8 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: In what ways can the equation of the inverse of an exponential function be written?

A: One way that the inverse of an exponential function can be written is in exponential form. For example, the inverse of the exponential function $y = a^x$ is $x = a^y$. Another way that the inverse can be written is in logarithmic form. For example, $x = a^y$ can be written as $y = \log_a x$. This means that a logarithm is an exponent. Specifically, $\log_a x$ means "the exponent that must be applied to a to get x." Since $x = a^y$ is equivalent to $y = \log_a x$, this exponent is y.

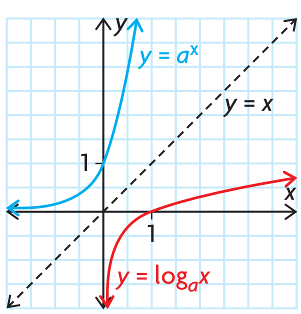
Study Aid

- See Lesson 8.1.
- Try Mid-Chapter Review Questions 1 and 2.

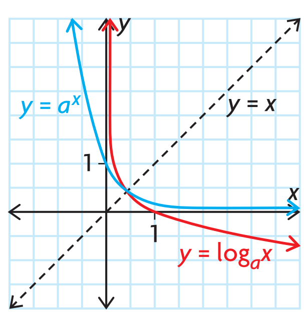
Q: What does the graph of a logarithmic function of the form $y = \log_a x$ look like and what are its characteristics?

A: The general shape of the graph of a logarithmic function depends on the value of its base.

When $a > 1$, the exponential function is an increasing function, and the logarithmic function is also an increasing function.



When $0 < a < 1$, the exponential function is a decreasing function, and the logarithmic function is also a decreasing function.



- The y-axis is the vertical asymptote for the logarithmic function. The x-axis is the horizontal asymptote for the exponential function.
- The x-intercept of the logarithmic function is 1, while the y-intercept of the exponential function is 1.
- The domain of the logarithmic function is $\{x \in \mathbf{R} \mid x > 0\}$, since the range of the exponential function is $\{y \in \mathbf{R} \mid y > 0\}$.
- The range of the logarithmic function is $\{y \in \mathbf{R}\}$, since the domain of the exponential function is $\{x \in \mathbf{R}\}$.

Study **Aid**

- See Lesson 8.2, Examples 1 and 2.
- Try Mid-Chapter Review Questions 3 and 4.

Q: How does varying the parameters of the equation $y = a\log(k(x - d)) + c$ affect the graph of the parent function, $y = \log x$?

A: The value of the parameter a determines whether there is a vertical stretch or compression. The value of k determines whether there is a horizontal stretch or compression. The value of d indicates a horizontal translation, and the value of c indicates a vertical translation. If a is negative, there is a reflection of the parent function $y = \log x$ in the x-axis. If k is negative, there is a reflection of the parent function $y = \log x$ in the y-axis.

Study **Aid**

- See Lesson 8.3, Examples 2 and 3.
- Try Mid-Chapter Review Questions 7, 8, and 9.

Q: How do you evaluate a logarithm?

A1: A logarithm of a number indicates the exponent to which the base must be raised to get the number.

For example, $\log_4 64$ means "the exponent to which you must raise 4 to get 64." The answer is 3.

A2: If the logarithm involves base 10, a calculator can be used to determine its value; $\log_{10}25 = \log 25 \doteq 1.3979$.

Study **Aid**

- See Lesson 8.4, Examples 4, 5, and 6.
- Try Mid-Chapter Review Questions 10 to 13.

A3: If the logarithm has a base other than 10, use the relationship $\log_a b = \frac{\log b}{\log a}$ and a calculator to determine its value;

$\log_2 15 = \frac{\log 15}{\log 2} \doteq 3.9069$.

Q: How do you simplify expressions that contain logarithms?

A: If the logarithms are written with the same base, you can simplify them using the laws of logarithms that correspond to the relevant exponent laws.

The log of a product can be expressed as a sum of the logs; for example, $\log_5(6 \times 7) = \log_5 6 + \log_5 7$.

The log of a quotient can be expressed as the difference of the logs; for example, $\log_7\left(\frac{25}{6}\right) = \log_7 25 - \log_7 6$.

The logarithm of a power can be expressed as the product of the exponent of the power and the logarithm of the base of the power; for example, $\log_3 4^6 = 6\log_3 4$.

PRACTICE Questions

Lesson 8.1

1. Express in logarithmic form.
- **a)** $y = 5^x$
- **b)** $y = \left(\frac{1}{3}\right)^x$
- **c)** $x = 10^y$
- **d)** $m = p^q$

2. Express in exponential form.
- **a)** $y = \log_3 x$
- **b)** $y = \log x$
- **c)** $k = \log m$
- **d)** $t = \log_s r$

Lesson 8.2

3. Describe the transformations of the parent function $y = \log x$ that result in $f(x)$.
- **a)** $f(x) = 2 \log x - 4$
- **b)** $f(x) = -\log 3x$
- **c)** $f(x) = \frac{1}{4} \log \frac{1}{4}x$
- **d)** $f(x) = \log [2(x - 2)]$
- **e)** $f(x) = \log (x + 5) + 1$
- **f)** $f(x) = 5 \log (-x) - 3$

4. Given the parent function $y = \log_3 x$, write the equation of the function that results from each set of transformations.
- **a)** vertical stretch by a factor of 4, followed by a reflection in the x-axis
- **b)** horizontal translation 3 units to the left, followed by a vertical translation 1 unit up
- **c)** vertical compression by a factor of $\frac{2}{3}$, followed by a horizontal stretch by a factor of 2
- **d)** vertical stretch by a factor of 3, followed by a reflection in the y-axis and a horizontal translation 1 unit to the right

5. State the coordinates of the image point of (9, 2) for each of the transformed functions in question 4.

6. How does the graph of $f(x) = 2 \log_2 x + 2$ compare with the graph of $g(x) = \log_2 x$?

Lesson 8.3

7. Evaluate.
- **a)** $\log_3 81$
- **b)** $\log_4 \frac{1}{16}$
- **c)** $\log_5 1$
- **d)** $\log_{\frac{2}{3}} \frac{27}{8}$

8. Evaluate to three decimal places.
- **a)** $\log 4$
- **b)** $\log 45$
- **c)** $\log 135$
- **d)** $\log 300$

9. Evaluate the value of each expression to three decimal places.
- **a)** $\log_2 21$
- **b)** $\log_5 117$
- **c)** $\log_7 141$
- **d)** $\log_{11} 356$

Lesson 8.4

10. Express as a single logarithm.
- **a)** $\log 7 + \log 4$
- **b)** $\log 5 - \log 2$
- **c)** $\log_3 11 + \log_3 4 - \log_3 6$
- **d)** $\log_p q + \log_p q$

11. Evaluate.
- **a)** $\log_{11} 33 - \log_{11} 3$
- **b)** $\log_7 14 + \log_7 3.5$
- **c)** $\log_5 100 + \log_5 \frac{1}{4}$
- **d)** $\log_{\frac{1}{2}} 72 - \log_{\frac{1}{2}} 9$
- **e)** $\log_4 \sqrt[3]{16}$
- **f)** $\log_3 9\sqrt{27}$

12. Describe how the graph of $f(x) = \log x^3$ is related to the graph of $g(x) = \log x$.

13. Use a calculator to evaluate each expression to two decimal places.
- **a)** $\log 4^8$
- **b)** $\log \sqrt{40}$
- **c)** $\log 9^4$
- **d)** $\log 200 \div \log 50$
- **e)** $(\log 20)^2$
- **f)** $5 \log 5$