1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 1.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1, 2, and 3.

Q: What is a function, and which of its representations is the best for solving problems and making predictions?

A: A function is a relation between two variables, in which each input has a unique output. Functions can be represented using words, graphs, numbers, and algebra.

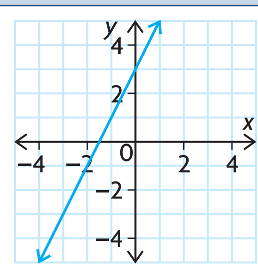
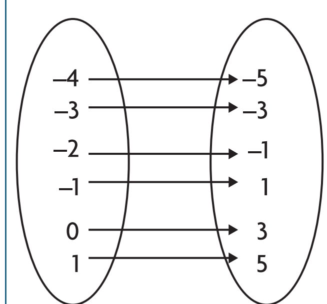
Word Example	Graphical Example	Numerical Example		Algebraic Example
One number is three more than twice another number.		Table of values:	Mapping diagram: 	$f(x) = 2x + 3$

Table of values:

x	y
−4	−5
−3	−3
−2	−1
−1	1
0	3
1	5

The algebraic model is the most useful and most accurate. If you know the value of one variable, you can substitute this value into the function to create an equation, which can then be solved using an appropriate strategy. This leads to an accurate answer. Both numerical and graphical models are limited in their use because they represent the function for only small intervals of the domain and range. When using a graphical model, it may be necessary to interpolate or extrapolate. This can lead to approximate answers.

Study *Aid*

- See Lesson 1.2.
- Try Mid-Chapter Review Questions 4 and 5.

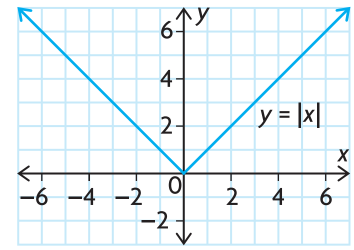
Q: What is the absolute value function, and what are the characteristics of its graph?

A: The absolute value function is $f(x) = |x|$. On a number line, $|x|$ is the distance of any value, x, from the origin. The absolute value function consists of two linear pieces, each defined by a different equation:

$$f(x) = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

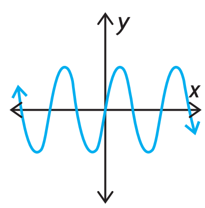
This function has the following characteristics:

- x-intercept: $x = 0$
- y-intercept: $y = 0$
- domain: $D = \{x \in \mathbf{R}\}$; range: $\mathbf{R} = \{y \in \mathbf{R} \mid y \geq 0\}$
- interval of decrease: $(-\infty, 0)$; interval of increase: $(0, \infty)$
- end behaviour: As $x \rightarrow \infty$, $y \rightarrow \infty$; as $x \rightarrow -\infty$, $y \rightarrow \infty$.

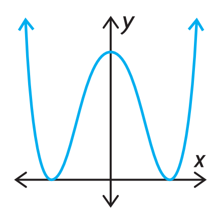


Q: What is the difference between an odd function and an even function, and how are the parent functions differentiated by this characteristic?

A: The graph of an odd function has rotational symmetry about the origin.



The graph of an even function is symmetric about the y-axis.



To test algebraically whether a function is odd or even, substitute $-x$ for x and simplify:

- If $f(-x) = -f(x)$, then the function is odd.
- If $f(-x) = f(x)$, then the function is even.

Odd Parent Functions: $f(x) = x, f(x) = \frac{1}{x}, f(x) = \sin x$

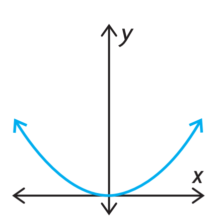
Even Parent Functions: $f(x) = x^2, f(x) = |x|, f(x) = \cos x$

Study Aid

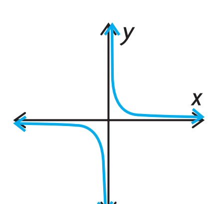
- See Lesson 1.3, Examples 3 and 4.
- Try Mid-Chapter Review Questions 6, 7, and 8.

Q: What is a discontinuity, and what is a continuous function?

A: A discontinuity is a break in the graph of a function. A function is continuous if it has no discontinuities; that is, no holes or breaks in its graph over its entire domain.



a continuous function



The function $y = \frac{1}{x}$ has a discontinuity at $x = 0$.

Study Aid

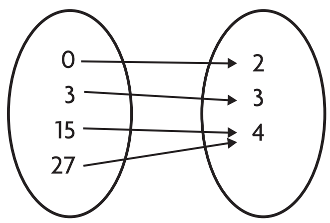
- See Lesson 1.3.
- Try Mid-Chapter Review Question 9.

PRACTICE Questions

Lesson 1.1

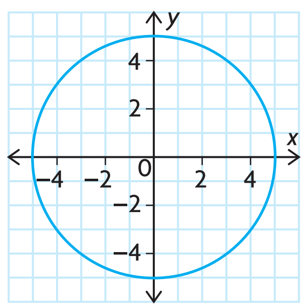
1. Determine whether each relation is a function, and state its domain and range.

a)



b) $y = 2x + 3$

c)



d) $\{(2, 7), (1, 3), (2, 6), (10, -1)\}$

2. The height of a bungee jumper above the ground is modelled by the following data.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Height (m)	50	40	30	20	10	20	30	40	45	35	25

a) Is the relationship between height and time a function? Explain.
b) What is the domain?
c) What is the range?

3. Determine the domain and range for each of the following and state whether it is a function:

a) $f(x) = 3x + 1$
b) $x^2 + y^2 = 9$
c) $y = \sqrt{5 - x}$
d) $x^2 - y = 2$

Lesson 1.2

4. Arrange the following values in order, from least to greatest:
$|-3|, -|3|, |5|, |-4|, |0|$

5. Sketch the graph of each function.

a) $f(x) = |x| + 3$
b) $f(x) = |x| - 2$
c) $f(x) = |-2x|$
d) $f(x) = |0.5x|$

Lesson 1.3

6. Determine a parent function that matches each set of characteristics.

a) The graph is neither even nor odd, and as $x \to \infty, y \to \infty$.
b) $(-\infty, 0)$ and $(0, \infty)$ are both intervals of decrease.
c) The domain is $[0, \infty)$.

7. Determine algebraically if each function is even, odd, or neither.

a) $f(x) = |2x|$
b) $f(x) = (-x)^2$
c) $f(x) = x + 4$
d) $f(x) = 4x^5 + 3x^3 - 1$

8. Each set of characteristics describes a parent function that has been shifted. Draw a possible graph, and state whether the graph is continuous.

a) There is a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 3$.
b) The range is $\{f(x) \in \mathbf{R} | -3 \le f(x) \le -1\}$.
c) The interval of increase is $(-\infty, \infty)$, and there is a horizontal asymptote at $y = -10$.

9. Sketch a graph that has the following characteristics:

- The function is odd.
- The function is continuous.
- The function has zeros at $x = -3$, 0, and 3.
- The function is increasing on the intervals $x \in (-\infty, -2)$ or $x \in (2, \infty)$.
- The function is decreasing on the interval $x \in (-2, 2)$.