5 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

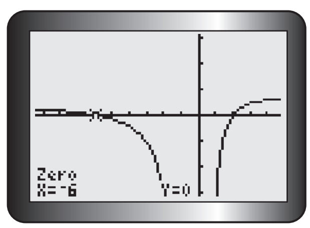
- See Lesson 5.4, Examples 1, 2, and 3.
- Try Chapter Review Questions 7, 8, and 9.

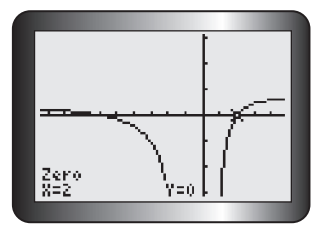
Q: How do you solve and verify a rational equation such as $\frac{3x - 8}{2x - 1} = \frac{x - 4}{x + 1}$?

A: You can solve a simple rational equation algebraically by multiplying each term in the equation by the lowest common denominator and then solving the resulting polynomial equation.

For example, to solve $\frac{3x - 8}{2x - 1} = \frac{x - 4}{x + 1}$, multiply the equation by $(2x - 1)(x + 1)$, where $x \neq -1$ or $\frac{1}{2}$. Then solve the resulting polynomial equation.

To verify your solutions, you can graph the corresponding function, $f(x) = \frac{3x - 8}{2x - 1} - \frac{x - 4}{x + 1}$, using graphing technology and determine the zeros of f.





The zeros are -6 and 2, so the solution to the equation is $x = -6$ or 2.

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- See Lesson 5.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 10 and 11.

Q: How do you solve a rational inequality, such as $\frac{x - 2}{x + 1} > \frac{x - 6}{x - 2}$?

A1: You can solve a rational inequality algebraically by creating and solving an equivalent linear or polynomial inequality with zero on one side. For factorable polynomial inequalities of degree 2 or more, use a table to identify the positive/negative intervals created by the zeros and vertical asymptotes of the rational expression.

A2: You can use graphing technology to graph the functions on both sides of the inequality, determine their intersection and the locations of all vertical asymptotes, and then note the intervals of x that satisfy the inequality.

Q: How do you determine the average or instantaneous rate of change of a rational function?

A: You can determine average and instantaneous rates of change of a rational function at points within the domain of the function using the same methods that are used for polynomial functions.

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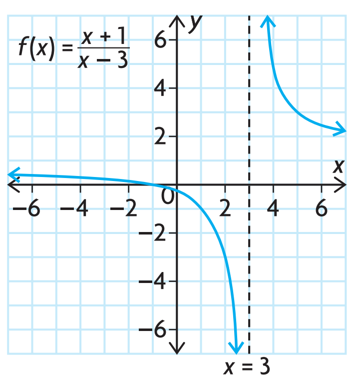
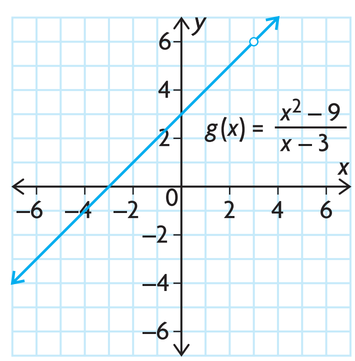
- See Lesson 5.6, Examples 1, 2, and 3.
- Try Chapter Review Questions 12, 13, and 14.

Q: When is it not possible to determine the average or instantaneous rate of change of a rational function?

A: You cannot determine the average and instantaneous rates of change of a rational function at a point where the graph has a hole or a vertical asymptote. You can only calculate the instantaneous rate of change at a point where the rational function is defined and where a tangent line can be drawn. A rational function is not defined at a point where there is a hole or a vertical asymptote. For example, $f(x) = \frac{x+1}{x-3}$ and $g(x) = \frac{x^2-9}{x-3}$ are rational functions that are not defined at $x = 3$.



The graph of $f(x)$ has a vertical asymptote at $x = 3$.



The graph of $g(x)$ has a hole at $x = 3$.

You cannot draw a tangent line on either graph at $x = 3$, so you cannot determine an instantaneous rate of change at this point.

PRACTICE Questions

Lesson 5.1

1. For each function, determine the domain and range, intercepts, positive/negative intervals, and increasing and decreasing intervals. Use this information to sketch a graph of the reciprocal function.

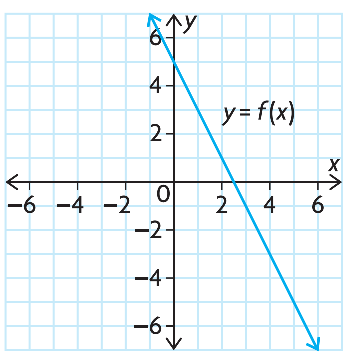
a) $f(x) = 3x + 2$

b) $f(x) = 2x^2 + 7x - 4$

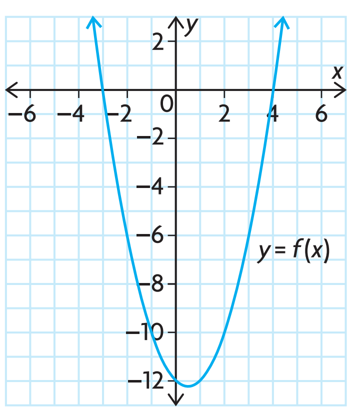
c) $f(x) = 2x^2 + 2$

2. Given the graphs of $f(x)$ below, sketch the graphs of $y = \frac{1}{f(x)}$.

a)



b)



Lesson 5.2

3. For each function, determine the equations of any vertical asymptotes, the locations of any holes, and the existence of any horizontal or oblique asymptotes.

a) $y = \dfrac{1}{x + 17}$

b) $y = \dfrac{2x}{5x + 3}$

c) $y = \dfrac{3x + 33}{-4x^2 - 42x + 22}$

d) $y = \dfrac{3x^2 - 2}{x - 1}$

Lesson 5.3

4. The population of locusts in a Prairie a town over the last 50 years is modelled by the function $f(x) = \frac{75x}{x^2 + 3x + 2}$. The locust population is given in hundreds of thousands. Describe the locust population in the town over time, where x is time in years.

5. For each function, determine the domain, intercepts, asymptotes, and positive/negative intervals. Use these characteristics to sketch the graph of the function. Then describe where the function is increasing or decreasing.

a) $f(x) = \dfrac{2}{x + 5}$

b) $f(x) = \dfrac{4x - 8}{x - 2}$

c) $f(x) = \dfrac{x - 6}{3x - 18}$

d) $f(x) = \dfrac{4x}{2x + 1}$

6. Describe how you can determine the behaviour of the values of a rational function on either side of a vertical asymptote.

Lesson 5.4

7. Solve each equation algebraically, and verify your solution using a graphing calculator.

a) $\frac{x-6}{x+2}=0$

b) $15x+7=\frac{2}{x}$

c) $\frac{2x}{x-12}=\frac{-2}{x+3}$

d) $\frac{x+3}{-4x}=\frac{x-1}{-4}$

8. A group of students have volunteered for the student council car wash. Janet can wash a car in m minutes. Rodriguez can wash a car in $m-5$ minutes, while Nick needs the same amount of time as Janet. If they all work together, they can wash a car in about 3.23 minutes. How long does Janet take to wash a car?

9. The concentration of a toxic chemical in a spring-fed lake is given by the equation $c(x)=\frac{50x}{x^2+3x+6}$, where c is given in grams per litre and x is the time in days. Determine when the concentration of the chemical is 6.16 g/L.

Lesson 5.5

10. Use an algebraic process to find the solution set of each inequality. Verify your answers using graphing technology.

a) $-x+5<\frac{1}{x+3}$

b) $\frac{55}{x+16}>-x$

c) $\frac{2x}{3x+4}>\frac{x}{x+1}$

d) $\frac{x}{6x-9}\leq\frac{1}{x}$

11. A biologist predicted that the population of tadpoles in a pond could be modelled by the function $f(t)=\frac{40t}{t^2+1}$, where t is given in days. The function that actually models the tadpole population is $g(t)=\frac{45t}{t^2+8t+7}$. Determine where $g(t)>f(t)$.

Lesson 5.6

12. Estimate the slope of the line that is tangent to each function at the given point. At what point(s) is it not possible to draw a tangent line?

a) $f(x)=\frac{x+3}{x-3}$, where $x=4$

b) $f(x)=\frac{2x-1}{x^2+3x+2}$, where $x=1$

13. The concentration, c, of a drug in the bloodstream t hours after the drug was taken orally is given by $c(t)=\frac{5t}{t^2+7}$, where c is measured in milligrams per litre.

a) Calculate the average rate of change in the drug's concentration during the first 2 h since ingestion.

b) Estimate the rate at which the concentration of the drug is changing after exactly 3 h.

c) Graph $c(t)$ on a graphing calculator. When is the concentration of the drug increasing the fastest in the bloodstream? Explain.

14. Given the function $f(x)=\frac{2x}{x-4}$, determine the coordinates of a point on $f(x)$ where the slope of the tangent line equals the slope of the secant line that passes through $A(5, 10)$ and $B(8, 4)$.

15. Describe what happens to the slope of a tangent line on the graph of a rational function as the x-coordinate of the point of tangency

a) gets closer and closer to the vertical asymptote.

b) grows larger in both the positive and negative direction.