Chapter Self-Test

1. Prove that
$$\frac{1-2\sin^2 x}{\cos x + \sin x} + 2\sin \frac{x}{2}\cos \frac{x}{2} = \cos x.$$

2. Solve the following equation:
$$\cos 2x + 2 \sin^2 x - 3 = -2$$
, where $0 \le x \le 2\pi$.

3. Determine the solution(s) for each of the following equations, where
$$0 \le x \le 2\pi$$
.

a)
$$\cos x = \frac{\sqrt{3}}{2}$$

$$b) \quad \tan x = -\sqrt{3}$$

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$$\cos x = \frac{\sqrt{3}}{2}$$
 b) $\tan x = -\sqrt{3}$ c) $\sin x = -\frac{\sqrt{2}}{2}$

- **4.** The quadratic trigonometric equation $a \cos^2 x + b \cos x 1 = 0$ has the solutions $\frac{\pi}{3}$, π , and $\frac{5\pi}{3}$ in the interval $0 \le x \le 2\pi$. What are the values of a and b?
- **5.** The depth of the ocean at a swim buoy can be modelled by the function $d(t) = 4 + 2 \sin \left(\frac{\pi}{6}t\right)$, where d is the depth of water in metres and t is the time in hours, if $0 \le t \le 24$. Consider a day when t = 0 represents midnight. Determine when the depth of water is 3 m.
- **6.** Nina needs to find the cosine of $\frac{11\pi}{4}$. If she knows the sine and cosine of π , as well as the sine and cosine of $\frac{7\pi}{4}$, how can she find the cosine of $\frac{11\pi}{4}$? What is her answer?

7. Solve
$$3 \sin x + 2 = 1.5$$
, where $0 \le x \le 2\pi$.

- **8.** The tangent of the acute angle α is 0.75, and the tangent of the acute angle β is 2.4. Without using a calculator, determine the value of $\sin (\alpha - \beta)$ and $\cos (\alpha + \beta)$.
- **9.** The angle x lies in the interval $\frac{\pi}{2} \le x \le \pi$, and $\sin^2 x = \frac{4}{9}$. Determine the value of each of the following. Round your answers to four decimal places.

a)
$$\sin 2x$$

c)
$$\cos \frac{x}{2}$$

b)
$$\cos 2x$$

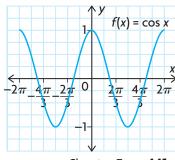
d)
$$\sin 3x$$

10. Use the graph of $f(x) = \cos x$ to estimate the solution of each of the following trigonometric equations in the interval $-2\pi \le x \le 2\pi$.

a)
$$2 - 14 \cos x = -5$$

b)
$$9 - 22 \cos x - 1 = 19$$

c)
$$2 + 7.5 \cos x = -5.5$$



Chapter 7