4 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 4.1, Examples 1 to 4.
- Try Mid-Chapter Review Questions 1, 2, and 3.

Q: How can you solve a polynomial equation?

A1: You can use an algebraic strategy using the corresponding polynomial function, the factor theorem, and division to factor the polynomial. Set each factor equal to zero and solve for the independent variable. You will need to use the quadratic formula if one of the factors is a nonfactorable quadratic.

For example, to solve the equation

$2x^4 + x^3 - 19x^2 - 14x + 24 = 0$, let $f(x) = 2x^4 + x^3 - 19x^2 - 14x + 24$. Possible values of x that make $f(x) = 0$ are numbers of the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient. Some possible values in this case are:

$$\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1, \pm 2}$$

Since $f(-2) = 0$, by the factor theorem, $(x + 2)$ is a factor of $f(x)$. Determine $f(x) \div (x + 2)$ to find the other factor.

$$\begin{array}{r|rrrrr} -2 & 2 & 1 & -19 & -14 & 24 \\ & \downarrow & -4 & 6 & 26 & -24 \\ \hline & 2 & -3 & -13 & 12 & 0 \end{array}$$

Now, $f(x) = (x + 2)(2x^3 - 3x^2 - 13x + 12)$.

Possible values of x that make the cubic polynomial 0 are numbers of the form:

$$\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12,}{\pm 2}$$

Since $f(3) = 0$, $(x - 3)$ is also a factor of $f(x)$. Divide the cubic polynomial by $(x - 3)$ to determine the other factor.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -13 & 12 \\ & \downarrow & 6 & 9 & -12 \\ \hline & 2 & 3 & -4 & 0 \end{array}$$

So, $f(x) = (x + 2)(x - 3)(2x^2 + 3x - 4)$.

$$x + 2 = 0 \text{ or } x - 3 = 0 \text{ or } 2x^2 + 3x - 4 = 0$$

$$x = -2 \text{ or } x = 3$$

Since $2x^2 + 3x - 4$ is not factorable, use the quadratic formula to determine the other zeros.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)}$$

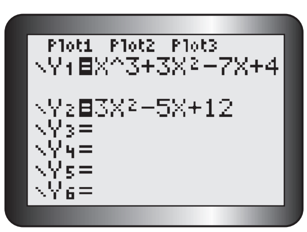
$$x = \frac{-3 \pm \sqrt{41}}{4}$$

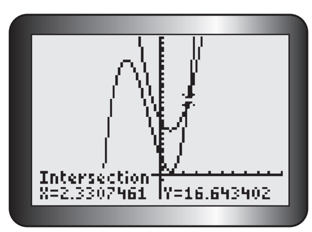
$$x \doteq -2.35 \text{ or } 0.85$$

The equation has four roots: $x = -2$, $x \doteq -2.35$, $x \doteq 0.85$, and $x = 3$.

A2: You can use a graphing strategy to find the zeros of a rearranged equation, or graph both sides of the equation separately and then determine the point(s) of intersection.

For example, to solve the equation $x^3 + 3x^2 - 7x + 4 = 3x^2 - 5x + 12$, enter both polynomials in the equation editor of the graphing calculator and then graph both corresponding functions. Use the intersect operation to determine the point of intersection of the two graphs.





The solution is $x \doteq 2.33$.

Q: How can you solve a linear inequality?

A: You solve a linear inequality using inverse operations in much the same way you would solve a linear equation. If at any time you multiply or divide the inequality by a negative number, you must reverse the inequality sign.

Study Aid

- See Lesson 4.2, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 4 to 8.

PRACTICE Questions

Lesson 4.1

1. Determine the solutions for each of the following.

a) $0 = -2x^3(2x - 5)(x - 4)^2$

b) $0 = (x^2 + 1)(2x + 4)(x + 2)$

c) $x^3 - 4x^2 = 7x - 10$

d) $0 = (x^2 - 2x - 24)(x^2 - 25)$

e) $0 = (x^3 + 2x^2)(x + 9)$

f) $-x^4 = -13x^2 + 36$

2. Jude is diving from a cliff into the ocean. His height above sea level in metres is represented by the function $h(t) = -5(t - 0.3)^2 + 25$, where t is measured in seconds.

a) Expand the height function.

b) How high is the cliff?

c) When does Jude hit the water?

d) Determine where the function is negative. What is the significance of the negative values?

3. Chris makes an open-topped box from a 30 cm by 30 cm piece of cardboard by cutting out equal squares from the corners and folding up the flaps to make the sides. What are the dimensions of each square, to the nearest hundredth of a centimetre, so that the volume of the resulting box is 1000 cm^3?

Lesson 4.2

4. Solve the following inequalities algebraically and plot the solution on a number line.

a) $2x - 4 < 3x + 7$

b) $-x - 4 \leq x + 4$

c) $-2(x - 4) \geq 16$

d) $2(3x - 7) > 3(7x - 3)$

5. Solve and state your solution using inverval notation.

$$2x < \frac{3x + 6}{2} \leq 4 + 2x$$

6. Create a linear inequality with both a constant and a linear term on each side and that has each of the following as a solution.

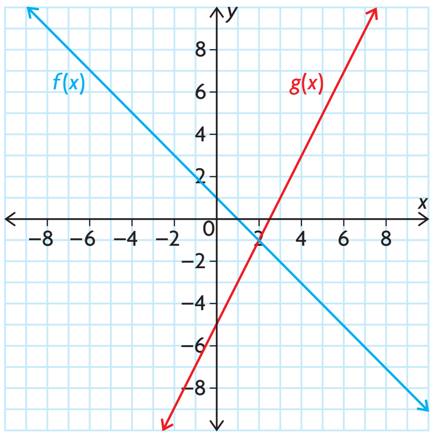
a) $x > 7$

b) $x \in (-\infty, -8)$

c) $-1 \leq x \leq 7$

d) $x \in [3, \infty)$

7. Consider the following functions.



a) Find the equations of the lines depicted.

b) Solve the inequality $f(x) < g(x)$ by examining the graph.

c) Confirm your solution by solving the inequality algebraically.

8. The New Network cell phone company charges \$20 a month for service and \$0.02 per minute of talking time. The My Mobile company charges \$15 a month for service and \$0.03 per minute of talking time.

a) Write expressions for the total bill of each company.

b) Set up an inequality that can be used to determine for what amount of time (in minutes) My Mobile is the better plan.

c) Solve your inequality.

d) Why did you have to put a restriction on the algebraic solution from part c)?