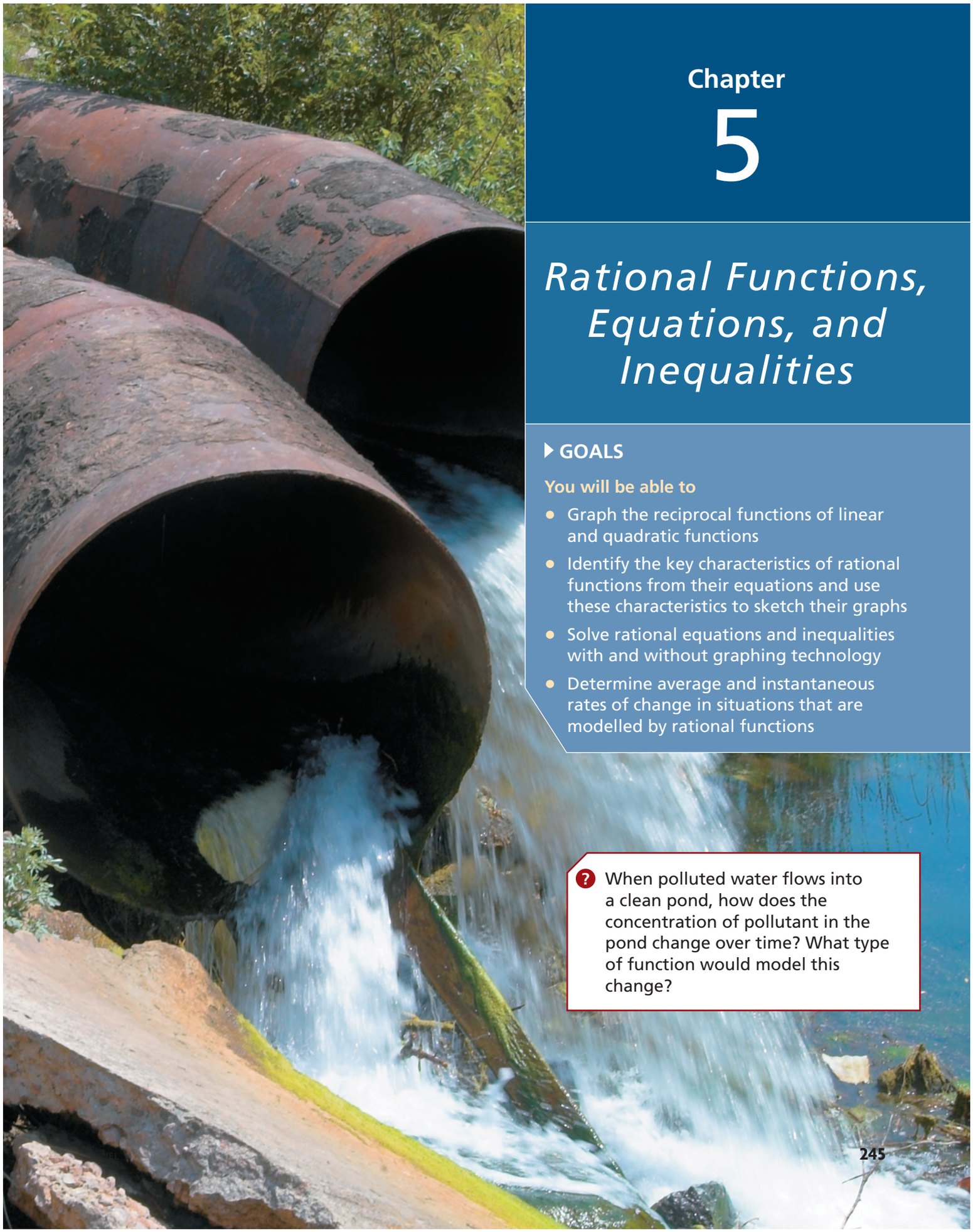

Chapter

5

Rational Functions, Equations, and Inequalities

GOALS

You will be able to

- Graph the reciprocal functions of linear and quadratic functions
- Identify the key characteristics of rational functions from their equations and use these characteristics to sketch their graphs
- Solve rational equations and inequalities with and without graphing technology
- Determine average and instantaneous rates of change in situations that are modelled by rational functions

? When polluted water flows into a clean pond, how does the concentration of pollutant in the pond change over time? What type of function would model this change?

5 Getting Started

Study *Aid*

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
1	R-3
2, 3, 4, 8	R-4
7	R-8

SKILLS AND CONCEPTS You Need

1. Factor each expression.

a) $x^2 - 3x - 10$

b) $3x^2 + 12x - 15$

c) $16x^2 - 49$

d) $9x^2 - 12x + 4$

e) $3a^2 + a - 30$

f) $6x^2 - 5xy - 21y^2$

2. Simplify each expression. State any restrictions on the variables, if necessary.

a) $\dfrac{12 - 8s}{4}$

b) $\dfrac{6m^2n^4}{18m^3n}$

c) $\dfrac{9x^3 - 12x^2 - 3x}{3x}$

d) $\dfrac{25x - 10}{5(5x - 2)^2}$

e) $\dfrac{x^2 + 3x - 18}{9 - x^2}$

f) $\dfrac{a^2 + 4ab - 5b^2}{2a^2 + 7ab - 15b^2}$

3. Simplify each expression, and state any restrictions on the variable.

a) $\dfrac{3}{5} \times \dfrac{7}{9}$

b) $\dfrac{2x}{5} \div \dfrac{x^2}{15}$

c) $\dfrac{x^2 - 4}{x - 3} \div \dfrac{x + 2}{12 - 4x}$

d) $\dfrac{x^3 + 4x^2}{x^2 - 1} \times \dfrac{x^2 - 5x + 6}{x^2 - 3x}$

4. Simplify each expression, and state any restrictions on the variable.

a) $\dfrac{2}{3} + \dfrac{6}{7}$

b) $\dfrac{3x}{4} + \dfrac{5x}{6}$

c) $\dfrac{1}{x} + \dfrac{4}{x^2}$

d) $\dfrac{5}{x - 3} - \dfrac{2}{x}$

e) $\dfrac{2}{x - 5} + \dfrac{y}{x^2 - 25}$

f) $\dfrac{6}{a^2 - 9a + 20} - \dfrac{8}{a^2 - 2a - 15}$

5. Solve and check.

a) $\dfrac{5x}{8} = \dfrac{15}{4}$

b) $\dfrac{x}{4} + \dfrac{1}{3} = \dfrac{5}{6}$

c) $\dfrac{4x}{5} - \dfrac{3x}{10} = \dfrac{3}{2}$

d) $\dfrac{x + 1}{2} - \dfrac{2x - 1}{3} = -1$

6. Sketch the graph of the reciprocal function $f(x) = \dfrac{1}{x}$ and describe its characteristics. Include the domain and range, as well as the equations of the asymptotes.

7. List the transformations that need to be applied to $y = \frac{1}{x}$ to graph each of the following reciprocal functions. Then sketch the graph.

a) $f(x) = \frac{1}{x + 3}$

b) $f(x) = \frac{2}{x - 1}$

c) $f(x) = -\frac{1}{2x} - 3$

d) $f(x) = \frac{2}{-3(x - 2)} + 1$

8. Describe the steps that are required to divide two rational expressions. Use your description to simplify $\frac{9y^2 - 4}{4y - 12} \div \frac{9y^2 + 12 + 4}{18 - 6y}$.

APPLYING What You Know

Painting Houses

Tony can paint the exterior of a house in six working days. Rebecca takes nine days to complete the same painting job.

? How long will Rebecca and Tony take to paint a similar house, if they work together?

A. What fraction of the job can Tony complete in one day? What fraction of the job can Rebecca complete?

B. Write a numerical expression to represent the fraction of the job that Rebecca and Tony can complete in one day, if they work together.

C. Let x represent the number of days that Rebecca and Tony, working together, will take to complete the job. Explain why $\frac{1}{x}$ represents the fraction of the job Rebecca and Tony will complete in one day when they work together.

D. Use your answers for parts B and C to write an equation. Determine the **lowest common denominator** for the **rational expressions** in your equation. Rewrite the equation using the lowest common denominator.

E. Solve the equation you wrote in part D by collecting like terms and comparing the numerators on the two sides of the equation.

F. What is the amount of time Rebecca and Tony will take to paint a similar house, when they work together?