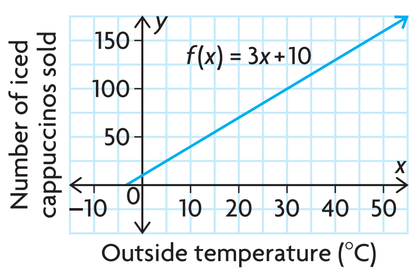
1.7 Exploring Operations with Functions

GOAL

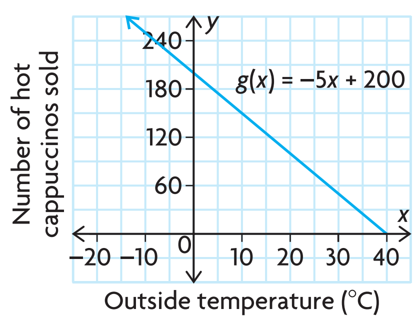
Explore the properties of the sum, difference, and product of two functions.

A popular coffee house sells iced cappuccino for \$4 and hot cappuccino for \$3. The manager would like to predict the relationship between the outside temperature and the total daily revenue from each type of cappuccino sold. The manager discovers that every 1 °C increase in temperature leads to an increase in the sales of cold drinks by three cups per day and to a decrease in the sales of hot drinks by five cups per day.

The function $f(x) = 3x + 10$ can be used to model the number of iced cappuccinos sold.



The function $g(x) = -5x + 200$ can be used to model the number of hot cappuccinos sold.



In both functions, x represents the daily average outside temperature. In the first function, $f(x)$ represents the daily average number of iced cappuccinos sold. In the second function, $g(x)$ represents the daily average number of hot cappuccinos sold.

? How does the outside temperature affect the daily revenue from cappuccinos sold?

A. Make a table of values for each function, with the temperature in intervals of 5°, from 0° to 40°.

B. What does $h(x) = f(x) + g(x)$ represent?

C. Simplify $h(x) = (3x + 10) + (-5x + 200)$.

D. Make a table of values for the function in part C, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $h(x)$ compare?

E. What does $h(x) = f(x) - g(x)$ represent?

F. Simplify $h(x) = (3x + 10) - (-5x + 200)$.

G. Make a table of values for the function in part F, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $h(x)$ compare?

H. What does $R(x) = 4f(x) + 3g(x)$ represent?

I. Simplify $R(x) = 4(3x + 10) + 3(-5x + 200)$.

J. Make a table of values for the function in part I, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $R(x)$ compare?

K. How does temperature affect the daily revenue from cappuccinos sold?

Reflecting

L. Explain how the sum function, $h(x)$, would be different if
- **a)** both $f(x)$ and $g(x)$ were increasing functions
- **b)** both $f(x)$ and $g(x)$ were decreasing functions

M. What does the function $k(x) = g(x) - f(x)$ represent? Is its graph identical to the graph of $h(x) = f(x) - g(x)$? Explain.

N. Determine the function $h(x) = f(x) \times g(x)$. Does this function have any meaning in the context of the daily revenue from cappuccinos sold? Explain how the table of values for this function is related to the tables of values you made in part A.

O. If you are given the graphs of two functions, explain how you could create a graph that represents
- **a)** the sum of the two functions
- **b)** the difference between the two functions
- **c)** the product of the two functions

In Summary

Key Idea

- If two functions have domains that overlap, they can be added, subtracted, or multiplied to create a new function on that shared domain.

Need to Know

- Two functions can be added, subtracted, or multiplied graphically by adding, subtracting, or multiplying the values of the dependent variable for identical values of the independent variable.
- Two functions can be added, subtracted, or multiplied algebraically by adding, subtracting, or multiplying the expressions for the dependent variable and then simplifying.
- The properties of each original function have an impact on the properties of the new function.

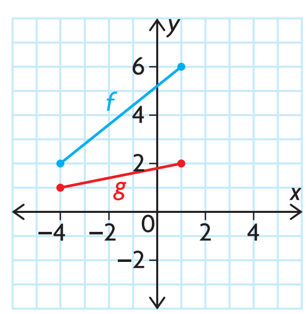
FURTHER Your Understanding

1. Let $f = \{(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)\}$ and $g = \{(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)\}$. Determine:

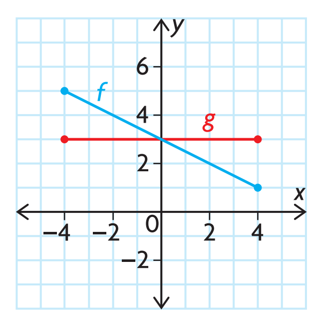
a) $f + g$ **b)** $f - g$ **c)** $g - f$ **d)** fg

2. Use the graphs of f and g to sketch the graphs of $f + g$.

a)

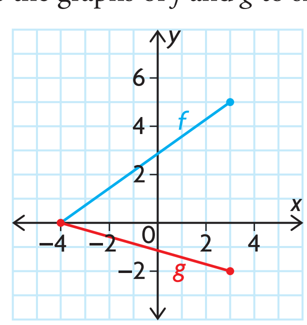


b)

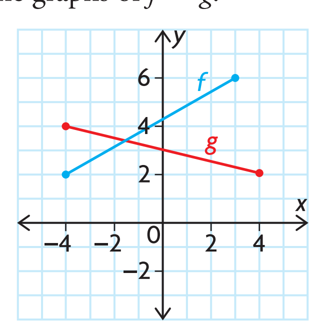


3. Use the graphs of f and g to sketch the graphs of $f - g$.

a)

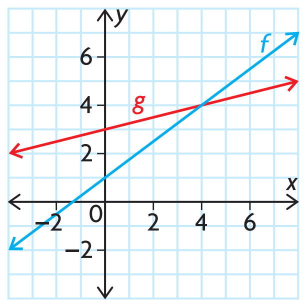


b)

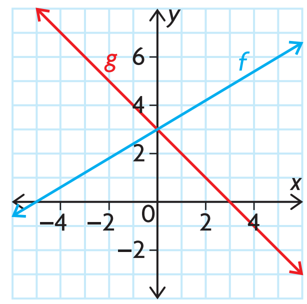


4. Use the graphs of f and g to sketch the graphs of fg.

a)



b)



5. Determine the equation of each new function, and then sketch its graph.

a) $h(x) = f(x) + g(x)$, where $f(x) = x^2$ and $g(x) = -x^2$

b) $p(x) = m(x) - n(x)$, where $m(x) = x^2$ and $n(x) = -7x + 12$

c) $r(x) = s(x) + t(x)$, where $s(x) = |x|$ and $t(x) = 2^x$

d) $a(x) = b(x) \times c(x)$, where $b(x) = x$ and $c(x) = x^2$

6. a) Using the graphs you sketched in question 5, compare and contrast the relationship between the properties of the original functions and the properties of the new function.

b) Which properties of the original functions determined the properties of the new function?

7. Let $f(x) = x + 3$ and $g(x) = -x^2 + 5$, $x \in \mathbf{R}$.

a) Sketch each graph on the same set of axes.

b) Make a table of values for $-3 \leq x \leq 3$, and determine the corresponding values of $h(x) = f(x) \times g(x)$.

c) Use the table to sketch $h(x)$ on the same axes. Describe the shape of the graph.

d) Determine the algebraic model for $h(x)$. What is its degree?

e) What is the domain of $h(x)$? How does this domain compare with the domains of $f(x)$ and $g(x)$?

8. Let $f(x) = x^2 + 2$ and $g(x) = x^2 - 2$, $x \in \mathbf{R}$.

a) Sketch each graph on the same set of axes.

b) Make a table of values for $-3 \leq x \leq 3$, and determine the corresponding values of $h(x) = f(x) \times g(x)$.

c) Use the table to sketch $h(x)$ on the same axes. Describe the shape of the graph.

d) Determine the algebraic model for $h(x)$. What is its degree?

e) What is the domain of $h(x)$?