5 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can you use the graph of a linear or quadratic function to graph its reciprocal function?

A: If you have the graph of a linear or quadratic function, you can draw the graph of its reciprocal function as follows:

1. Draw a vertical asymptote for the reciprocal function at each zero of the original function. The x-axis is a horizontal asymptote for the reciprocal function, unless the original function is a constant function.
2. The reciprocal function has the same positive/negative intervals that the original function has, so you can shade the regions where there will be no graph.
3. Mark any points where the y-value of the original function is 1 or -1. The reciprocal function also goes through these points.
4. The y-intercept of the reciprocal function is the reciprocal of the y-intercept of the original function. Determine and mark the y-intercept of the reciprocal function.
5. If the original function is quadratic, the reciprocal function has a local maximum/minimum at the same x-value as the vertex of the quadratic function. The y-value of this local maximum/minimum is the reciprocal of the y-value of the vertex. Determine and mark the local maximum/minimum point.
6. Draw the pieces of the graph of the reciprocal function through the marked points, approaching the asymptotes.

Study *Aid*

- See Lesson 5.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 and 2.

Q: What are rational functions, and what are the characteristics of their graphs?

A: Rational functions are quotients of polynomial functions. Their equations have the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$. Graphs of rational functions may have vertical, horizontal, or oblique asymptotes. Some rational functions have holes in their graphs.

Study *Aid*

- See Lesson 5.2.
- Try Mid-Chapter Review Question 3.

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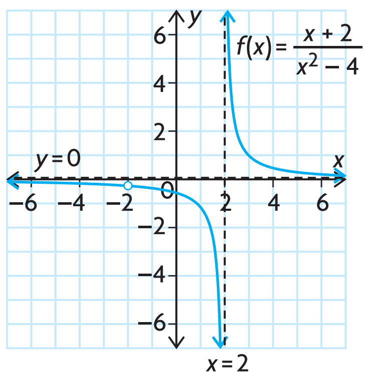
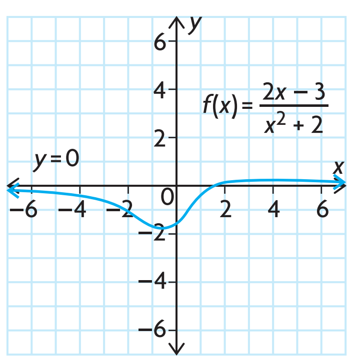
- See Lesson 5.2.
- Try Mid-Chapter Review Question 4.





Q: Most rational functions have one or more discontinuities. Where and why do these discontinuities occur? When is a rational function continuous?

A: If the polynomial function in the denominator of a rational function has one or more zeros, the rational function will be discontinuous at these points. If a value of x can be zero in both the numerator and the denominator of a rational function (that is, if the numerator and denominator have a common linear factor), the result is a hole. This type of discontinuity is called a point discontinuity.

If a zero in the denominator does not correspond to a zero in the numerator, there will be a vertical asymptote at the x-value. This is called an infinite discontinuity.

For example, $f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)}$ has a point discontinuity where $x = -2$ because -2 is a zero of both the denominator, $q(x) = x^2 - 4$, and the numerator, $p(x) = x + 2$. The graph of $f(x)$ has an infinite discontinuity where $x = 2$ because 2 is a zero of $q(x)$ but not of $p(x)$. The graph also has a hole at $x = -2$ and a vertical asymptote at $x = 2$. Note that $\frac{p(-2)}{q(-2)} = \frac{0}{0}$, but $\frac{p(2)}{q(2)} = \frac{4}{0}$.

If the polynomial function in the denominator of a rational function does not have any zeros, the rational function is continuous. Its graph is a smooth curve, with no breaks.

For example, $f(x) = \frac{2x-3}{x^2+2}$ is a continuous rational function with a horizontal asymptote at $y = 0$.

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- See Lesson 5.3, Example 2.
- Try Mid-Chapter Review Questions 5 to 8.

Q: How do you determine the equations of the vertical and horizontal asymptotes of a rational function of the form $f(x) = \frac{b}{cx+d}$ and $f(x) = \frac{ax+b}{cx+d}$?

A: You can determine the equations of the vertical and horizontal asymptotes directly from the equation of a rational function of the form $f(x) = \frac{b}{cx+d}$ or $f(x) = \frac{ax+b}{cx+d}$. The vertical asymptote occurs at the zero of the function in the denominator. The equation of the vertical asymptote is $x = -\frac{d}{c}$. The horizontal asymptote describes the end behaviour of the function when $x \to \pm\infty$.

All rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ have $y = \frac{a}{c}$ as a horizontal asymptote.

All rational functions of the form $f(x) = \frac{b}{cx+d}$ have $y = 0$ (the x-axis) as a horizontal asymptote.

PRACTICE Questions

Lesson 5.1

1. State the reciprocal of each function, and determine the locations of any vertical asymptotes.

a) $f(x) = x - 3$
b) $f(q) = -4q + 6$
c) $f(z) = z^2 + 4z - 5$
d) $f(d) = 6d^2 + 7d - 3$

2. For each function, determine the domain and range, intercepts, positive/negative intervals, and intervals of increase/decrease. Use this information to sketch the graphs of the function and its reciprocal.

a) $f(x) = 4x + 6$
b) $f(x) = x^2 - 4$
c) $f(x) = x^2 + 6$
d) $f(x) = -2x - 4$

Lesson 5.2

3. Different characteristics of the graph of a rational function are created by different characteristics of the function. List at least four characteristics of a graph and the characteristic of the function that causes each one. Make sure that you include a characteristic of a continuous rational function in your list.

4. For each function, determine the equations of any vertical asymptotes, the locations of any holes, and the existence of any horizontal asymptotes (other than the x-axis) or oblique asymptotes.

a) $y = \dfrac{x}{x - 2}$

b) $y = \dfrac{x - 1}{3x - 3}$

c) $y = \dfrac{-7x}{4x + 2}$

d) $y = \dfrac{x^2 + 2}{x - 6}$

e) $y = \dfrac{1}{x^2 + 2x - 15}$

Lesson 5.3

5. List the functions that had a horizontal asymptote in question 4, and give the equation of the horizontal asymptote.

6. For each function, determine the domain, intercepts, asymptotes, and positive/negative intervals. Use these characteristics to sketch the graph of the function. Then describe where the function is increasing or decreasing.

a) $f(x) = \dfrac{5}{x - 6}$

b) $f(x) = \dfrac{3x}{x + 4}$

c) $f(x) = \dfrac{5x + 10}{x + 2}$

d) $f(x) = \dfrac{x - 2}{2x - 1}$

7. Kevin is trying to develop a reciprocal function to model some data that he has. He wants the horizontal asymptote to be $y = 7$. He also wants the graph to decrease and approach $y = 7$ as x approaches infinity, so he chooses the equation $y = \frac{7x + 6}{x}$. Then he decides that he needs the vertical asymptote to be $x = -1$, so he changes the equation to $y = \frac{7x + 6}{x + 1}$. What happened to the graph of Kevin's function? Did it give him the result he wanted? Explain why or why not.

8. For the function $f(x) = \frac{7x - m}{2 - nx}$, find the values of m and n such that the vertical asymptote is at $x = 6$ and the x-intercept is 5.

9. Create a rational function that has a domain of $\{x \in \mathbf{R} | x \neq -2\}$ and no vertical asymptote. Describe the graph of this function.