 + 3x + 33$

d) $3x^6 - 11x^5 - 9x^4 + 47x^3 - 46x + 14$

13. 13

14. a) $(x + 1)(x - 8)(x + 2)$

b) $(x - 4)(2x + 3)(x + 3)$

c) $x(x - 2)(x - 3)(3x - 4)$

d) $(x - 1)(x + 4)(x + 4)(x + 4)$

15. a) $(x - 2)(4x + 5)(2x - 1)$

b) $(2x + 5)(x - 2)(x + 3)$

c) $(x - 3)(x - 3)(x - 3)(x + 2)$

d) $(2x + 1)(2x + 1)(x - 3)(x + 3)$

16. a) $(4x - 3)(16x^2 + 12x + 9)$

b) $(8x - 5)(64x^2 + 40x + 25)$

c) $(7x - 12)(49x^2 + 84x + 144)$

d) $(11x - 1)(121x^2 + 11x + 1)$

17. a) $(10x + 7)(100x^2 - 70x + 49)$

b) $(12x + 5)(144x^2 - 60x + 25)$

c) $(3x + 11)(9x^2 - 33x + 121)$

d) $(6x + 13)(36x^2 - 78x + 169)$

18. a) $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

b) $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$

c) Both methods produce factors of $(x - y)$ and $(x + y)$; however, the other factors are different. Since the two factorizations must be equal to each other, this means that $(x^4 + x^2y^2 + y^4)$ must be equal to $(x^2 + xy + y^2)(x^2 - xy + y^2)$.

## Chapter Self-Test, p. 186

**1.** a) $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, where $a_0, a_1, \dots, a_n$ are real numbers and $n$ is a whole number. The degree of the function is $n$; the leading coefficient is $a_n$.
b) $n - 1$
c) $n$
d) odd degree function
e) even degree function with a negative leading coefficient

**2.** $y = (x + 4)(x + 2)(x - 2)$

**3.** a) $(x - 9)(x + 8)(2x - 1)$
b) $(3x - 4)(3x^2 + 9x + 79)$

**4.** more zeros

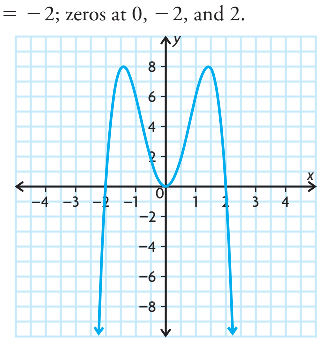
**5.** $-5 < x < -3$; $x > 1$

**6.** yes

**7.** a) $y = 5(2(x - 2))^3 + 4$
b) $(2.5, 9)$

**8.** $x + 5$

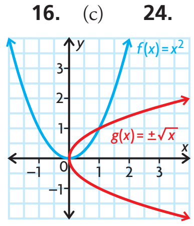
**9.** $a = -2$; zeros at 0, $-2$, and 2.



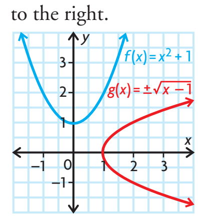
## Cumulative Review Chapters 1–3, pp. 188–191

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1.** | (b) | **9.** | (c) | **17.** | (a) | **25.** | (c) |
| **2.** | (a) | **10.** | (d) | **18.** | (d) | **26.** | (c) |
| **3.** | (c) | **11.** | (a) | **19.** | (b) | **27.** | (d) |
| **4.** | (b) | **12.** | (a) | **20.** | (c) | **28.** | (b) |
| **5.** | (b) | **13.** | (c) | **21.** | (b) | **29.** | (c) |
| **6.** | (d) | **14.** | (d) | **22.** | (b) | **30.** | (c) |
| **7.** | (d) | **15.** | (c) | **23.** | (b) | **31.** | (c) |
| **8.** | (a) | **16.** | (c) | **24.** | (a) | | |

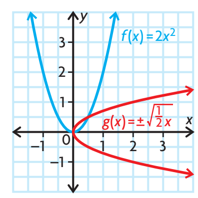
**32.** a)




b) Answers may vary. For example, vertical translation up produces horizontal translation of the inverse to the right.



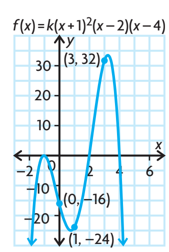

Vertical stretch produces horizontal stretch of inverse.




c) Answers may vary. For example, if the vertex of the inverse is $(a, b)$, restrict the value of $y$ to either $y \geq b$ or $y \leq b$.

**33.** Answers may vary. For example, average rates of change vary between $-2$ and 4, depending on the interval; instantaneous rates of change are 9 at $(0, 1)$, 0 at $(1, 5)$, $-3$ at $(2, 3)$, 0 at $(3, 1)$, 9 at $(4, 5)$; instantaneous rate of change is 0 at maximum $(1, 5)$ and at minimum $(3, 1)$.

**34.** a) $f(x) = -2(x + 1)^2(x - 2)(x - 4)$
b) $p = 32$
c) As $x \to \pm\infty$, $f(x) \to -\infty$; zeros: $-1$, 2, and 4
d) $-16$
e)



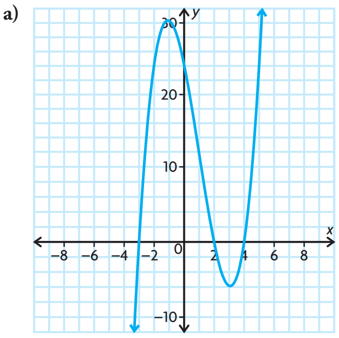

# Chapter 4

## Getting Started, pp. 194–195

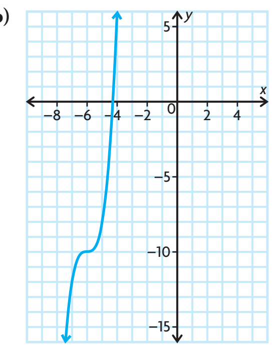
**1.** a) 3 c) 1
b) 5 d) $\frac{64}{11}$

**2.** a) $x(x + 6)(x - 5)$
b) $(x - 4)(x^2 + 4x + 16)$
c) $3x(2x + 3)(4x^2 - 6x + 9)$
d) $(x + 3)(x - 3)(2x + 7)$

**3.** a)



b)



**4.** 2 and 5

**5.** a) 3 and $-3$ c) $-\frac{2}{3}$ and $\frac{5}{2}$
b) $-10$ and 2 d) 0.3452 and $-4.345$

**6.** a) $(3, 7)$; Answers may vary. For example, the change in distance over time from $t = 3$ to $t = 7$ is greater than at other intervals of time.
b) $\frac{1}{3}$ m/s; $\frac{3}{4}$ m/s
c) Answers may vary. For example, away; Erika's displacement, or distance from the sensor, is increasing.

**7.** a) 2 s
b) 4.75 m/s
c) $-10.245$ m/s

**8.** a) Disagree; You could use the quadratic formula to solve $y = x^3 + 4x^2 + 3x$ because it equals $x(x^2 + 4x + 3)$.
b) Disagree; $y = (x + 3)^2(x - 2)$ is a cubic equation that will have two roots.
c) Disagree; The equation $y = x^3$ will only pass through two quadrants.
d) Agree; All polynomials are continuous and all polynomials have a $y$-intercept.
e) Disagree; $f(-3) = 9$
f) Agree; The instantaneous rates of change will tell you whether the graph is increasing, decreasing, or not changing at those points.

## Lesson 4.1, pp. 204–206

**1.** a) 0, 1, $-2$, 2 d) $-6$, $\frac{5}{2}$
b) $-\frac{3}{2}$, $\frac{5}{4}$, $-7$ e) 0, $-3$, 3
c) 3, $-5$, 4 f) $-5$, $-2$, 6