3 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can you tell whether an expression is a polynomial?

A: A polynomial is an expression in which the coefficients are real numbers and the exponents on the variables are whole numbers.

For example, consider the following expressions:

$3x^2 - 5x^3 + \frac{1}{2}x, \quad \sqrt{x} + 4x^2 - 3, \quad \frac{x+3}{2x-5}, \quad \sqrt{5}x^2 + 8x - 10$

Only two of these expressions, $3x^2 - 5x^3 + \frac{1}{2}x$ and $\sqrt{5}x^2 + 8x - 10$, are polynomials.

Study Aid

- See Lesson 3.1.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can you describe the characteristics of the graph of a polynomial function by looking at its equation?

A: The degree of the function and the sign of the leading coefficient can be used to determine the end behaviours of the graph.

- If the degree of the function is odd and the leading coefficient is
  - negative, then the function extends from above the x-axis to below the x-axis
  - positive, then the function extends from below the x-axis to above the x-axis
- If the degree of the function is even and the leading coefficient is
  - negative, then both ends of the function are below the x-axis
  - positive, then both ends of the function are above the x-axis
- For any polynomial function, the maximum number of turning points is one less than the degree of the function.
- If the degree of the function is
  - odd, then there must be an even number of turning points
  - even, then there must be an odd number of turning points

Study Aid

- See Lesson 3.2, Example 1.
- Try Mid-Chapter Review Questions 3 and 4.

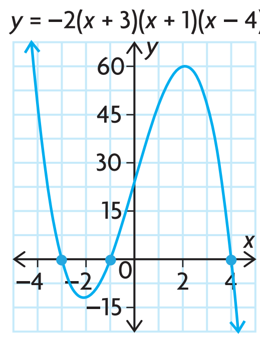
Q: How can you sketch the graph of a polynomial function that is in factored form?

A: The factors of the function can be used to determine the real roots of the corresponding polynomial equation. These roots are the x-intercepts of the graph. Use other characteristics of the function, such as end behaviours, turning points, and the order of each factor, to approximate the shape of the graph.

Study Aid

- See Lesson 3.3, Example 2.
- Try Mid-Chapter Review Questions 5, 6, and 7.

For example, to sketch the graph of $y = -2(x + 3)(x + 1)(x - 4)$, first determine the x-intercepts. They are -3, -1, and 4. Because the order of each factor is 1, the graph has a linear shape near each zero. Because the leading coefficient is -2 and the degree is 3, the graph extends from the second quadrant to the fourth quadrant. There are, at most, two turning points.

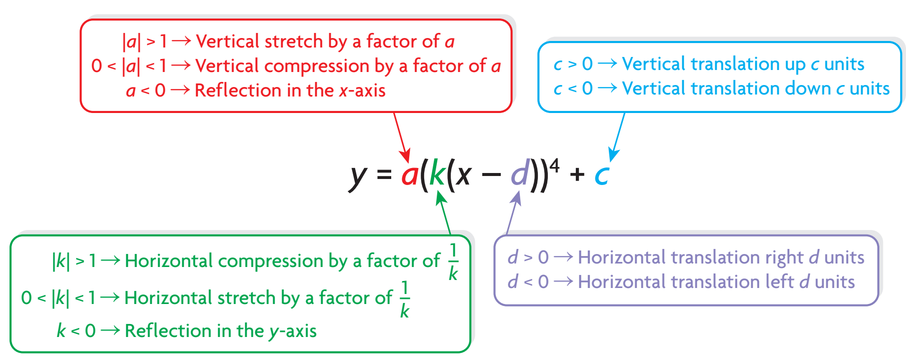


Study | Aid

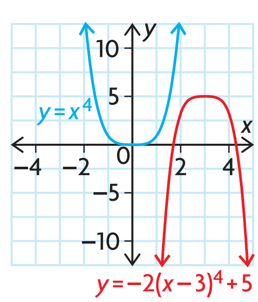
- See Lesson 3.4, Examples 1 and 2.
- Try Mid-Chapter Review Questions 8 and 9.

Q: How can you sketch the graph of a polynomial function using transformations?

A: If the equation is in the form $y = a(k(x - d))^n + c$, then transform the graph of $y = x^n$ as follows:



For example, to sketch the graph of $y = -2(x - 3)^4 + 5$, vertically stretch the graph of $y = x^4$ by a factor of 2, reflect it through the x-axis, and then translate it 3 units to the right and 5 units up. As a result of these transformations, every point (x, y) on the graph of $y = x^4$ changes to $(x + 3, -2y + 5)$.



PRACTICE Questions

Lesson 3.1

1. Determine whether or not each function is a polynomial function. If it is not a polynomial function, explain why.

a) $f(x) = \frac{2}{3}x^4 + x^2 - 1$

b) $f(x) = x^{\frac{5}{2}} - 7x^2 + 3$

c) $f(x) = \sqrt{10x^3} - 16x^2 + 15$

d) $f(x) = \frac{x^2 + 4x + 2}{x - 2}$

2. For each of the following, give an example of a polynomial function that has the characteristics described.

a) a function of degree 3 that has four terms

b) a function of degree 4 that has three terms

c) a function of degree 6 that has two terms

d) a function of degree 5 that has five terms

Lesson 3.2

3. State the end behaviours of each of the following functions.

a) $f(x) = -11x^3 + x^2 - 2$

b) $f(x) = 70x^2 - 67$

c) $f(x) = x^3 - 1000$

d) $f(x) = -13x^4 - 4x^3 - 2x^2 + x + 5$

4. State whether each function has an even number of turning points or an odd number of turning points.

a) $f(x) = 6x^3 + 2x$

b) $f(x) = -20x^6 - 5x^3 + x^2 - 17$

c) $f(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$

d) $f(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$

Lesson 3.3

5. Sketch a possible graph of each of the following functions.

a) $f(x) = -(x - 8)(x + 1)$

b) $f(x) = 3(x + 3)(x + 3)(x - 1)$

c) $f(x) = (x + 2)(x - 4)(x + 2)(x - 4)$

d) $f(x) = -4(2x + 5)(x - 2)(x + 4)$

6. If the value of k is unknown, which of the following characteristics of the graph of $f(x) = k(x + 14)(x - 13)(x + 15)(x - 16)$ cannot be determined: the x-intercepts, the shape of the graph near each zero, the end behaviours, or the maximum number of turning points?

7. Determine the equation of the polynomial function that has the following zeros and passes through the point $(7, 5000)$: $x = 2$ (order 1), $x = -3$ (order 2), and $x = 5$ (order 1).

Lesson 3.4

8. Describe the transformations that were applied to $y = x^4$ to get each of the following functions.

a) $y = -25(3(x + 4))^4 - 60$

b) $y = 8\left(\frac{3}{4}x\right)^4 + 43$

c) $y = (-13x + 26)^4 + 13$

d) $y = \frac{8}{11}(-x)^4 - 1$

9. Describe the transformations that were applied to $y = x^3$ to produce the following graph.