7 Chapter Self-Test

1. Prove that $\frac{1 - 2\sin^2 x}{\cos x + \sin x} + 2\sin\frac{x}{2}\cos\frac{x}{2} = \cos x$.

2. Solve the following equation: $\cos 2x + 2\sin^2 x - 3 = -2$, where $0 \le x \le 2\pi$.

3. Determine the solution(s) for each of the following equations, where $0 \le x \le 2\pi$.

a) $\cos x = \frac{\sqrt{3}}{2}$ **b)** $\tan x = -\sqrt{3}$ **c)** $\sin x = -\frac{\sqrt{2}}{2}$

4. The quadratic trigonometric equation $a\cos^2 x + b\cos x - 1 = 0$ has the solutions $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$ in the interval $0 \le x \le 2\pi$. What are the values of a and b?

5. The depth of the ocean at a swim buoy can be modelled by the function $d(t) = 4 + 2\sin\left(\frac{\pi}{6}t\right)$, where d is the depth of water in metres and t is the time in hours, if $0 \le t \le 24$. Consider a day when $t = 0$ represents midnight. Determine when the depth of water is 3 m.

6. Nina needs to find the cosine of $\frac{11\pi}{4}$. If she knows the sine and cosine of π, as well as the sine and cosine of $\frac{7\pi}{4}$, how can she find the cosine of $\frac{11\pi}{4}$? What is her answer?

7. Solve $3\sin x + 2 = 1.5$, where $0 \le x \le 2\pi$.

8. The tangent of the acute angle α is 0.75, and the tangent of the acute angle β is 2.4. Without using a calculator, determine the value of $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$.

9. The angle x lies in the interval $\frac{\pi}{2} \le x \le \pi$, and $\sin^2 x = \frac{4}{9}$. Determine the value of each of the following. Round your answers to four decimal places.

a) $\sin 2x$ **c)** $\cos\frac{x}{2}$

b) $\cos 2x$ **d)** $\sin 3x$

10. Use the graph of $f(x) = \cos x$ to estimate the solution of each of the following trigonometric equations in the interval $-2\pi \le x \le 2\pi$.

a) $2 - 14\cos x = -5$

b) $9 - 22\cos x - 1 = 19$

c) $2 + 7.5\cos x = -5.5$