6 Chapter Review

FREQUENTLY ASKED Questions

Study Aid

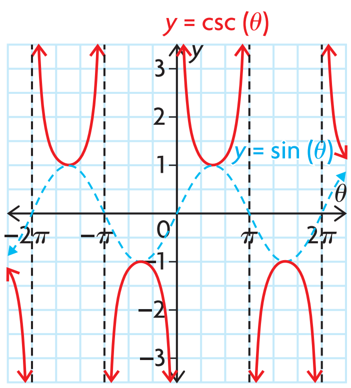
- See Lesson 6.5.
- Try Chapter Review Question 13.

Q: What do the graphs of the reciprocal trigonometric functions look like, and what are their defining characteristics?

A: Each of the primary trigonometric graphs has a corresponding reciprocal function:

Cosecant

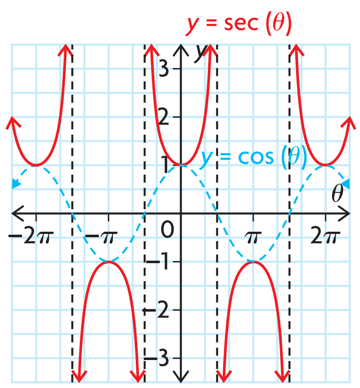
$$y = \csc x$$
$$y = \frac{1}{\sin x}$$



- has vertical asymptotes at the points where $\sin x = 0$
- has a period of 2π radians, the same period as $y = \sin x$
- has the domain $\{x \in \mathbf{R} \mid x \neq n\pi, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R} \mid |y| \geq 1\}$

Secant

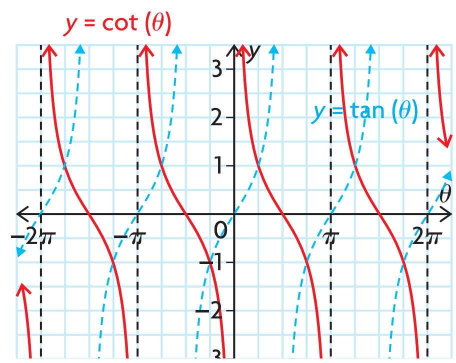
$$y = \sec x$$
$$y = \frac{1}{\cos x}$$



- has vertical asymptotes at the points where $\cos x = 0$
- has a period of 2π radians, the same period as $y = \cos x$
- has the domain $\{x \in \mathbf{R} \mid x \neq (2n - 1)\frac{\pi}{2}, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R} \mid |y| \geq 1\}$

Cotangent

$$y = \cot x$$
$$y = \frac{1}{\tan x}$$
$$y = \cot(x)$$



- has vertical asymptotes at the points where $y = \tan x$ crosses the x-axis
- has zeros at the points where $y = \tan x$ has asymptotes
- has a period of π, the same period as $y = \tan x$
- has the domain $\{x \in \mathbf{R} \mid x \neq n\pi, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R}\}$

Study Aid

- See Lesson 6.6, Example 1.
- Try Chapter Review Questions 14, 15, and 16.

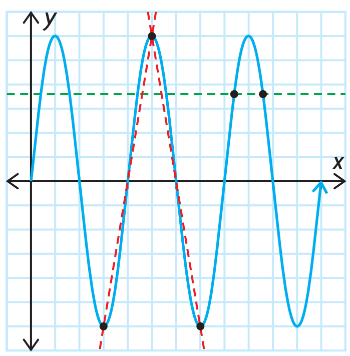
Q: How can you use a sinusoidal function to model a periodic situation?

A: If you are given a description of a periodic situation, draw a rough sketch of one cycle. If you are given data, create a scatter plot. Based on the graph, decide whether you will use a sine model or a cosine model. Use these graphs to determine the equation of the axis, the vertical translation, c, and the amplitude, a, of the function.

Use the period to help you determine k. Determine the horizontal translation, d, that must be applied to a key point on the parent function to map its corresponding location on the model. Use the parameters you found to write the equation in the form $y = a \sin (k(x - d)) + c$ or $y = a \cos (k(x - d)) + c$.

Q: Does the average rate of change of a sinusoidal function have any unique characteristics?

A:



For a sinusoidal function,

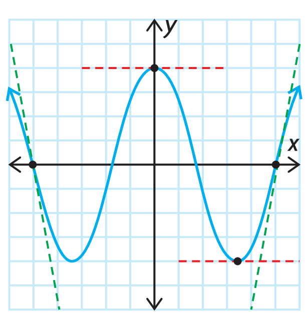
- the average rate of change is zero on any interval where the values of the function are the same
- the absolute value of the average rate of change on the intervals between a maximum and a minimum and between a minimum and a maximum are equal

Study Aid

- See Lesson 6.7, Example 2.
- Try Chapter Review Questions 17 and 19.

Q: Do the instantaneous rates of change of a sinusoidal function have any unique characteristics?

A:



For a sinusoidal function, the instantaneous rate of change is

- zero at any maximum or minimum
- at its least value halfway between a maximum and a minimum
- at its greatest value halfway between a minimum and a maximum

Study Aid

- See Lesson 6.7, Examples 1 and 3.
- Try Chapter Review Question 18.

PRACTICE Questions

Lesson 6.1

1. An arc 33 m long subtends a central angle of a circle with a radius of 16 m. Determine the measure of the central angle in radians.

2. A circle has a radius of 75 cm and a central angle of $\frac{14\pi}{15}$. Determine the arc length.

3. Convert each of the following to exact radian measure and then evaluate to one decimal.

a) $20°$ **c)** $160°$
b) $-50°$ **d)** $420°$

4. Convert each of the following to degree measure.

a) $\frac{\pi}{4}$ **c)** $\frac{8\pi}{3}$
b) $-\frac{5\pi}{4}$ **d)** $-\frac{2\pi}{3}$

Lesson 6.2

5. For each of the following values of $\sin \theta$, determine the measure of θ if $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

a) $\frac{1}{2}$ **c)** $\frac{\sqrt{2}}{2}$
b) $-\frac{\sqrt{3}}{2}$ **d)** $-\frac{1}{2}$

6. If $\cos \theta = \frac{-5}{13}$ and $0 \le \theta \le 2\pi$, determine

a) $\tan \theta$
b) $\sec \theta$
c) the possible values of θ to the nearest tenth

7. A tower that is 65 m high makes an obtuse angle with the ground. The vertical distance from the top of the tower to the ground is 59 m. What obtuse angle does the tower make with the ground, to the nearest hundredth of a radian?

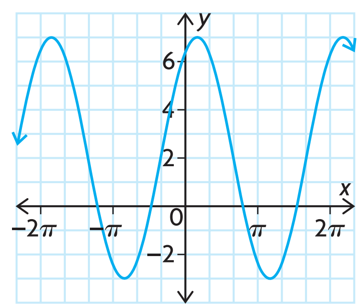
Lesson 6.3

8. State the period of the graph of each function, in radians.

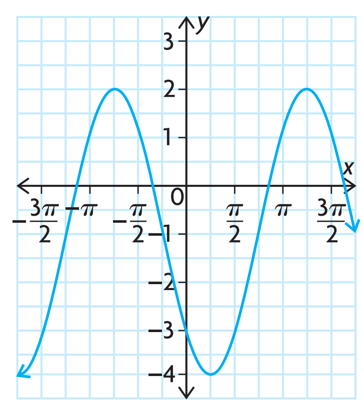
a) $y = \sin x$ **c)** $y = \tan x$
b) $y = \cos x$

Lesson 6.4

9. The following graph is a sine curve. Determine the equation of the graph.



10. The following graph is a cosine curve. Determine the equation of the graph.



11. State the transformations that have been applied to $f(x) = \cos x$ to obtain each of the following functions.

a) $f(x) = -19 \cos x - 9$
b) $f(x) = \cos\left(10\left(x + \frac{\pi}{12}\right)\right)$
c) $f(x) = \frac{10}{11} \cos\left(x - \frac{\pi}{9}\right) + 3$
d) $f(x) = -\cos(-x + \pi)$

12. The current, I, in amperes, of an electric circuit is given by the function $I(t) = 4.5 \sin (120\pi t)$, where t is the time in seconds.

a) Draw a graph that shows one cycle.

b) What is the singular period?

c) At what value of t is the current a maximum in the first cycle?

d) When is the current a minimum in the first cycle?

Lesson 6.5

13. State the period of the graph of each function, in radians.

a) $y = \csc x$

b) $y = \sec x$

c) $y = \cot x$

Lesson 6.6

14. A bumblebee is flying in a circular motion within a vertical plane, at a constant speed. The height of the bumblebee above the ground, as a function of time, can be modelled by a sinusoidal function. At $t = 0$, the bumblebee is at its lowest point above the ground.

a) What does the amplitude of the sinusoidal function represent in this situation?

b) What does the period of the sinusoidal function represent in this situation?

c) What does the equation of the axis of the sinusoidal function represent in this situation?

d) If a reflection in the horizontal axis was applied to the sinusoidal function, was the sine function or the cosine function used?

15. The population of a ski-resort town, as a function of the number of months into the year, can be described by a cosine function. The maximum population of the town is about 15 000 people, and the minimum population is about 500 people. At the beginning of the year, the population is at its greatest. After six months, the population reaches its lowest number of people. What is the equation of the cosine function that describes the population of this town?

16. A weight is bobbing up and down on a spring attached to a ceiling. The data in the following table give the height of the weight above the floor as it bobs. Determine the sine function that models this situation.

***t* (s)**	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2
***h*(*t*) (cm)**	120	136	165	180	166	133	120	135	164	179	165	133

Lesson 6.7

17. State two intervals in which the function $y = 7 \sin \left(\frac{1}{5}x\right) + 2$ has an average rate of change that is

a) zero

b) a negative value

c) a positive value

18. State two points where the function $y = \frac{1}{4} \cos (4\pi x) - 3$ has an instantaneous rate of change that is

a) zero

b) a negative value

c) a positive value

19. A person's blood pressure, $P(t)$, in millimetres of mercury (mm Hg), is modelled by the function $P(t) = 100 - 20 \cos \left(\frac{8\pi}{3}t\right)$, where t is the time in seconds.

a) What is the period of the function?

b) What does the value of the period mean in this situation?

c) Calculate the average rate of change in a person's blood pressure on the interval $t \in [0.2, 0.3]$.

d) Estimate the instantaneous rate of change in a person's blood pressure at $t = 0.5$.