

Chapter

2

Functions: Understanding Rates of Change

GOALS

You will be able to

- Calculate an average rate of change of a function given a table of values, a graph, or an equation
- Estimate the instantaneous rate of change of a function given a table of values, a graph, or an equation
- Interpret the average rate of change of a function over a given interval
- Interpret the instantaneous rate of change of a function at a given point
- Solve problems that involve rate of change

At what point on this hill is the speed of the roller coaster the fastest?

Getting Started

Study **Aid**

• For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
1	R-5
3	R-6
4	R–8
6	R-12

SKILLS AND CONCEPTS You Need

- 1. Determine the slope of the line through each pair of points.
 a) A (2, 3) and B (5, 7)
 b) C (3, -1) and D (-4, 5)
- **2.** Calculate the finite differences for each table, identify the type of function that each table represents, and provide a reason for your choice.

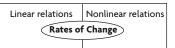
a)	x	1	2	3	4	5	6
	У	1	- 1	- 5	-13	-29	-61

b)	x	1	2	3	4	5	6
	У	0	11	28	51	80	115

- **3.** Determine the zeros for each of the following functions. **a)** $g(x) = 2x^2 - x - 6$ **c)** $j(x) = \sin(x - 45^\circ), 0^\circ \le x \le 360^\circ$ **b)** $h(x) = 2^x - 1$ **d)** $k(x) = 2\cos x, -360^\circ \le x \le 0^\circ$
- 4. Given y = f(x), describe how the graph of f(x) is transformed in each of the following functions.

$a) y = \frac{1}{2}f(x)$	c) $y = -3f(x) + 7$
b) $y = 2f(x - 4)$	d) $y = 5f(x - 3) - 2$

- **5.** Suppose you invest \$1000 in a savings account that pays 8%/a compounded annually.
 - a) Write an equation for the amount of money you will have after *t* years.
 - b) How much money will you have after three years?
 - c) Does the amount of money in your account increase at a constant rate each year? Explain.
- 6. The height above the ground of one of the seats of a Ferris wheel, in metres, can be modelled by the function $h(t) = 8 + 7 \sin (15^{\circ}t)$, where t is measured in seconds.
 - a) What is the maximum and minimum height reached by any seat?
 - **b**) How long does one seat on this ride take to rotate back to its starting point?
 - c) After 30 s, what will the height of the seat be?
- **7.** Create a chart to show what you know about rates of change in **linear** and **nonlinear relations**.



Getting Started

APPLYING What You Know

Safe Driving

It is important for drivers to know how much time they need to come to a safe stop. The time needed to stop depends on the speed of the vehicle.

The following table gives safe stopping distances on dry pavement.

Speed (km/h)	Reaction-Time Distance (m)	Braking Distance (m)	Overall Stopping Distance (m)
0	0.00	0.00	0.00
20	8.33	1.77	10.10
40	16.67	7.09	23.77
60	25.00	15.96	40.96
80	33.33	28.38	61.71
100	41.67	44.35	86.02



What might be realistic reaction-time distances, braking distances, and overall stopping distances for speeds of 70 km/h and 120 km/h?

- **A.** What type of function best models the reaction-time distances for the given speeds? Explain how you know.
- **B.** Sketch a graph and determine an equation for the type of function you chose in part A, using the data in the table.
- **C.** What type of function best models the braking distances for the given speeds? Explain how you know.
- **D.** Sketch a graph and determine an equation for the type of function you chose in part C, using the data in the table.
- **E.** Do any of the three distances in the table increase at a constant rate? Explain.
- **F.** What other factors, in addition to the speed of the vehicle, may affect the overall stopping distance?
- **G.** Use the graphs and equations you found to predict the reaction times, braking distances, and overall stopping distances, in metres, for speeds of 70 km/h and 120 km/h.