7 Chapter Review

FREQUENTLY ASKED Questions

Study Aid

- See Lesson 7.4, Examples 1 to 5.
- Try Chapter Review Questions 7, 8, and 9.

Q: What is the difference between a trigonometric equation and a trigonometric identity, and how can you prove that a given equation is an identity?

A: A trigonometric equation is true for one, several, or many values of the variable it contains. A trigonometric identity is an equation that involves trigonometric ratios and is true for *all* values of the variables for which the expressions on both sides are defined.

To prove that an equation is an identity, you can use algebraic manipulation on one or both sides of the equation until one side is identical to the other side. This often involves a variety of strategies, such as

- rewriting the expressions using known identities
- rewriting the expressions using compound angle formulas and double angle formulas
- using a common denominator or factoring where possible

To prove that an equation is *not* an identity, you can use a counterexample. If any value, when substituted, results in LS $\neq$ RS, then the equation is *not* an identity.

Study Aid

- See Lesson 7.5, Examples 1 to 4.
- Try Chapter Review Question 10.

Q: How can you solve a linear trigonometric equation?

A1: You can solve a linear trigonometric equation algebraically, using special triangles, a calculator, a sketch of the graph of the corresponding function, and/or the CAST rule.

For example, to solve $2(\cos 2x + 1) = 3$ for $0 \leq x \leq 2\pi$, first rearrange the equation to isolate $\cos 2x$.

$$2\cos 2x + 2 = 3$$
$$2\cos 2x = 1$$
$$\cos 2x = \frac{1}{2}$$

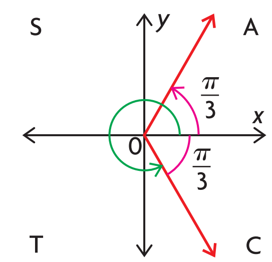
Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$ to determine the related acute angle of $2x$.

Using the 1, 2, $\sqrt{3}$ special triangle, the related angle is $\frac{\pi}{3}$.

Cosine is positive in quadrants I and IV.

$2x = \frac{\pi}{3}$ in quadrant I, so $x = \frac{\pi}{6}$.

$2x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ in quadrant IV, so $x = \frac{5\pi}{6}$.



$$\frac{\pi}{6} + \pi = \frac{7\pi}{6}$$

$$\frac{5\pi}{6} + \pi = \frac{11\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Cos $2x$ has a period of π, so add π to these solutions to determine the other solutions in the given domain.

A2: You can solve a linear trigonometric equation, or verify the solutions, using a graphing calculator.

One way to solve the equation $2(\cos 2x + 1) = 3$ is to enter $Y1 = 2(\cos 2x + 1)$ and $Y2 = 3$ and determine the intersection points.

Another way to solve the equation is to enter $Y1 = 2(\cos 2x + 1) - 3$ and determine the zeros.

Q: What strategies can you use to solve a quadratic trigonometric equation?

Study Aid

- See Lesson 7.6, Examples 1, 2, and 3.
- Try Chapter Review Questions 11, 12, and 13.

A1: You can often factor a quadratic trigonometric equation, and then solve the resulting two linear trigonometric equations.

For example, to solve $2 \tan^2 x - \tan x - 6 = 0$, factor the left side so that $(2 \tan x + 3)(\tan x - 2) = 0$. Solve the two linear equations, $2 \tan x + 3 = 0$ and $\tan x - 2 = 0$.

If it is not factorable, you can use the quadratic formula, then solve the resulting two linear equations.

A2: You may need to use a Pythagorean identity, compound angle formula, or double angle formula to create a quadratic equation that contains only a single trigonometric function whose arguments all match.

A3: You can use a graphing calculator to solve or verify the solutions. Graph the functions defined by the two sides of the equation and determine the intersection points. You can also create a single function of the form $f(x) = 0$, graph it, and determine its zeros.

PRACTICE Questions

Lesson 7.1

1. State a trigonometric ratio that is equivalent to each of the following trigonometric ratios.

a) $\sin \frac{3\pi}{10}$

b) $\cos \frac{6\pi}{7}$

c) $-\sin \frac{13\pi}{7}$

d) $-\cos \frac{8\pi}{7}$

2. Write an equation that is equivalent to $y = -5 \sin \left(x - \frac{\pi}{2}\right) - 8$, using the cosine function.

Lesson 7.2

3. Use a compound angle formula to determine a trigonometric expression that is equivalent to each of the following expressions.

a) $\sin \left(x - \frac{4\pi}{3}\right)$

b) $\cos \left(x + \frac{3\pi}{4}\right)$

c) $\tan \left(x + \frac{\pi}{3}\right)$

d) $\cos \left(x - \frac{5\pi}{4}\right)$

4. Evaluate each expression.

a) $\dfrac{\tan \frac{\pi}{12} + \tan \frac{7\pi}{4}}{1 - \tan \frac{\pi}{12} \tan \frac{7\pi}{4}}$

b) $\cos \frac{\pi}{9} \cos \frac{19\pi}{18} - \sin \frac{\pi}{9} \sin \frac{19\pi}{18}$

Lesson 7.3

5. Simplify each expression.

a) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

b) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

c) $1 - 2 \sin^2 \frac{3\pi}{8}$

d) $\dfrac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$

6. Determine the values of $\sin 2x$, $\cos 2x$, and $\tan 2x$, given

a) $\sin x = \frac{3}{5}$, and x is acute

b) $\cot x = -\frac{7}{24}$, and x is obtuse

c) $\cos x = \frac{12}{13}$, and $\frac{3\pi}{2} \le x \le 2\pi$

Lesson 7.4

7. Determine whether each of the following is a trigonometric equation or a trigonometric identity.

a) $\tan 2x = \dfrac{2 \sin x \cos x}{1 - 2 \sin^2 x}$

b) $\sec^2 x - \tan^2 x = \cos x$

c) $\csc^2 x - \cot^2 x = \sin^2 x + \cos^2 x$

d) $\tan^2 x = 1$

8. Prove that $\frac{1 - \sin^2 x}{\cot^2 x} = 1 - \cos^2 x$ is a trigonometric identity.

9. Prove that $\frac{2 \sec^2 x - 2 \tan^2 x}{\csc x} = \sin 2x \sec x$ is a trigonometric identity.

Lesson 7.5

10. Solve each trigonometric equation in the interval $0 \le x \le 2\pi$.

a) $\dfrac{2}{\sin x} + 10 = 6$

b) $-\dfrac{5 \cot x}{2} + \dfrac{7}{3} = -\dfrac{1}{6}$

c) $3 + 10 \sec x - 1 = -18$

Lesson 7.6

11. **a)** Solve the equation $y^2 - 4 = 0$.

b) Solve $\csc^2 x - 4 = 0$ in the interval $0 \le x \le 2\pi$.

12. Solve each equation for x in the interval $0 \le x \le 2\pi$.

a) $2 \sin^2 x - \sin x - 1 = 0$

b) $\tan^2 x \sin x - \dfrac{\sin x}{3} = 0$

c) $\cos^2 x + \left(\dfrac{1 - \sqrt{2}}{2}\right) \cos x - \dfrac{\sqrt{2}}{4} = 0$

d) $25 \tan^2 x - 70 \tan x = -49$

13. Solve the equation $\frac{1}{1 + \tan^2 x} = -\cos x$ for x in the interval $0 \le x \le 2\pi$.