Chapter

1

Functions: Characteristics and Properties

▶ GOALS

You will be able to

- Review and consolidate your knowledge of the properties and characteristics of functions and their inverses

- Review and consolidate your knowledge of graphing functions using transformations

- Investigate the characteristics of piecewise functions

? What type of function can be used to model the height of a golf ball during its flight, and what information about the relationship between height and time can be found using this function?
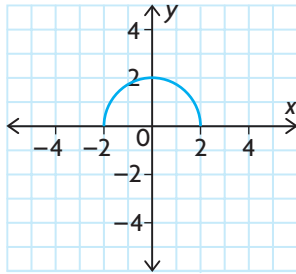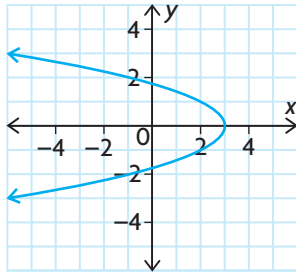
1 Getting Started

SKILLS AND CONCEPTS *You Need*

1. Evaluate $f(x) = x^2 + 3x - 4$ for each of the following values.

   a) $f(2)$ b) $f(-1)$ c) $f\left(\dfrac{1}{4}\right)$ d) $f(a + 1)$

2. Factor each of the following expressions.

   a) $x^2 + 2xy + y^2$ c) $(x + y)^2 - 64$
   b) $5x^2 - 16x + 3$ d) $ax + bx - ay - by$

3. State the **transformations** that are applied to each **parent function**, resulting in the given transformed function. Sketch the graphs of the parent function and transformed function.

   a) $f(x) = x^2, y = f(x - 3) + 2$ c) $g(x) = \sin x, y = -2g(0.5x)$
   b) $f(x) = 2^x, y = f(x - 1) + 2$ d) $g(x) = \sqrt{x}, y = -2g(2x)$

4. State the **domain** and **range** of each function.

   a)

   b) $f(x) = x^2 - 6x - 10$

   c) $y = \dfrac{1}{x}$

   d) $y = 3 \sin x$

   e) $g(x) = 10^x$

5. Which of the following represent functions? Explain.

   a)

   b) $y = 2(x - 1)^2 + 3$

   c) $y = \pm\sqrt{x} - 4$

   d) $y = 2^x - 4$

   e) $y = \cos(2(x - 30°) + 1)$

6. Consider the **relation** $y = x^3$.
   a) If $(2, n)$ is a point on its graph, determine the value of n.
   b) If $(m, 20)$ is a point on its graph, determine m correct to two decimal places.

7. A function can be described or defined in many ways. List these different ways, and explain how each can be used to determine whether a relation is a function.

APPLYING What You Know

Modelling the Height of a Football

During a football game, a football is thrown by a quarterback who is 2 m tall. The football travels through the air for 4 s before it is caught by the wide receiver.

? **What function can be used to model the height of the football above the ground over time?**

A. Explain why the variables time, t, in seconds and height, $h(t)$, in metres are good choices to model this situation.

B. What is $h(0)$? What does it mean in the context of this situation?

C. What happens at $t = 2$ s?

D. What happens at $t = 4$ s?

E. Explain why each of the following functions is *not* a good model for this situation. Support your claim with reasons and a well-labelled sketch.
  i) $h(t) = -5t(t - 4)$
  ii) $h(t) = -5(t - 4)^2 + 2$
  iii) $h(t) = 5t^2 + 4t - 3$

F. Determine a model that can be used to represent the height of the football, given this additional information:
  • The ball reached a maximum height of 22 m.
  • The wide receiver who caught the ball is also 2 m tall.

G. Use your model from part F to graph the height of the football over the duration of its flight.