3 Chapter Review

FREQUENTLY ASKED Questions

Q: How can you divide polynomials?

A: You can divide polynomials using an algorithm similar to long division with numbers. If the divisor is a binomial, then you can use synthetic division.

For example, you can divide $3x^3 + 2x - 17$ by $x - 2$ as follows:

Using Synthetic Division

$(x - 2) \rightarrow k = 2$

$3x^3 + 2x - 17$
$= 3x^3 + 0x^2 + 2x - 17$

$$\begin{array}{r|cccc} 2 & 3 & 0 & 2 & -17 \\ & \downarrow & 6 & 12 & 28 \\ \hline & 3 & 6 & 14 & 11 \end{array}$$

$3x^3 + 2x - 17$
$= (x - 2)(3x^2 + 6x + 14) + 11$

Using Regular Polynomial Division

$$\begin{array}{rl} & 3x^2 + 6x + 14 \\ x - 2 \overline{)} & 3x^3 + 0x^2 + 2x - 17 \\ 3x^2(x - 2) \rightarrow & \underline{3x^3 - 6x^2} \\ & 6x^2 + 2x \\ 6x(x - 2) \rightarrow & \underline{6x^2 - 12x} \\ & 14x - 17 \\ 14(x - 2) \rightarrow & \underline{14x - 28} \\ & 11 \end{array}$$

$3x^3 + 2x - 17 = (x - 2)(3x^2 + 6x + 14) + 11$

Study Aid
- See Lesson 3.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 10, 11, and 12.

Q: How do you factor a polynomial of degree 3 or greater?

A1: Use the factor theorem to determine one factor of the polynomial, and then divide to determine the other factors.

For example, to factor $x^3 - 6x^2 - 13x + 42$, let $f(x) = x^3 - 6x^2 - 13x + 42$ and determine the first possible factor by finding a number the makes $f(x) = 0$.

Possibilities: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

$f(2) = (2)^3 - 6(2)^2 - 13(2) + 42 = 0$, so $x - 2$ is a factor.

Use synthetic division to find the other factor.

$$\begin{array}{r|cccc} 2 & 1 & -6 & -13 & 42 \\ & \downarrow & 2 & 8 & -42 \\ \hline & 1 & -4 & -21 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 - 4x - 21)$ Factor the quotient.
$= (x - 2)(x - 7)(x + 3)$

Study Aid
- See Lesson 3.6, Example 2.
- Try Chapter Review Questions 14 and 15.

A2: Factor using the sum or difference of cubes pattern when appropriate:

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

For example, you can factor $27x^3 - 64$ as follows:
$27x^3 - 64$
$= (3x)^3 - (4)^3$
$= (3x - 4)(9x^2 + 12x + 16)$

Study Aid
- See Lesson 3.7, Examples 2 and 3.
- Try Chapter Review Questions 16, 17, and 18.

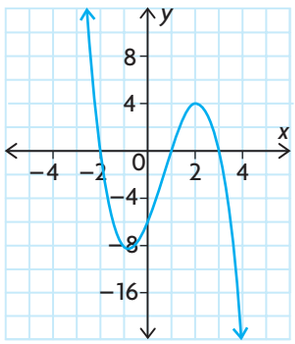
PRACTICE Questions

Lesson 3.1

1. Draw the graph of a polynomial function that has all of the following characteristics:
- $f(2) = 10, f(-3) = 0$, and $f(4) = 0$
- The y-intercept is 0.
- $f(x) > 0$ when $x < -3$ and $0 < x < 4$.
- $f(x) < 0$ when $-3 < x < 0$ and $x > 4$.
- The range is the set of real numbers.

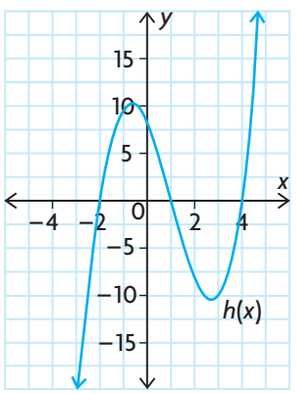
Lesson 3.2

2. Describe the end behaviours of this function.

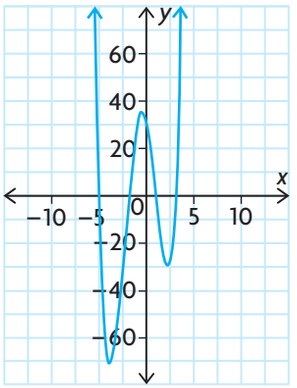


3. State the possible degree of each function, the sign of the leading coefficient, and the number of turning points.

a)



b)



Lesson 3.3

4. For each of the following, write the equations of three cubic functions that have the given zeros and belong to the same family of functions.

a) $-3, 6, 4$ **c)** $-7, 2, 3$

b) $5, -1, -2$ **d)** $9, -5, -4$

5. For each of the following, write the equations of three quartic functions that have the given zeros and belong to the same family of functions.

a) $-6, 2, 5, 8$ **c)** $0, -1, 9, 10$

b) $4, -8, 1, 2$ **d)** $-3, 3, -6, 6$

6. Sketch the graph of $f(x) = (x - 3)(x + 2)(x + 5)$ using the zeros and end behaviours.

7. Determine the equation of the function with zeros at ± 1 and -2, and a y-intercept of -6. Then sketch the function.

Lesson 3.4

8. Describe the transformations that were applied to $y = x^2$ to obtain each of the following functions.

a) $y = -2(x - 1)^2 + 23$

b) $y = \left(\frac{12}{13}(x + 9)\right)^2 - 14$

c) $y = x^2 - 8x + 16$

d) $y = \left(x + \frac{3}{7}\right)\left(x + \frac{3}{7}\right)$

e) $y = 40(-7(x - 10))^2 + 9$

9. The function $y = x^3$ has undergone each of the following sets of transformations. List three points on the resulting function.

a) vertically stretched by a factor of 25, horizontally compressed by a factor of $\frac{5}{6}$, horizontally translated 3 units to the right

b) reflected in the y-axis, horizontally stretched by a factor of 7, vertically translated 19 units down

c) reflected in the x-axis, vertically compressed by a factor of $\frac{6}{11}$, horizontally translated 5 units to the left, vertically translated 16 units up

d) vertically stretched by a factor of 100, horizontally stretched by a factor of 2, vertically translated 14 units up

e) reflected in the y-axis, vertically translated 45 units down

f) reflected in the x-axis, horizontally compressed by a factor of $\frac{7}{10}$, horizontally translated 12 units to the right, vertically translated 6 units up

Lesson 3.5

10. Calculate each of the following using long division.

a) $(2x^3 + 5x^2 + 3x - 4) \div (x + 5)$

b) $(x^4 + 4x^3 - 3x^2 - 6x - 7) \div (x^2 - 8)$

c) $(2x^4 - 2x^2 + 3x - 16) \div (x^3 + 3x^2 + 3x - 3)$

d) $(x^5 - 8x^3 - 7x - 6) \div (x^4 + 4x^3 + 4x^2 - x - 3)$

11. Divide each polynomial by $x + 2$ using synthetic division.

a) $2x^3 + 5x^2 - x - 5$

b) $3x^3 + 13x^2 + 17x + 3$

c) $2x^4 + 5x^3 - 16x^2 - 45x - 18$

d) $2x^3 + 4x^2 - 5x - 4$

12. Each divisor was divided into another polynomial, resulting in the given quotient and remainder. Determine the dividend.

a) divisor: $x - 9$, quotient: $2x^2 + 11x - 8$, remainder: 3

b) divisor: $4x + 3$, quotient: $x^3 - 2x + 7$, remainder: -4

c) divisor: $3x - 4$, quotient: $x^3 + 6x^2 - 6x - 7$, remainder: 5

d) divisor: $3x^2 + x - 5$, quotient: $x^4 - 4x^3 + 9x - 3$, remainder: $2x - 1$

Lesson 3.6

13. Without dividing, determine the remainder when $x^3 + 2x^2 - 6x + 1$ is divided by $x + 2$.

14. Factor each polynomial using the factor theorem.

a) $x^3 - 5x^2 - 22x - 16$

b) $2x^3 + x^2 - 27x - 36$

c) $3x^4 - 19x^3 + 38x^2 - 24x$

d) $x^4 + 11x^3 + 36x^2 + 16x - 64$

15. Factor fully.

a) $8x^3 - 10x^2 - 17x + 10$

b) $2x^3 + 7x^2 - 7x - 30$

c) $x^4 - 7x^3 + 9x^2 + 27x - 54$

d) $4x^4 + 4x^3 - 35x^2 - 36x - 9$

Lesson 3.7

16. Factor each difference of cubes.

a) $64x^3 - 27$

b) $512x^3 - 125$

c) $343x^3 - 1728$

d) $1331x^3 - 1$

17. Factor each sum of cubes.

a) $1000x^3 + 343$

b) $1728x^3 + 125$

c) $27x^3 + 1331$

d) $216x^3 + 2197$

18. **a)** Factor the expression $x^6 - y^6$ completely by treating it as a difference of squares.

b) Factor the same expression by treating it as a difference of cubes.

c) Explain any similarities or differences in your final results.