Chapter

8

Exponential and Logarithmic Functions

GOALS

You will be able to

- Relate logarithmic functions to exponential functions
- Describe the characteristics of logarithmic functions and their graphs
- Evaluate logarithms and simplify logarithmic expressions
- Solve exponential and logarithmic equations
- Use exponential and logarithmic functions to solve problems involving exponential growth and decay, and applications of logarithmic scales

Understanding the Richter Scale

Richter Magnitude	Equivalent Kilograms of TNT	Extra Information
0–1	0.6–20 kg of dynamite	We cannot feel these.
2	600 kg of dynamite	Smallest quake people can normally feel.
3	20 000 kg of dynamite	People near the epicentre feel this quake.
4	60 000 kg of dynamite	This will cause damage around the epicentre. It is the same as a small fission bomb.
5	20 000 000 kg of dynamite	Damage done to weak buildings in the area of the epicentre.
6	60 000 000 kg of dynamite	Can cause great damage around the epicentre.
7	20 billion kg of dynamite	Creates enough energy to heat New York City for one year. Can be detected all over the world. Causes serious damage.
8	60 billion kg of dynamite	Causes death and major destruction. Destroyed San Francisco in 1906.
9	20 trillion kg of dynamite	Rare, but would cause unbelievable damage!

? The Richter scale is used to measure earthquake intensity. What type of function do you think the Richter scale might be related to?

8 Getting Started

Study Aid

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
1, 2, 3	R-1
4, 8	R-7, R-8
6, 7	R-7

SKILLS AND CONCEPTS You Need

1. Rewrite each expression in an equivalent form, and then evaluate.

a) 5^{-2} **c)** $36^{\frac{1}{2}}$ **e)** $-121^{\frac{1}{2}}$

b) 11^{0} **d)** $125^{\frac{1}{3}}$ **f)** $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

2. Simplify each expression, and then evaluate.

a) $(3^5)(3^2)$ **c)** $\frac{10^9}{10^6}$ **e)** $(8^{\frac{1}{3}})^2$

b) $(-2)^{12}(-2)^{-10}$ **d)** $\frac{(7^6)(7^{-3})}{7^{-1}}$ **f)** $\frac{(4^{\frac{3}{4}})(4^{\frac{1}{4}})}{4^{\frac{1}{2}}}$

3. Simplify.

a) $(2m)^3$ **c)** $(16x^6)^{\frac{1}{2}}$ **e)** $(-d^4)\left(\frac{c}{d}\right)^2$

b) $(a^4b^5)^{-2}$ **d)** $\frac{x^5y^2}{x^2y}$ **f)** $\left((x^3)^{-\frac{1}{3}}\right)^{-1}$

4. Sketch a graph of each of the following exponential functions. State the domain, range, y-intercept, and the equation of the horizontal asymptote of each function.

a) $y = 2^x$ **b)** $y = \left(\frac{1}{2}\right)^x$ **c)** $y = 3^{2x} - 2$

5. **a)** Determine the equation of the inverse of each of the following functions.

i) $f(x) = 3x - 6$ **iii)** $f(x) = 6x^3$

ii) $f(x) = x^2 - 5$ **iv)** $f(x) = (x - 4)^2 + 3$

b) Which of the inverses you found in part a) are also functions?

6. A bacteria culture doubles every 4 h. If there are 100 bacteria in the culture initially, determine how many bacteria there will be after

a) 12 h **c)** 3.5 days

b) 1 day **d)** 1 week

7. The population of a town is declining at a rate of 1.2% per year. If the population was 15 000 in 2005, what will the population be in 2020?

8. Use a table like this to compare the graphs of $y = 3(2^x)$ and $y = 3\left(\frac{1}{2}\right)^x$.

Similarities	Differences

APPLYING What You Know

Underwater Light Intensity

For every metre below the surface of the ocean, the light intensity at the surface is reduced by 2.4%. A particular underwater camera requires at least 40% of the light at the surface of the ocean to operate.

? What is the maximum depth at which the camera can successfully take photographs underwater?

A. Explain why the function $P = 100(0.976)^m$ gives the percent of light remaining at a depth of m metres below the surface of the ocean.

B. Graph P as a function of m.

C. Determine a reasonable domain and range for this function. What restrictions might have to be placed on the domain and range?

D. Determine the light intensity at a depth of 12 m.

E. At what depth is the light intensity reduced to 40% of the intensity at the surface of the ocean? Explain how you determined your answer.

F. The water in the western end of Lake Ontario is murky, and the light intensity is reduced by 3.6%/m. Write the function that represents the percent, P, of light remaining at a depth of m metres below the surface.

G. Graph the function you created in part F.

H. Compare this graph with your graph in part B. How are the graphs alike? How are they different?

I. What is the maximum depth at which the camera could take photographs in the murky water of Lake Ontario?