



Chapter

Trigonometric Identities and Equations

GOALS

You will be able to

- Recognize equivalent trigonometric relationships
- Use compound angle formulas to determine the exact values of trigonometric ratios that involve sums, differences, and products of special angles
- Prove trigonometric identities using a variety of strategies
- Solve trigonometric equations using a variety of strategies

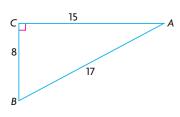
Global temperatures have increased by an average of 1 °C in the past 100 years. Ocean levels are rising by 1 cm to 2 cm every year. How do temperatures vary from month to month? How do ocean levels in a harbour vary from hour to hour? What types of functions model these types of variation?

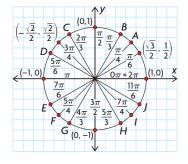
Getting Started

Study Aid

• For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix/ Lesson
1	R-6
3	R-10
4, 5, 6	6.2
7	R-14
8	R-12





SKILLS AND CONCEPTS You Need

- 1. Solve each equation to two decimal places where necessary.
 - a) 3x 7 = 5 9xb) $2(x + 3) - \frac{x}{4} = \frac{1}{2}$ c) $x^2 - 5x - 24 = 0$ d) $6x^2 + 11x = 10$ e) $x^2 + 2x - 1 = 0$ f) $3x^2 = 3x + 1$
- 2. Show that the line segment from A(1, 0) to $B(2, \frac{1}{2})$ is the same length as the line segment from $C(-\frac{1}{2}, 5)$ to D(0, 6).
- **3.** Given $\triangle ABC$ shown,
 - a) state the six trigonometric ratios for $\angle A$
 - **b**) determine the measure of $\angle A$ in **radians**, to one decimal place
 - c) determine the measure of $\angle B$ in **degrees**, to one decimal place
- **4.** P(-2, 2) lies on the terminal arm of an angle in standard position.
 - a) Sketch the **principal angle**, θ .
 - b) Determine the value of the related acute angle in radians.
 - c) Determine the value of θ in radians.
- **5.** a) Determine the coordinates of each missing point on the unit circle shown.
 - **b**) Determine:

i)
$$\cos\left(\frac{3\pi}{4}\right)$$
 ii) $\sin\left(\frac{11\pi}{6}\right)$ iii) $\cos(\pi)$ iv) $\csc\left(\frac{\pi}{6}\right)$

- 6. Given $\tan x = -\frac{3}{4}$, where $0 \le x \le 2\pi$,
 - a) state the other five trigonometric ratios as fractions
 - **b**) determine the value(s) of *x*, to one decimal place
- 7. State whether each relationship is true or false.
 - a) $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cos \theta \neq 0$ b) $\sin^2 \theta + \cos^2 \theta = 1$ c) $\cos^2 \theta = \sin^2 \theta - 1$ c) $\sin^2 \theta + \cos^2 \theta = 1$ c) $\sin^2 \theta + \sin^2 \theta = \sec^2 \theta$

c)
$$\sec \theta = \frac{1}{\sin \theta}$$
, $\sin \theta \neq 0$ f) $\cot \theta = \frac{\cos \theta}{\sin \theta}$, $\sin \theta \neq 0$

8. Create a flow chart that shows how transformations can be used to sketch the graph of a sinusoidal function in the form $y = a \sin (k(x - d)) + c$.

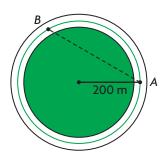
Getting Started

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APPLYING What You Know

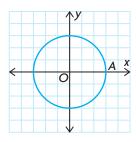
Going for a Run

Julie goes for a daily run in her local park. She parks her bike at point A and runs five times around the playing field, in a counterclockwise direction. The radius of the path that she runs is 200 m. This morning, she ran one-third of the way around the field, to point B, before realizing that she had left her heart-rate monitor on her bike. She ran in a straight line across the field, back to her bike, to get her monitor.



• graph paper

- How far did Julie run when she went across the field, back to her bike?
- **A.** Draw a circle (centred at the origin) on graph paper, as shown, to represent the path that Julie runs. Write the coordinates of point *A*.



- **B.** Mark point *B* one-third of the way around the circle from point *A*. What is the radian measure of $\angle AOB$? Write the coordinates $(r \cos \theta, r \sin \theta)$ of point *B* in terms of this angle.
- **C.** Use the distance formula, $d = \sqrt{(x_2 x_1)^2 + (y_2 y_1)^2}$, to calculate the distance from *A* to *B*.
- **D.** What kind of triangle is $\triangle AOB$? What are the lengths of AO and BO?
- **E.** Verify your answer in part C using the cosine law.
- **F.** How far did Julie run when she went across the field, back to her bike, to get her heart-rate monitor?