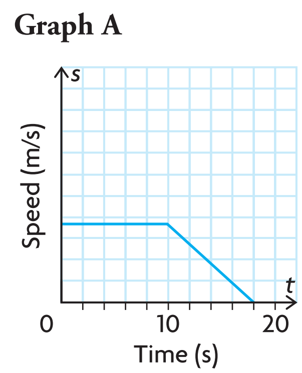
2 Chapter Review

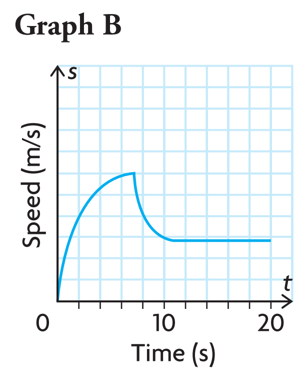
FREQUENTLY ASKED Questions

Study Aid

- See Lesson 2.4, Examples 1, 2, and 3.
- Try Chapter Review Questions 8, 9, and 10.

Q: What descriptions could be given to produce the following speed versus time graphs? Explain.





A: Graph A: A person walks at the same rate for 10 s and then slows down and comes to a stop at 18 s. This is shown in the graph because the horizontal line means that the person is walking at the same rate, and the straight line with a negative slope means that the person is slowing down at a constant rate.

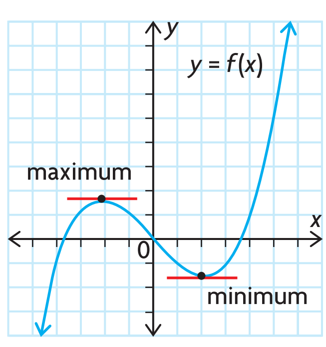
Graph B: A person walks, increasing speed at a variable rate for 8 s and then decreasing speed at a variable rate. From 11 s to 20 s, the person walks at the same rate.

Study Aid

- See Lesson 2.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 11 and 13.

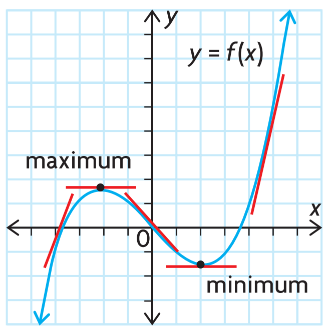
Q: How can you verify, for a given value of the independent variable, where a maximum or minimum occurs using rate of change calculations?

A: Check to see if the instantaneous rate of change is equal to zero at any point where a maximum or minimum might occur. If it does, then a maximum or minimum could occur there. Graphically, the tangent line must be horizontal at this point.



If the instantaneous rate of change is positive before the point where the rate of change is zero, and negative after, then a maximum occurs. Graphically, the tangent lines must have a positive slope before the maximum point and a negative slope after.

If the instantaneous rate of change is negative before the point where the rate of change is zero, and positive after, then a minimum occurs. Graphically, the tangent lines must have a negative slope before the minimum point and a positive slope after.



Q: When solving problems that require you to estimate the value for the instantaneous rate of change in a relationship at a specific point, what does the sign of this estimated value indicate?

A: The sign of the estimated value of the instantaneous rate of change gives you information about what is happening to the values of the dependent variable in the relationship at that exact point in time. If the instantaneous rate of change is positive (indicated graphically by a tangent line that rises from left to right), then the values for the dependent variable are increasing. If the instantaneous rate of change is negative (indicated graphically by a tangent line that falls from left to right), then the values for the dependent variable are decreasing.

Q: Can the difference quotient $\frac{f(a+h)-f(a)}{h}$ be used to determine both average and instantaneous rates of change?

A: Yes. For any function $y = f(x)$, the difference quotient provides a formula for calculating the average rate of change between two points $(a, f(a))$ and $(a + h, f(a + h))$. In both the case of average and instantaneous rate of change, h is the difference between the values for the independent variable that define the interval on which the rate of change is being calculated. In the case of instantaneous rate of change, h is made arbitrarily small so that this interval is close to 0. The calculation approximates the instantaneous rate of change when two points on $y = f(x)$ are chosen that are very, very close to each other.

Study Aid

- See Lesson 2.5, Example 2.
- Try Chapter Review Question 12.

PRACTICE Questions

Lesson 2.1

1. The following table shows the daily number of watches sold at a shop and the amount of money made from the sales.

Number of Watches (w)	Revenue (r) ($)
25	437.50
17	297.50
20	350.00
12	210.00
24	420.00

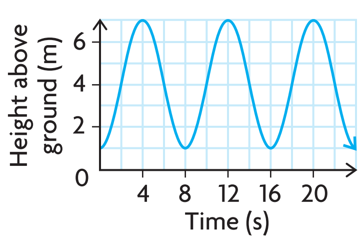
a) Does the data in the table appear to follow a linear relation? Explain.

b) Graph the data. How does the graph compare with your hypothesis?

c) What is the average rate of change in revenue from $w = 20$ to $w = 25$?

d) What is the cost of one watch, and how does this cost relate to the graph?

2. The graph shows the height above the ground of a person riding a Ferris wheel.



a) Calculate the average rate of change in height on the interval [0, 4].

b) Calculate the average rate of change in height on the interval [4, 8].

c) Discuss the similarities and differences in your answers to parts a) and b).

3. A company is opening a new office. The initial expense to set up the office is $10 000, and the company will spend another $2500 each month in utilities until the new office opens.

a) Write the equation that represents the company's total expenses in terms of months until the office opens.

b) What is the average rate of change in the company's expenses from $3 \le m \le 6$?

c) Do you expect this rate of change to vary? Why or why not?

Lesson 2.2

4. An investment's value, $V(t)$, is modelled by the function $V(t) = 2500(1.15)^t$, where t is the number of years after funds are invested.

a) To find the instantaneous rate of change in the value of the investment at $t = 4$, what intervals on either side of 4 would you choose? Why?

b) Use your intervals from part a) to find the instantaneous rate of change in the value of the investment at $t = 4$.

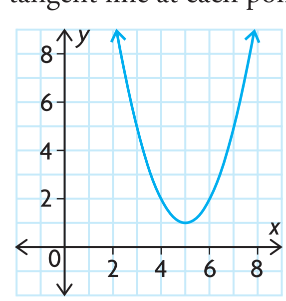
5. The height, in centimetres, of a piston attached to a turning wheel at time t, in seconds, is modelled by the equation $y = 2 \sin(120°t)$.

a) Examine the equation, and select a strategy for finding the instantaneous rate of change in the piston's height at $t = 12$ s.

b) Use your strategy from part a) to find the instantaneous rate of change at $t = 12$ s.

Lesson 2.3

6. For the graph shown, estimate the slope of the tangent line at each point.



a) (4, 2)

b) (5, 1)

c) (7, 5)

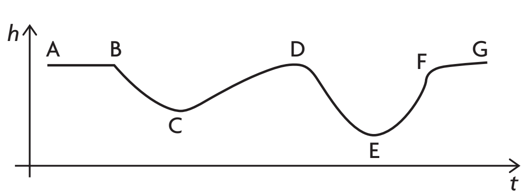
7. Use a graphing calculator to graph the equation $y = 5x^2 + 3x + 7$. Then use your calculator to estimate the instantaneous rate of change for each value of x.
   a) $x = -4$
   b) $x = -2$
   c) $x = -0.3$
   d) $x = 2$

Lesson 2.4

8. A sculptor makes a vase for flowers. The radius and circumference of the vase increase as the height of the vase increases. The vase is filled with water. Draw a possible graph of the height of the water as time increases.

9. A newspaper carrier delivers papers on her bicycle. She bikes to the first neighbourhood at a rate of 10 km/h. She slows down at a constant rate over a period of 7 s, to a speed of 5 km/h, so that she can deliver her papers. After travelling at this rate for 3 s, she sees one of her customers and decides to stop. She slows at a constant rate until she stops. It takes her 6 s to stop.
   a) Draw a graph of the newspaper carrier's rate over time for the time period after she arrives at the first neighbourhood.
   b) What is the average rate of change in speed over the first 7 s?
   c) What is the average rate of change in speed from second 7 to 12 seconds.
   d) What is the instantaneous rate of change in speed at 12 s?

10. The graph shows the height of a roller coaster versus time. Describe how the vertical speed of the roller coaster will vary as it travels along the track from A to G. Sketch a graph to show the vertical speed of the roller coaster.



Lesson 2.5

11. A maximum or minimum is given for each of the following functions. Select a strategy, and verify whether the point given is a maximum or a minimum.
    a) $f(x) = x^2 - 10x + 7;\ (5, -18)$
    b) $g(x) = -x^2 - 6x - 4;\ (-3, 5)$
    c) $h(x) = -2x^2 + 68x + 75;\ (17, 653)$
    d) $j(x) = \sin(-2x);\ (45°, -1)$
    e) $k(x) = -4\cos(x + 25);\ (-25°, -4)$
    f) $m(x) = \frac{1}{20}(x^3 + 2x^2 - 15x);\ \left(-3, \frac{9}{5}\right)$

12. a) For each function, find the equation for the slope of the secant line between any general point on the function $(a + h, f(a + h))$ and the given x-coordinate of another point.
       i) $f(x) = x^2 - 30x;\ a = 2$
       ii) $g(x) = -4x^2 - 56x + 16;\ a = -1$
    b) Use each slope equation you found in part a) to estimate the slope of the tangent line at the point with the given x-coordinate.

13. a) Explain how the instantaneous rates of change differ on either side of a maximum point of a function.
    b) Explain how the instantaneous rates of change differ on either side of a minimum point of a function.

14. a) Use graphing technology to graph $f(x) = x^4 - 2x^2$.
    b) Use the graph to estimate the locations of the maximum and minimum values of this function.
    c) Explain how tangent lines can be used to verify the locations you identified in part b).
    d) Confirm your estimates by using the maximum and minimum operations on the graphing calculator.