

## Chapter

# 3

## *Polynomial Functions*

### ► GOALS

#### You will be able to

- Identify and describe key characteristics of polynomial functions
- Divide one polynomial by another polynomial
- Factor polynomial expressions
- Solve problems that involve polynomial equations and inequalities graphically and algebraically

**?** A fractal object displays properties of self-similarity. The fractal shown was created using a computer, the polynomial function  $f(z) = 35z^9 - 180z^7 + 378z^5 - 420z^3 + 315z$ , and a process called iteration. How can you estimate the number of zeros that this polynomial function has?

# 3

## Getting Started

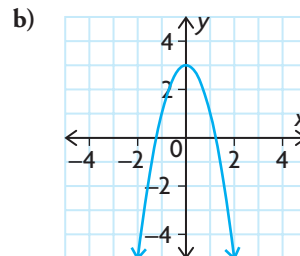
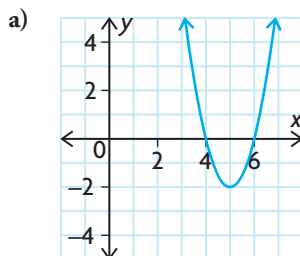
### Study Aid

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
1	R-2
2	R-3
3	R-6
4	R-8
5	R-9

### SKILLS AND CONCEPTS You Need

- Expand and simplify each of the following expressions.
  - $2x^2(3x - 11)$
  - $(x - 4)(x + 6)$
  - $4x(2x - 5)(3x + 2)$
  - $(5x - 4)(x^2 + 7x - 8)$
- Factor each of the following expressions completely.
  - $x^2 + 3x - 28$
  - $2x^2 - 18x + 28$
- Solve each of the following equations. Round your answer to two decimal places, if necessary.
  - $3x + 7 = x - 5$
  - $(x + 3)(2x - 9) = 0$
  - $x^2 + 11x + 24 = 0$
  - $6x^2 + 22x = 8$
- Describe the transformations that must be applied to  $y = x^2$  to create the graph of each of the following functions.
  - $y = \frac{1}{4}(x - 3)^2 + 9$
  - $y = \left(\frac{1}{2}x\right)^2 - 7$
- Write the equation of each function shown below.



- Graph each of the following functions.
  - $y = 3(x + 5)^2 - 4$
  - $y = 2x^2 - 12x + 5$
- Use finite differences to classify each set of data as linear, quadratic, or other.

a) 

x	y
-2	56.4
-1	50.6
0	45
1	39.6
2	34.4

b) 

x	y
-2	11
-1	5
0	2
1	7
2	13

c) 

x	y
-2	2
-1	6
0	18
1	54
2	162

d) 

x	y
-2	7
-1	6.5
0	6
1	5.5
2	5

- Create a concept web that shows the connections between each of the following for the function  $f(x) = 3x^2 + 24x + 36$ : the  $y$ -intercept, factored form, vertex form, axis of symmetry, direction of opening, zeroes, minimum value, value of the discriminant, and translations of the parent function.  
On each arrow, write a brief description of the process you would use to obtain the information.

## APPLYING What You Know

### Examining Patterns

In the late 18th century, seven-year-old Carl Friedrich Gauss noticed a pattern that allowed him to determine the sum of the numbers from 1 to 100 very quickly. He realized that you could add 1 and 100, and then multiply by half of the largest number (50) to get 5050.

- ?** Are there formulas for calculating the sum of the first  $n$  natural numbers and the sums of consecutive squares of natural numbers?
- Copy and complete each table, then calculate the **finite differences** until they are constant.
  - Graph each relationship in part A on graph paper.
  - Use your graphs and finite differences to make a **conjecture** about the type of model that would fit the data in each table (linear, quadratic, or other).
  - Use a graphing calculator and the regression operation to verify your conjectures in part C.
  - Use the equations you found in part D to calculate the sum of the first five natural numbers and the sum of the squares of the first five natural numbers.
  - Verify that your calculations in part E are correct by comparing your sums with the values in both tables when  $n = 5$ .
  - Use the equation you found to verify that the sum of the natural numbers from 1 to 100 is 5050.
  - Use the equation you found to determine the sum of the squares of the natural numbers from 1 to 100.

### YOU WILL NEED

- graph paper

Table 1

$n$	Sum up to $n$ ( $f(n)$ )
1	1
2	$1 + 2 = 3$
3	$1 + 2 + 3 = 6$
4	
5	
6	
7	
8	
9	
10	

Table 2

$n$	Sum of the squares up to $n^2$ ( $g(n)$ )
1	1
2	$1^2 + 2^2 = 5$
3	$1^2 + 2^2 + 3^2 = 14$
4	
5	
6	
7	
8	
9	
10	