





Chapter

2

Functions: Understanding Rates of Change

► GOALS

You will be able to

- Calculate an average rate of change of a function given a table of values, a graph, or an equation
- Estimate the instantaneous rate of change of a function given a table of values, a graph, or an equation
- Interpret the average rate of change of a function over a given interval
- Interpret the instantaneous rate of change of a function at a given point
- Solve problems that involve rate of change

? At what point on this hill is the speed of the roller coaster the fastest?

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Getting Started

Study Aid

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
1	R-5
3	R-6
4	R-8
6	R-12

SKILLS AND CONCEPTS You Need

- Determine the slope of the line through each pair of points.
 - A (2, 3) and B (5, 7)
 - C (3, -1) and D (-4, 5)
- Calculate the finite differences for each table, identify the type of function that each table represents, and provide a reason for your choice.
 - | | | | | | | |
|---|---|----|----|-----|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 1 | -1 | -5 | -13 | -29 | -61 |
 - | | | | | | | |
|---|---|----|----|----|----|-----|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 0 | 11 | 28 | 51 | 80 | 115 |
- Determine the zeros for each of the following functions.
 - $g(x) = 2x^2 - x - 6$
 - $h(x) = 2^x - 1$
 - $j(x) = \sin(x - 45^\circ)$, $0^\circ \leq x \leq 360^\circ$
 - $k(x) = 2 \cos x$, $-360^\circ \leq x \leq 0^\circ$
- Given $y = f(x)$, describe how the graph of $f(x)$ is transformed in each of the following functions.
 - $y = \frac{1}{2}f(x)$
 - $y = 2f(x - 4)$
 - $y = -3f(x) + 7$
 - $y = 5f(x - 3) - 2$
- Suppose you invest \$1000 in a savings account that pays 8%/a compounded annually.
 - Write an equation for the amount of money you will have after t years.
 - How much money will you have after three years?
 - Does the amount of money in your account increase at a constant rate each year? Explain.
- The height above the ground of one of the seats of a Ferris wheel, in metres, can be modelled by the function $h(t) = 8 + 7 \sin(15^\circ t)$, where t is measured in seconds.
 - What is the maximum and minimum height reached by any seat?
 - How long does one seat on this ride take to rotate back to its starting point?
 - After 30 s, what will the height of the seat be?
- Create a chart to show what you know about rates of change in **linear** and **nonlinear relations**.

Linear relations	Nonlinear relations
Rates of Change	

APPLYING What You Know

Safe Driving

It is important for drivers to know how much time they need to come to a safe stop. The time needed to stop depends on the speed of the vehicle.

The following table gives safe stopping distances on dry pavement.

Speed (km/h)	Reaction-Time Distance (m)	Braking Distance (m)	Overall Stopping Distance (m)
0	0.00	0.00	0.00
20	8.33	1.77	10.10
40	16.67	7.09	23.77
60	25.00	15.96	40.96
80	33.33	28.38	61.71
100	41.67	44.35	86.02



- ? What might be realistic reaction-time distances, braking distances, and overall stopping distances for speeds of 70 km/h and 120 km/h?**
- What type of function best models the reaction-time distances for the given speeds? Explain how you know.
 - Sketch a graph and determine an equation for the type of function you chose in part A, using the data in the table.
 - What type of function best models the braking distances for the given speeds? Explain how you know.
 - Sketch a graph and determine an equation for the type of function you chose in part C, using the data in the table.
 - Do any of the three distances in the table increase at a constant rate? Explain.
 - What other factors, in addition to the speed of the vehicle, may affect the overall stopping distance?
 - Use the graphs and equations you found to predict the reaction times, braking distances, and overall stopping distances, in metres, for speeds of 70 km/h and 120 km/h.