# Mid-Chapter Review

## FREQUENTLY ASKED Questions

**Study Aid**

- See Lesson 6.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 1, 2, and 3.

**Q: How are radians and degrees related?**

**A:** Radians are determined by the relationship $\theta = \frac{a}{r}$, where $\theta$ is the angle subtended by arc length $a$ in a circle with radius $r$. One revolution creates an angle of 360°, or $2\pi$ radians. Since $360° = 2\pi$ radians, it follows that $180° = \pi$ radians. This relationship can be used to convert between the two measures.

- To convert from degrees to radians, multiply by $\frac{\pi}{180°}$.
- To convert from radians to degrees, either substitute 180° for $\pi$ or multiply by $\frac{180°}{\pi}$.

Here are three examples:

$$75° = 75° \times \frac{\pi}{180°} = \frac{5\pi}{12}$$

$$\frac{5\pi}{4} = \frac{5(180°)}{4} = 225°$$

$$3 \text{ radians} = \frac{3(180°)}{\pi} \doteq 171.887°$$

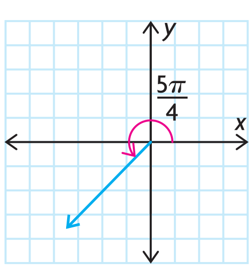
**Study Aid**

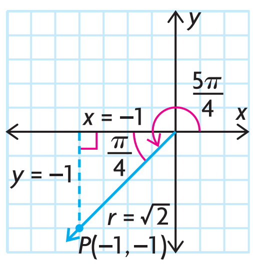
- See Lesson 6.2, Example 3.
- Try Mid-Chapter Review Questions 4 and 6.

**Q: How do you determine exact values of trigonometric ratios for multiples of special angles expressed in radians?**

**A:** An angle on the Cartesian plane is determined by rotating the terminal arm in either a clockwise or counterclockwise direction. The special triangles can be used to determine the coordinates of a point that lies on the terminal arm of the angle. Then, using the $x$, $y$, $r$ trigonometric definitions and the related angle, the exact values of the trigonometric ratios can be evaluated for multiples of angles $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.

For example, to determine the exact value of sec $\frac{5\pi}{4}$, sketch the angle in standard position. Determine the related angle. Since the terminal arm of $\frac{5\pi}{4}$ lies in the third quadrant, the related angle is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.






Sketch the 1, 1, $\sqrt{2}$ special triangle by drawing a vertical line from the point $(-1, -1)$ on the terminal arm to the negative $x$- axis. Use the values of $x$, $y$, and $r$ and the appropriate ratio to determine the value.

$$\sec \frac{5\pi}{4} = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

**Study Aid**

- See Lesson 6.4, Example 3.
- Try Mid-Chapter Review Questions 8 and 9.

**Q: How can transformations be used to graph sinusoidal functions?**

**A:** The graphs of functions of the form $f(x) = a \sin (k(x - d)) + c$ and $f(x) = a \cos (k(x - d)) + c$ are transformations of the parent functions $y = \sin (x)$ and $y = \cos (x)$, respectively.

In sinusoidal functions, the parameters $a$, $k$, $d$, and $c$ give the transformations to be applied, as well as the key characteristics of the graph.

- $|a|$ gives the vertical stretch/compression factor and the amplitude of the function.
- $\left|\frac{1}{k}\right|$ determines the horizontal stretch/compression factor, and $\left|\frac{2\pi}{k}\right|$ gives the period of the function.
- When $a$ is negative, the function is reflected in the $x$-axis. When $k$ is negative, the function is reflected in the $y$-axis.
- $d$ gives the horizontal translation.
- $c$ gives the vertical translation, and $y = c$ gives the equation of the horizontal axis of the function.

To sketch these functions, begin with the key points of the parent function. Apply any stretches/compressions and reflections first, and then follow them with any translations.

Alternatively, use the equation of the axis, amplitude, and period to sketch a graph of the form $f(x) = a \sin (x) + c$ or $f(x) = a \cos (x) + c$. Then apply the horizontal translation to the points of this graph, if necessary.

## *PRACTICE Questions*

**Lesson 6.1**

**1.** Convert each angle from radians to degrees. Express your answer to one decimal place, if necessary.

a) $\frac{\pi}{8}$  
b) $4\pi$  
c) $5$  
d) $\frac{11\pi}{12}$

**2.** Convert each angle from degrees to radians. Express your answer to one decimal place, if necessary.

a) $125°$  
b) $450°$  
c) $5°$  
d) $330°$  
e) $215°$  
f) $-140°$

**3.** A tire with a diameter of 38 cm rotates 10 times in 5 s.

a) What is the angle that the tire rotates through, in radians, from 0 s to 5 s?  
b) Determine the angular velocity of the tire.  
c) Determine the distance travelled by a pebble that is trapped in the tread of the tire.

**Lesson 6.2**

**4.** Sketch each angle in standard position, and then determine the exact value of the trigonometric ratio.

a) $\sin \frac{3\pi}{4}$  
b) $\sin \frac{11\pi}{6}$  
c) $\tan \frac{5\pi}{3}$  
d) $\tan \frac{5\pi}{6}$  
e) $\cos \frac{3\pi}{2}$  
f) $\cos \frac{4\pi}{3}$

**5.** The terminal arms of angles in standard position pass through the following points. Find the measure of each angle in radians, to the nearest hundredth.

a) $(-3, 14)$  
b) $(6, 7)$  
c) $(1, 9)$  
d) $(-5, -18)$  
e) $(2, 3)$  
f) $(4, -20)$

**6.** State an equivalent expression for each of the following expressions, in terms of the related acute angle.

a) $\sin\left(-\frac{7\pi}{6}\right)$  
b) $\cot \frac{7\pi}{4}$  
c) $\sec\left(-\frac{\pi}{2}\right)$  
d) $\cos\left(-\frac{5\pi}{6}\right)$

**Lesson 6.3**

**7.** State the $x$-intercepts and $y$-intercepts of the graph of each of the following functions.

a) $y = \sin x$  
b) $y = \cos x$  
c) $y = \tan x$

**Lesson 6.4**

**8.** Sketch the graph of each function on the interval $-2\pi \le x \le 2\pi$.

a) $y = \tan(x)$  
b) $y = 2\sin(-x) - 1$  
c) $y = \frac{5}{2}\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 3$  
d) $y = -\frac{1}{2}\cos\left(\frac{1}{2}x - \frac{\pi}{6}\right)$  
e) $y = 2\sin\left(-3\left(x - \frac{\pi}{2}\right)\right) + 4$  
f) $y = 0.4\sin(\pi - 2x) - 2.5$

**9.** The graph of the function $y = \sin x$ is transformed by vertically compressing it by a factor of $\frac{1}{3}$, reflecting it in the $y$-axis, horizontally compressing it by a factor of $\frac{1}{3}$, horizontally translating it $\frac{\pi}{8}$ units to the left, and vertically translating it 23 units down. Write the equation of the resulting graph.