Chapters 4–6 Cumulative Review

Multiple Choice

1. What are the solutions of $x^4 + 3x^3 = 4x^2 + 12x$?

a) $-2, 0, 3, 2$ **c)** $-3, 0, 4$

b) $-4, -3, 0$ **d)** $-3, -2, 0, 2$

2. Which cubic function has zeros at -1, 1, and 4 and passes through $(2, 36)$?

a) $f(x) = 2(x - 1)(x + 1)(x + 4)$

b) $f(x) = -6x^3 + 24x^2 + 6x - 24$

c) $f(x) = 36x^3 - 144x^2 - 6x + 144$

d) $f(x) = 6(x + 1)(x - 1)(x - 4)$

3. Which value is *not* a solution of $2 - 3x < x - 5$?

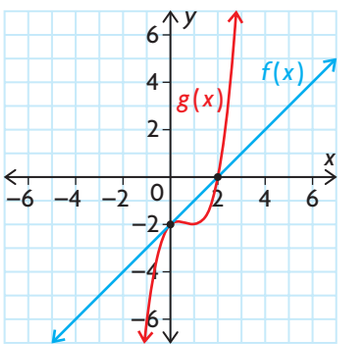
a) -2 **b)** 2 **c)** 3 **d)** 5

4. What is the solution of $-10 \leq 3x + 5 \leq 8$?

a) $-5 \leq x \leq \frac{13}{3}$ **c)** $-5 \leq x \leq 1$

b) $x \in (-5, 1)$ **d)** $x \in \left[-\frac{5}{3}, 1\right]$

5. On which interval is $f(x) < g(x)$?



a) $x > 2$ **c)** $x \in (-\infty, 0)$

b) $x < 0$ and $x > 2$ **d)** $x \in (0, 2)$

6. The height in metres of a diver above the pool's surface is given by $h(t) = -5t^2 + 3.5t + 10$, where t is in seconds. When is the diver more than 10.0 m above the pool?

a) $t < 1.5$ **c)** $t \in (0, 1)$

b) $t \in (0, 0.7)$ **d)** $0.7 < t < 1$

7. The instantaneous rate of change of a cubic function is positive for $x < 0$, negative for $0 < x < 2$, and positive for $x > 2$. Which is *not* a possible set of zeros for the function?

a) $x = 0, x = 1$

b) $x = -0.73, x = 1, x = 2.73$

c) $x = -3$

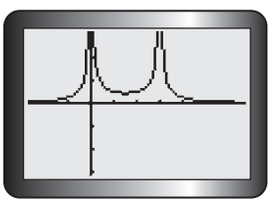
d) $x = -0.73, x = 2$

8. Which value is the best estimate of the instantaneous rate of change of the function $f(x) = 2x^3 - 4x^2 + 6x$ at the point $(0, 0)$?

a) -6.5 **b)** 0 **c)** 6.2 **d)** 5.5

9. Which is the graph of $y = \frac{1}{x^2 - 3x}$?

a)



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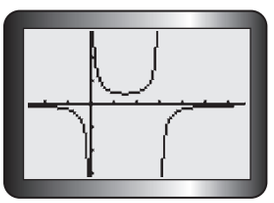

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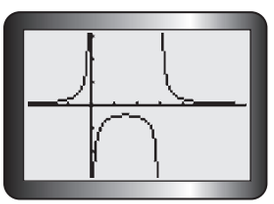

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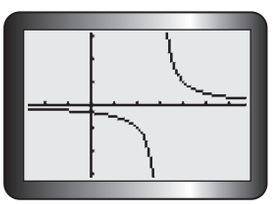


10. What type of asymptote(s) does $f(x) = \frac{1}{x^2 + 3x - 10}$ have?

a) only vertical
b) only horizontal
c) both vertical and horizontal
d) only oblique

11. Which function has a vertical asymptote at $x = 3$ and an oblique asymptote?

a) $f(x) = \frac{x - 3}{x^2 - 9}$
b) $g(x) = \frac{x^2 - 9}{x - 3}$
c) $h(x) = \frac{x + 3}{x - 3}$
d) $j(x) = \frac{x^2 + 9}{x - 3}$

12. Which function has domain $\{x \in \mathbf{R} | x \neq 3\}$ and is positive on $\{x \in \mathbf{R} | -2 < x < 3\}$?

a) $f(x) = \frac{x + 2}{3 - x}$
b) $g(x) = \frac{x + 2}{x - 3}$
c) $h(x) = \frac{x - 2}{x + 3}$
d) $j(x) = \frac{2 - x}{x + 3}$

13. How does the function $f(x) = \frac{2 - 3x}{5x - 3}$ behave as x approaches $\frac{3}{5}$ from the left?

a) $f(x) \to \infty$
b) $f(x) \to 0$
c) $f(x) \to \frac{1}{5}$
d) $f(x) \to -\infty$

14. What is the solution of $\frac{3 - 2x}{x + 2} = 3x$?

a) $x = 0, x = 1.5$
b) $x = -2, x = 0$
c) $x = -3, x = \frac{1}{3}$
d) $x = -\frac{1}{3}, x = 3$

15. When solving a rational equation such as $\frac{2 - 3x}{5x - 3} = \frac{x + 2}{5x}$, what is a possible first step?

a) Graph each side as a function.
b) Determine the zeros of the denominators.
c) Multiply all terms by the lowest common denominator.
d) any of the above

16. The inequality $2x - 3 \leq \frac{2}{x}$ is equivalent to

a) $\frac{(2x + 1)(x - 2)}{x} \leq 0$
b) $\frac{x(2x - 3)}{2} \leq 1$
c) $\frac{(2x - 1)(x + 2)}{x} \leq 0$
d) $\frac{(2x + 1)(x - 2)}{2} \leq 0$

17. For which interval(s) is the inequality $x - 3 > \frac{6}{x - 2}$ true?

a) $x \in (-\infty, 0)$ or $x \in (2, 5)$
b) $x \in (0, 5)$
c) $x < 0$ or $x > 5$
d) $0 < x < 2$ or $x > 5$

18. What is the slope of the line tangent to $y = \frac{3 - x}{2x}$ at $x = 1$?

a) $m = \frac{3}{2}$
b) $m = -\frac{3}{2}$
c) $m = 3$
d) $m = -3$

19. The position of an object moving along a straight line at time t seconds is given by $s(t) = \frac{2t + 1}{t - 4}$, where s is measured in metres. Which is the best estimate for the rate of change of s at $t = 3$ s?

a) -12 m/s
b) -9 m/s
c) -9.6 m/s
d) -7 m/s

20. A sector of a circle with a radius of 3 m has a central angle of $\frac{5\pi}{12}$. What is the perimeter of the sector?

a) $6\frac{5}{24}$ m
b) $\frac{5\pi}{4} + 6$ m
c) $\frac{5\pi}{2} + 6$ m
d) $\frac{5\pi}{4}$ m

21. Which of the following pairs of angles are equivalent?

a) $20°$ and $\frac{\pi}{9}$

b) $135°$ and $\frac{3\pi}{4}$

c) $-270°$ and $-\frac{3\pi}{2}$

d) all of the above

22. The point $(-4, 7)$ lies on the terminal arm of angle θ. What is the measure of θ in radians?

a) 4.19 **b)** 119.74 **c)** 2.09 **d)** 2.62

23. If $\sin\theta = -\frac{\sqrt{3}}{2}$, what are possible values of $\cos\theta$ and $\tan\theta$?

a) $\cos\theta = \frac{1}{2}$, $\tan\theta = -\sqrt{3}$

b) $\cos\theta = -\frac{1}{2}$, $\tan\theta = -\sqrt{3}$

c) $\cos\theta = -\frac{1}{2}$, $\tan\theta = -\frac{1}{\sqrt{3}}$

d) $\cos\theta = -\frac{1}{2}$, $\tan\theta = \frac{1}{\sqrt{3}}$

24. Which of the following values of x, where $x \in [0, 2\pi]$, satisfy $\sin x = 0.5$?

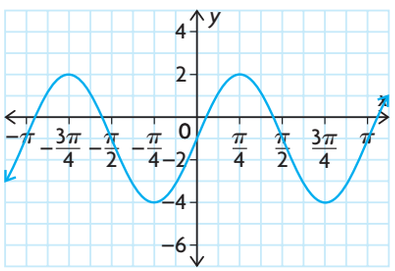
a) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

b) $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

c) $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

d) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

25. What is the equation of this transformation of the graph of $y = \sin x$?



a) $y = 3\sin(2(x + 1))$

b) $y = 3\sin(2x) - 1$

c) $y = 3\sin\left(\frac{1}{2}x\right) - 1$

d) $y = 2\sin(3x) - 1$

26. What transformations are needed to transform $y = \cos x$ into $y = \cos\left(\frac{1}{3}(x + 2\pi)\right)$?

a) horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 2π units left

b) horizontal stretch by a factor of 3, horizontal translation 2π units left

c) vertical compression by a factor of $\frac{1}{3}$, vertical translation 2 units up

d) horizontal stretch by a factor of 3, horizontal translation 2π units left

27. One blade of a wind turbine is at an angle of $-\frac{\pi}{4}$ to the upward vertical at time $t = 0$, and rotates counterclockwise one revolution every 2 seconds. The tip of the blade varies between 5 m and 41 m above the ground. Which equation is a model for the height, h, of the blade tip?

a) $h = 18\cos\left(\pi t + \frac{\pi}{4}\right) + 23$

b) $h = 41\cos\left(2\left(t + \frac{\pi}{4}\right)\right) - 5$

c) $h = 18\cos\left(\pi t - \frac{\pi}{4}\right) - 23$

d) $h = 41\cos\left(\pi\left(t + \frac{\pi}{4}\right)\right) - 36$

28. The instantaneous rate of change of $y = 2\sin(3x - \pi)$ is negative on which of the following intervals?

a) $\frac{\pi}{2} < x < \frac{5\pi}{6}$

b) $\frac{\pi}{2} < x < \frac{3\pi}{2}$

c) both a) and b)

d) neither a) nor b)

29. The population of blackflies at a lake in northern Ontario can be modelled by the function $P(t) = 23.7\cos\left(\frac{\pi}{6}(t - 7)\right) + 24.1$, where P is in millions and t is in months. Over which time interval is the average rate of change in the blackfly population the greatest?

a) $0 \le t \le 4$

b) $1 \le t \le 7$

c) $7 \le t \le 16$

d) $10 \le t \le 18$

Investigations

The Greatest Volume

30. An open top box is made by cutting corners out of a 50 cm by 40 cm piece of cardboard.

a) Determine a mathematical model that represents the volume of the box.

b) Determine the length of the sides of each square that must be cut that will result in a box with a volume of 6000 cm^3.

c) Determine the length of the sides of each square that must be cut that will result in a box with maximum volume.

d) Determine a range of sizes of the squares that can be cut from each corner that will result in a box with a volume of at least 1008 cm^3.

Combining Functions

31. Consider the polynomial functions $f(x) = x^2 - 5x + 6$ and $g(x) = x - 3$. Determine

a) the zeros of $f(x)$, $g(x)$, $\frac{f(x)}{g(x)}$, and $\frac{g(x)}{f(x)}$

b) the holes and asymptotes of $\frac{f(x)}{g(x)}$ and $\frac{g(x)}{f(x)}$, if any

c) any x-coordinate(s) where the tangents of $\frac{f(x)}{g(x)}$ and $\frac{g(x)}{f(x)}$ are perpendicular, and the equation(s) of the tangent(s) at such coordinates

Transformations of Trigonometric Functions

32. **a)** Investigate the effect of various types of transformations (i.e., stretches/compressions, reflections, and translations) of $y = \sin x$ on its zeros, maximum and minimum values, and instantaneous rates of change.

b) Repeat part a) for $y = \cos x$ and $y = \tan x$.