6.3 Exploring Graphs of the Primary Trigonometric Functions

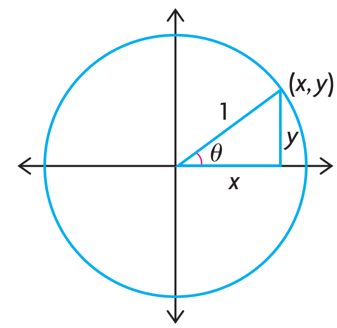
GOAL

Use radians to graph the primary trigonometric functions.

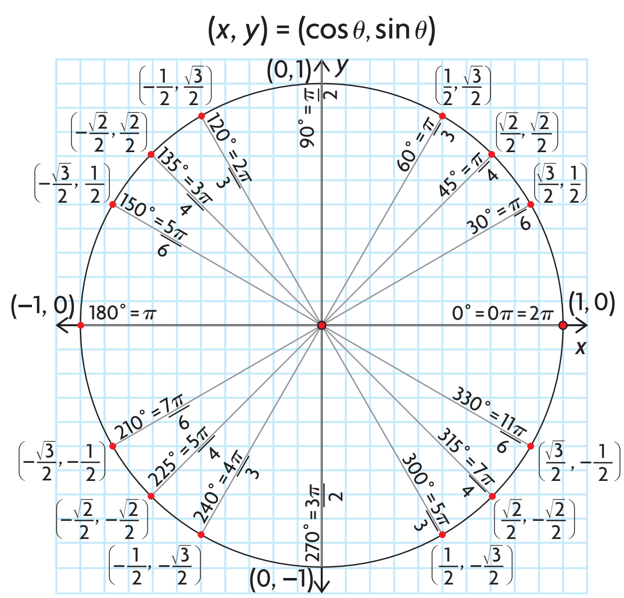
YOU WILL NEED

- graph paper
- graphing calculator

EXPLORE the Math



The unit circle is a circle that is centred at the origin and has a radius of 1 unit. On the unit circle, the sine and cosine functions take a particularly simple form: $\sin \theta = \frac{y}{1} = y$ and $\cos \theta = \frac{x}{1} = x$. The value of $\sin \theta$ is the y-coordinate of each point on the circle, and the value of $\cos \theta$ is the x-coordinate. As a result, each point on the circle can be represented by the ordered pair $(x, y) = (\cos \theta, \sin \theta)$, where θ is the angle formed between the positive x-axis and the terminal arm of the angle that passes through each point. For example, the point $\left(\cos \frac{\pi}{6}, \sin \frac{\pi}{6}\right)$ lies on the terminal arm of the angle $\frac{\pi}{6}$. Evaluating each trigonometric expression using the special triangles results in the ordered pair $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Repeating this process for other angles between 0 and 2π results in the following diagram:



? What do the graphs of the primary trigonometric functions look like when θ is expressed in radians?

A. Copy the following table. Complete the table using a calculator and the unit circle shown to approximate each value to two decimal places.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$									
$\cos\theta$									

θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$								
$\cos\theta$								

B. Plot the ordered pairs $(\theta, \sin\theta)$, and sketch the graph of the function $y = \sin\theta$. On the same pair of axes, plot the ordered pairs $(\theta, \cos\theta)$ and sketch the graph of the function $y = \cos\theta$.

C. State the domain, range, amplitude, equation of the axis, and period of each function.

D. Recall that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Use the values from your table for part A to calculate the value of $\tan\theta$. Use a calculator to confirm your results, to two decimal places.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\frac{\sin\theta}{\cos\theta}$									

θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\frac{\sin\theta}{\cos\theta}$								

E. What do you notice about the value of the tangent ratio when $\cos\theta = 0$? What do you notice about its value when $\sin\theta = 0$?

F. Based on your observations in part E, what characteristics does this imply for the graph of $y = \tan\theta$?

G. What do you notice about the value of the tangent ratio when $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}$, and $\pm\frac{7\pi}{4}$? Why does this occur?

H. On a new pair of axes, plot the ordered pairs $(\theta, \tan\theta)$ and sketch the graph of the function $y = \tan\theta$, where $0 \le \theta \le 2\pi$.

I. Determine the domain, range, amplitude, equation of the axis, and period of this function, if possible.

Reflecting

J. The tangent function is directly related to the slope of the line segment that joins the origin to each point on the unit circle. Explain why.

K. Where are the vertical asymptotes for the tangent graph located when $0 \le \theta \le 2\pi$, and what are their equations? Explain why they are found at these locations.

L. How does the period of the tangent function compare with the period of the sine and cosine functions?

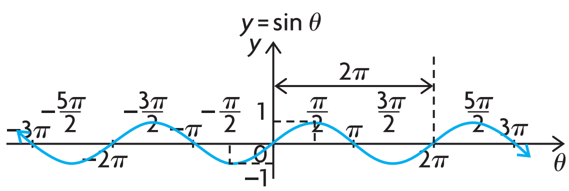
In Summary

Key Idea

- The graphs of the primary trigonometric functions can be summarized as follows:

Key points when $0 \le \theta \le 2\pi$

θ	$y = \sin(\theta)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

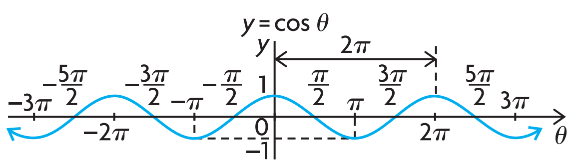


Period $= 2\pi$
Axis: $y = 0$
Amplitude $= 1$
Maximum value $= 1$
Minimum value $= -1$

$D = \{\theta \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \le y \le 1\}$

Key points when $0 \le \theta \le 2\pi$

θ	$y = \cos(\theta)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



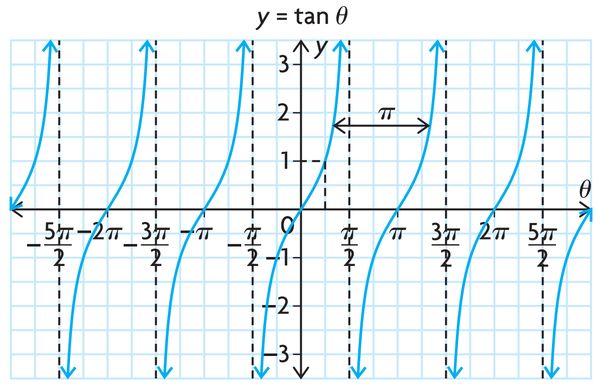
Period $= 2\pi$
Axis: $y = 0$
Amplitude $= 1$
Maximum value $= 1$
Minimum value $= -1$

$D = \{\theta \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \le y \le 1\}$

(continued)

Key points:

- y-intercept $= 0$
- θ-intercepts $= 0, \pm\pi, \pm 2\pi, \ldots$



Period $= \pi$

Axis: $y = 0$

Amplitude: undefined

No maximum or minimum values

Vertical asymptotes:
$\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

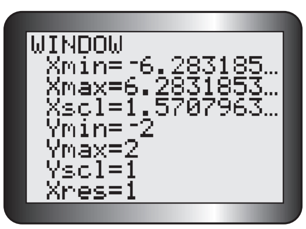
$D = \left\{\theta \in \mathbf{R} \,\middle|\, \theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\right\}$

$R = \{y \in \mathbf{R}\}$

FURTHER Your Understanding

1. **a)** Examine the graphs of $y = \sin\theta$ and $y = \cos\theta$. Create a table to compare their similarities and differences.

b) Repeat part a) using the graphs of $y = \sin\theta$ and $y = \tan\theta$.



2. **a)** Use a graphing calculator, in radian mode, to create the graphs of the trigonometric functions $y = \sin\theta$ and $y = \cos\theta$ on the interval $-2\pi \leq \theta \leq 2\pi$. To do this, enter the functions $Y1 = \sin\theta$ and $Y2 = \cos\theta$ in the equation editor, and use the window settings shown.

b) Determine the values of θ where the functions intersect.

c) The equation $t_n = a + (n - 1)d$ can be used to represent the general term of any arithmetic sequence, where a is the first term and d is the common difference. Use this equation to find an expression that describes the location of each of the following values for $y = \sin\theta$, where $n \in \mathbf{I}$ and θ is in radians.

i) θ-intercepts

ii) maximum values

iii) minimum values

3. Find an expression that describes the location of each of the following values for $y = \cos\theta$, where $n \in \mathbf{I}$ and θ is in radians.

a) θ-intercepts **b)** maximum values **c)** minimum values

4. Graph $y = \frac{\sin\theta}{\cos\theta}$ using a graphing calculator in radian mode. Compare your graph with the graph of $y = \tan\theta$.

5. Find an expression that describes the location of each of the following values for $y = \tan\theta$, where $n \in \mathbf{I}$ and θ is in radians.

a) θ-intercepts **b)** vertical asymptotes