4 Chapter Review

FREQUENTLY ASKED Questions

Study Aid

- See Lesson 4.3, Examples 1, 2, and 3.
- Try Chapter Review Questions 10 to 13.

Q: How do you solve a polynomial inequality?

A1: Sometimes you can use an algebraic strategy if the polynomial is factorable. Use inverse operations to make one side of the inequality equal to zero, factor the polynomial to determine its zeros, then test values to the left, between, and to the right of the zeros to determine which intervals will satisfy the inequality. This can be done using a number line or a factor table.

For example, to solve $3x^3 - 4x^2 - 3x - 10 > 2x^3 - 6x^2 + 6x + 8$

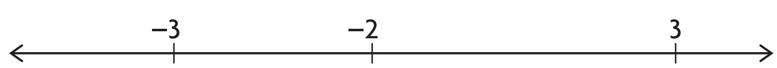
$$x^3 + 2x^2 - 9x - 18 > 0$$
$$x^2(x + 2) - 9(x + 2) > 0$$
$$(x^2 - 9)(x + 2) > 0$$
$$(x + 3)(x - 3)(x + 2) > 0$$

The equation $(x + 3)(x - 3)(x + 2) = 0$ has solutions $x = -3$, $x = 3$, or $x = -2$. These numbers divide the domain of real numbers into the following intervals:

$x < -3$, $-3 < x < -2$, $-2 < x < 3$, and $x > 3$

Substitute values that lie in each interval into the original inequality, $3x^3 - 4x^2 - 3x - 10 > 2x^3 - 6x^2 + 6x + 8$.

Let $f(x) = 3x^3 - 4x^2 - 3x - 10$ and let $g(x) = 2x^3 - 6x^2 + 6x + 8$.



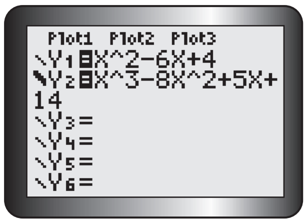
$x < -3$	$-3 < x < -2$	$-2 < x < 3$	$x > 3$
$f(-4) = -254$	$f(-2.5) = -74.375$	$f(1) = -14$	$f(4) = 106$
$g(-4) = -240$	$g(-2.5) = -75.75$	$g(1) = 10$	$g(4) = 64$
$f(x) < g(x)$	$f(x) > g(x)$	$f(x) < g(x)$	$f(x) > g(x)$

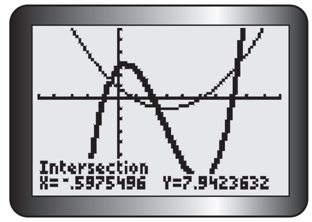
$3x^3 - 4x^2 - 3x - 10 > 2x^3 - 6x^2 + 6x + 8$ when $-3 < x < -2$ and $x > 3$.

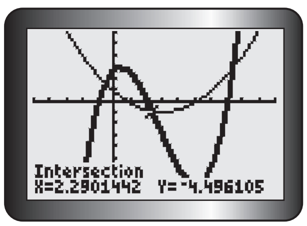
A2: You can always use a graphing strategy using one of the following methods.

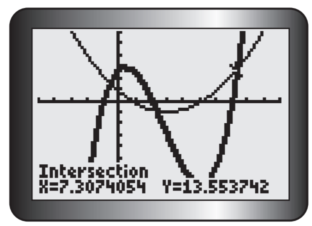
1. Treat each side of the inequality as two separate functions and graph them. Then determine their intersection points and identify the intervals for which one function is above or below the other, as required.
2. Create an equivalent inequality with zero on one side, and then identify the intervals created by the zeros of the graph of the corresponding function. Find where the graph lies above the x-axis (where $f(x) > 0$) or below (where $f(x) < 0$), as required.

For example, to solve $x^2 - 6x + 4 \geq x^3 - 8x^2 + 5x + 14$, use the graphing calculator to determine the intersection points for the functions.









The two functions intersect when $x \doteq -0.598$, 2.290, and 7.307.

Refer to the graph to see where Y1 is above Y2 on the intervals defined by these three points. For example, for points to the left of $x = -0.598$, Y1 is above Y2.

So, $x^2 - 6x + 4 \geq x^3 - 8x^2 + 5x + 14$ when $x \leq -0.598$ and when $2.290 \leq x \leq 7.307$.

Q: How do you calculate an average rate of change for a polynomial function?

A: The average rate of change is the slope of a secant that connects two points on the function. To calculate the average rate of change on the interval $x_1 \leq x \leq x_2$ for a function, $f(x)$, calculate the average rate of change, $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Study *Aid*

- See Lesson 4.4, Example 2.
- Try Chapter Review Questions 14, 17, and 18.

Q: How do you approximate the instantaneous rate of change for a polynomial function?

A1: You can calculate the average rate of change for a very small interval on either side of the point at which you wish to calculate the instantaneous rate of change using the difference quotient $\dfrac{f(a + h) - f(a)}{h}$, where h is a very small value.

A2: You can graph the function, either by hand or by using a graphing calculator, and draw a tangent line at the required point and estimate its slope.

Study *Aid*

- See Lesson 4.4, Examples 1 and 3.
- Try Chapter Review Questions 15, 16, and 18.

PRACTICE Questions

Lesson 4.1

1. Solve each of the following equations by factoring.

a) $x^4 - 16x^2 + 75 = 2x^2 - 6$
b) $2x^2 + 4x - 1 = x + 1$
c) $4x^3 - x^2 - 2x + 2 = 3x^3 - 2(x^2 - 1)$
d) $-2x^2 + x - 6 = -x^3 + 2x - 8$

2. Solve the equation algebraically, and check your solution graphically:
$18x^4 - 53x^3 + 52x^2 - 14x - 8 =$
$3x^4 - x^3 + 2x - 8$

3. a) Write the equation of a polynomial $f(x)$ that has a degree of 4, zeros at $x = 1, 2, -2$, and -1, and has a y-intercept of 4.
b) Determine the values of x where $f(x) = 48$.

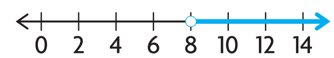
4. An open-topped box is made from a rectangular piece of cardboard, with dimensions of 24 cm by 30 cm, by cutting congruent squares from each corner and folding up the sides. Determine the dimensions of the squares to be cut to create a box with a volume of 1040 cm^3.

5. Between 1985 through 1995, the number of home computers, in thousands, sold in Canada is estimated by
$C(t) = 0.92(t^3 + 8t^2 + 40t + 400)$,
where t is in years and $t = 0$ for 1985.

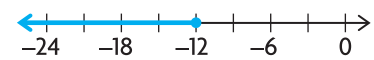
a) Explain why you can use this model to predict the number of home computers sold in 1993, but not to predict sales in 2005.
b) Explain how to find when the number of home computer sales in Canada reached 1.5 million, using this model.
c) In what year did home computer sales reach 1.5 million?

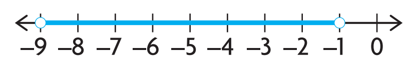
Lesson 4.2

6. For each number line given, write an inequality with both constant and linear terms on each side that has the corresponding solution.

a)


b)


c)


d)


7. Solve the following inequalities algebraically. State your answers using interval notation.

a) $2(4x - 7) > 4(x + 9)$
b) $\dfrac{x - 4}{5} \geq \dfrac{2x + 3}{2}$
c) $-x + 2 > x - 2$
d) $5x - 7 \leq 2x + 2$

8. Solve the following inequalities. State your answers using set notation.

a) $-3 < 2x + 1 < 9$
b) $8 \leq -x + 8 \leq 9$
c) $6 + 2x \geq 0 \geq -10 + 2x$
d) $x + 1 < 2x + 7 < x + 5$

9. A phone company offers two options. The first plan is an unlimited calling plan for \$34.95 a month. The second plan is a \$20.95 monthly fee plus \$0.04 a minute for call time.

a) When is the unlimited plan a better deal?
b) Graph the situation to confirm your answer from part a).

Lesson 4.3

10. Select a strategy and determine the interval(s) for which each inequality is true.

a) $(x + 1)(x - 2)(x + 3)^2 < 0$

b) $\dfrac{(x - 4)(2x + 3)}{5} \geq \dfrac{2x + 3}{5}$

c) $-2(x - 1)(2x + 5)(x - 7) > 0$

d) $x^3 + x^2 - 21x + 21 \leq 3x^2 - 2x + 1$

11. Determine algebraically where the intervals of the function are positive and negative.
$f(x) = 2x^4 - 2x^3 - 32x^2 - 40x$

12. Solve the following inequality using graphing technology:
$x^3 - 2x^2 + x - 3 \geq 2x^3 + x^2 - x + 1$

13. In Canada, hundreds of thousands of cubic metres of wood are harvested each year. The function
$f(x) = 1135x^4 - 8197x^3 + 15\,868x^2 - 2157x + 176\,608, 0 \leq x \leq 4$, models the volume harvested, in cubic metres, from 1993 to 1997. Estimate the intervals (in years and months) when less than 185 000 m^3 were harvested.

Lesson 4.4

14. For each of the following functions, determine the average rate of change in $f(x)$ from $x = 2$ to $x = 7$, and estimate the instantaneous rate of change at $x = 5$.

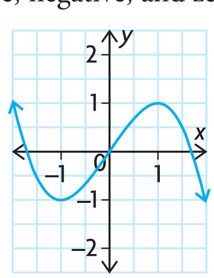
a) $f(x) = x^2 - 2x + 3$

b) $h(x) = (x - 3)(2x + 1)$

c) $g(x) = 2x^3 - 5x$

d) $v(x) = -x^4 + 2x^2 - 5x + 1$

15. Given the following graph, determine the intervals of x where the instantaneous rate of change is positive, negative, and zero.



16. The height in metres of a projectile is modelled by the function $h(t) = -5t^2 + 25$, where t is the time in seconds.

a) Find the point when the object hits the ground.

b) Find the average rate of change from the point when the projectile is launched $(t = 0)$ to the point in which it hits the ground.

c) Estimate the object's speed at the point of impact.

17. Consider the function $f(x) = 2x^3 + 3x - 1$.

a) Find the average rate of change from $x = 3$ to $x = 3.0001$.

b) Find the average rate of change from $x = 2.9999$ to $x = 3$.

c) Why are your answers so similar? Estimate the instantaneous rate of change at $x = 3$.

18. The incidence of lung cancer in Canadians per 100 000 people is shown below.

Year	Males	Females
1975	73.1	14.7
1980	83.2	21.7
1985	93.2	30.9
1990	92.7	36.5
1995	84.7	40.8
2000	78.6	46.4

Source: Cancer Bureau, Health Canada

a) Use regression to determine a cubic function to represent the curve of best fit for both the male and female data.

b) According to your models, when will more females have lung cancer than males?

c) Was the incidence of lung cancer changing at a faster rate in the male or female population during the period fom 1975 to 2000? Justify your answer.

d) Was the incidence of lung cancer changing at a faster rate in the male or female population in 1998? Justify your answer.