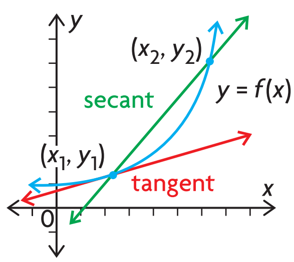
2 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What is the difference between the average rate of change and the instantaneous rate of change?

A: The average rate of change of a quantity represented by a dependent variable occurs over an interval of the independent variable. The instantaneous rate of change of a quantity represented by a dependent variable occurs at a single value of the independent variable. As a result, average rate of change can be represented graphically using secant lines, while instantaneous rate of change can be represented graphically using tangent lines.



Study Aid

- See Lesson 2.1, Example 1.
- Try Mid-Chapter Review Question 1.

Q: How do you determine the average rate of change?

A1: To determine the average rate of change from a table of values or from the equation of any function $y = f(x)$, over the interval between the x-coordinates of points (x_1, y_1) and (x_2, y_2), divide the change in y by the change in x.

$$\begin{aligned}\text{Average rate of change} &= \frac{\text{change in } y}{\text{change in } x}\\ &= \frac{\Delta y}{\Delta x}\\ &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{f(x_2) - f(x_1)}{x_2 - x_1}\end{aligned}$$

A2: To determine the average rate of change from the graph of a function, calculate the slope of the secant line that passes through the two points that define the interval on the graph. The slope is equivalent to the average rate of change on the defined interval.

Study Aid

- See Lesson 2.1, Examples 2 to 4.
- Try Mid-Chapter Review Questions 1 and 2.

Study Aid

- See Lesson 2.2, Examples 1, 2, and 3, and Lesson 2.3.
- Try Mid-Chapter Review Questions 2 to 5.

Q: How can you estimate the instantaneous rate of change?

A1: Calculate the average rate of change for values that are very close to the location where the instantaneous rate of change occurs. You can use preceding and following intervals, or you can use centred intervals. Use your results to find the trend and then estimate the instantaneous rate of change.

A2: Calculate the average rate of change using the difference quotient with the location where the instantaneous rate of change occurs $(a, f(a))$ and a general point $(a + h, f(a + h))$:

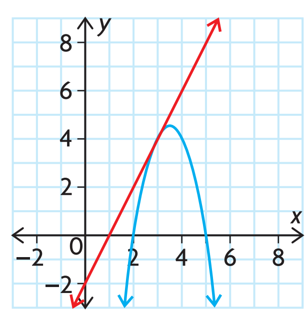
$$\frac{f(a + h) - f(a)}{h}$$

Choose values for h that are very small.

A3: Draw a tangent line at the point where the instantaneous rate of change occurs. Calculate the slope of this line.

For example, to estimate the instantaneous rate of change of $f(x) = -2x^2 + 14x - 20$ at the point $(3, 4)$, graph $f(x)$ and draw a tangent line at $(3, 4)$.
Use the points $(3, 4)$ and $(1, 0)$ on the tangent line to calculate the slope of the tangent line.

$$\text{Slope} = \frac{0 - 4}{1 - 3}$$
$$= 2$$



So the instantaneous rate of change in y with respect to x is about 2.

PRACTICE Questions

Lesson 2.1

1. The following table gives the amount of water that is used on a farm during the first six months of the year.

Month	Volume (1000 of m^3)
January	3.00
February	3.75
March	3.75
April	4.00
May	5.10
June	5.50

a) Plot the data in the table on a graph.
b) Find the rate of change in the volume of water used between consecutive months.
c) Between which two months is the change in the volume of water used the greatest?
d) Determine the average rate of change in the volume of water used between March and June.

Lesson 2.2

2. A city's population (in tens of thousands) is modelled by the function $P(t) = 1.2(1.05)^t$, where t is the number of years since 2000. Examine the equation for this function and its graph.
a) What can you conclude about the average rate of change in population between consecutive years as time increases?
b) Estimate the instantaneous rate of change in population in 2010.

3. The height of a football that has been kicked can be modelled by the function $h(t) = -5t^2 + 20t + 1$, where $h(t)$ is the height in metres and t is the time in seconds.
a) What is the average rate of change in height on the interval $0 \leq t \leq 2$ and on the interval $2 \leq t \leq 4$?
b) Use the information given in part a) to find the time for which the instantaneous rate of change in height is 0 m/s. Verify your response.

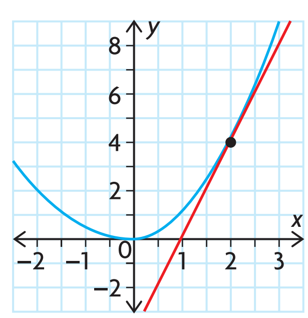
4. The movement of a certain glacier can be modelled by $d(t) = 0.01t^2 + 0.5t$, where d is the distance, in metres, that a stake on the glacier has moved, relative to a fixed position, t days after the first measurement was made. Estimate the rate at which the glacier is moving after 20 days.

5. Create a graphic organizer, such as a web diagram, mind map, or concept map, for rate of change. Include both average rate of change and instantaneous rate of change in your graphic organizer.

Lesson 2.3

6. Create a table to estimate the slope of the tangent to $y = x^3 + 1$ at $P(2, 9)$. Be sure to approach P from both directions.

7. Estimate the slope of the tangent line in the graph of this function.



8. Explain what the answer for question 7 represents.

9. Graph the function $f(x) = 0.5x^2 + 5x - 15$ using your graphing calculator. Estimate the instantaneous rate of change for each value of x.
a) $x = -5$
b) $x = -1$
c) $x = 0$
d) $x = 3$