7 Mid-Chapter Review

FREQUENTLY ASKED Questions

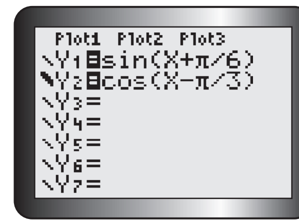
Q: How can you identify equivalent trigonometric expressions?

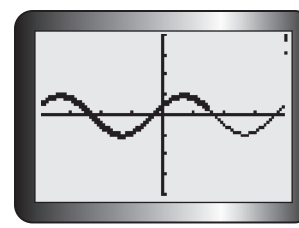
Study Aid

- See Lesson 7.1.
- Try Mid-Chapter Review Questions 1 and 2.

A1: Compare the graphs of the corresponding trigonometric functions on a graphing calculator. If the graphs appear to be identical, then the expressions may be equivalent.

For example, to see if $\sin\left(x + \frac{\pi}{6}\right)$ is the same as $\cos\left(x - \frac{\pi}{3}\right)$, graph the functions $f(x) = \sin\left(x + \frac{\pi}{6}\right)$ and $g(x) = \cos\left(x - \frac{\pi}{3}\right)$ on the same screen. If you use a bold line for the second function, you will see it drawing in over the first graph.





Since the graphs appear to coincide, you can make the conjecture that $f(x) = g(x)$. It follows that $\sin\left(x + \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{3}\right)$. This can be confirmed by analyzing both functions. Both functions have a period of 2π. As well, $f(x) = \sin\left(x + \frac{\pi}{6}\right)$ is the sine function translated $\frac{\pi}{6}$ to the left, while $g(x) = \cos\left(x - \frac{\pi}{3}\right)$ is the cosine function translated $\frac{\pi}{3}$ to the right. These transformations of the parent functions result in the same function over their entire domains.

A2: Use some of the following strategies:

- the reflective property of even and odd functions
- translations of a function by an amount that is equal to a multiple of its period
- combinations of other transformations
- the relationship between trigonometric ratios of complementary angles in a right triangle
- the relationship between a principal angle in standard position on the Cartesian plane and its related angles

A3: Use compound angle formulas.

Study Aid

- See Lesson 7.2, Example 4.
- Try Mid-Chapter Review Questions 3 and 4.

For example, to identify a trigonometric expression that is equivalent to $\cos\left(x - \frac{\pi}{4}\right)$, use the subtraction formula for cosine.

$$\begin{aligned}\cos\left(x - \frac{\pi}{4}\right) &= \cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}\\ &= (\cos x)\left(\frac{1}{\sqrt{2}}\right) + (\sin x)\left(\frac{1}{\sqrt{2}}\right)\\ &= \frac{1}{\sqrt{2}}(\cos x + \sin x)\end{aligned}$$

> **Study Aid**
> - See Lesson 7.2, Example 1.
> - Try Mid-Chapter Review Questions 5 and 6.

Q: How can you determine the exact values of trigonometric ratios for angles other than the special angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$, and their multiples?

A: You can combine special angles by adding or subtracting them, and then use compound angle formulas to determine trigonometric ratios for the new angle.

For example, consider $\frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$.

Determine $\sin \frac{7\pi}{12}$ by finding

$$\begin{aligned}\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) &= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}\\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{1 + \sqrt{3}}{2\sqrt{2}}\end{aligned}$$

> **Study Aid**
> - See Lesson 7.3, Example 2.
> - Try Mid-Chapter Review Questions 8 to 12.

Q: Given a trigonometric ratio for θ, how would you calculate trigonometric ratios for 2θ?

A: You can use double angle formulas.

For example, if you know that $\cos\theta = \frac{2}{5}$, you can calculate $\cos 2\theta$ using the formula

$$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1\\ &= 2\left(\frac{2}{5}\right)^2 - 1\\ &= \frac{8}{25} - 1\\ &= -\frac{17}{25}\end{aligned}$$

To calculate $\sin 2\theta$ and $\tan 2\theta$, you need to consider the quadrant in which θ lies. If $\cos\theta$ is positive, θ can be in quadrant I or quadrant IV. This means you need to calculate two answers for both $\sin 2\theta$ and $\tan 2\theta$.

PRACTICE Questions

Lesson 7.1

1. For each of the following trigonometric ratios, state an equivalent trigonometric ratio.

a) $\cos \frac{\pi}{16}$

b) $\sin \frac{7\pi}{9}$

c) $\tan \frac{9\pi}{10}$

d) $-\cos \frac{2\pi}{5}$

e) $-\sin \frac{9\pi}{7}$

f) $\tan \frac{3\pi}{4}$

2. Use the sine function to write an equation that is equivalent to $y = -6 \cos \left(x + \frac{\pi}{2}\right) + 4$.

Lesson 7.2

3. Use a compound angle addition formula to determine a trigonometric expression that is equivalent to each of the following expressions.

a) $\cos \left(x + \frac{5\pi}{3}\right)$

b) $\sin \left(x + \frac{5\pi}{6}\right)$

c) $\tan \left(x + \frac{5\pi}{4}\right)$

d) $\cos \left(x + \frac{4\pi}{3}\right)$

4. Use a compound angle subtraction formula to determine a trigonometric expression that is equivalent to each of the following expressions.

a) $\sin \left(x - \frac{11\pi}{6}\right)$

b) $\tan \left(x - \frac{\pi}{3}\right)$

c) $\cos \left(x - \frac{7\pi}{4}\right)$

d) $\sin \left(x - \frac{2\pi}{3}\right)$

5. Evaluate each expression.

a) $\dfrac{\tan \frac{8\pi}{9} - \tan \frac{5\pi}{9}}{1 + \tan \frac{8\pi}{9} \tan \frac{5\pi}{9}}$

b) $\sin \frac{299\pi}{298} \cos \frac{\pi}{298} - \cos \frac{299\pi}{298} \sin \frac{\pi}{298}$

c) $\sin 50° \cos 20° - \cos 50° \sin 20°$

d) $\sin \frac{3\pi}{8} \cos \frac{\pi}{8} + \cos \frac{3\pi}{8} \sin \frac{\pi}{8}$

6. Simplify each expression.

a) $\dfrac{2 \tan x}{1 - \tan^2 x}$

b) $\sin \frac{x}{5} \cos \frac{4x}{5} + \cos \frac{x}{5} \sin \frac{4x}{5}$

c) $\cos \left(\frac{\pi}{2} - x\right)$

d) $\sin \left(\frac{\pi}{2} + x\right)$

e) $\cos \left(\frac{\pi}{4} + x\right) + \cos \left(\frac{\pi}{4} + x\right)$

f) $\tan \left(x - \frac{\pi}{4}\right)$

7. The expression $a \cos x + b \sin x$ can be expressed in the form $R \cos (x - \alpha)$, where $R = \sqrt{a^2 + b^2}$, $\cos \alpha = \frac{a}{R}$, and $\sin \alpha = \frac{b}{R}$. Use this information to write an expression that is equivalent to $\sqrt{3} \cos x - 3 \sin x$.

Lesson 7.3

8. Evaluate each expression.

a) $2 \cos^2 \frac{2\pi}{3} - 1$

b) $2 \sin \frac{11\pi}{12} \cos \frac{11\pi}{12}$

c) $\cos^2 \frac{7\pi}{8} - \sin^2 \frac{7\pi}{8}$

d) $1 - 2 \sin^2 \left(\frac{\pi}{2}\right)$

9. The angle x lies in the interval $\pi \le x \le \frac{3\pi}{2}$, and $\cos^2 x = \frac{10}{11}$. Without using a calculator, determine the value of each trigonometric ratio.

a) $\sin x$

b) $\cos x$

c) $\sin 2x$

d) $\cos 2x$

10. Given $\sin x = \frac{3}{5}$ and $0 \le x \le \frac{\pi}{2}$, find $\sin 2x$ and $\cos 2x$.

11. Given $\sin x = \frac{5}{13}$ and $0 \le x \le \frac{\pi}{2}$, find $\sin 2x$.

12. Given $\cos x = -\frac{4}{5}$ and $\pi \le x \le \frac{3\pi}{2}$, find $\tan 2x$.