8 Chapter Review

FREQUENTLY ASKED Questions

Q: How do you solve an exponential equation?

A1: All exponential equations can be solved using the following property:

If $\log_a M = \log_a N$, then $M = N$.

Take the logarithm of both sides of an exponential equation using a base of 10. Then use the power rule for logs to simplify the equation.

A2: Some exponential equations can be solved by using this property:

If $a^x = a^y$, then $x = y$, where $a > 0$, and $a \neq 1$.

Write both sides of an exponential equation with the same base, and set the exponents equal to each other.

A3: If graphing technology is available, treat both sides of an exponential equation as functions, and graph them simultaneously. The x-coordinate of the point of intersection of the two functions is the solution to the equation. There can be more than one solution.

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- See Lesson 8.5, Examples 1 to 4.
- Try Chapter Review Questions 10 to 13.

Q: How do you solve an equation that contains logarithms?

A1: If there is a single logarithm in the equation, isolate the log term and then rewrite the equation in exponential form to solve it.

A2: If there is more than one term with a logarithm in the equation, simplify the equation using the laws of logarithms. The equation can then be expressed in exponential form to solve it. If there are terms with logs on both sides of the equation, use the following property:

If $\log_a M = \log_a N$, then $M = N$, where a, M, $N > 0$.

Study *Aid*

- See Lesson 8.6, Examples 1 to 3.
- Try Chapter Review Questions 14, 15, and 16.

Q: How do you compare two values on a logarithmic scale?

A: A logarithmic scale increases exponentially, usually by powers of 10. This means that each value on a logarithmic scale is an increase of 10 times the previous value. To compare the values, use the ratio rather than the difference.

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- See Lesson 8.7, Example 1.
- Try Chapter Review Questions 17 to 20.

PRACTICE Questions

Lesson 8.1

1. Determine the inverse of each function. Express your answers in logarithmic form.

a) $y = 4^x$
b) $y = a^x$
c) $y = \left(\frac{3}{4}\right)^x$
d) $m = p^q$

Lesson 8.2

2. Describe the transformations that must be applied to the parent function $y = \log x$ to obtain each of the following functions.

a) $f(x) = -3 \log (2x)$
b) $f(x) = \log (x - 5) + 2$
c) $f(x) = \frac{1}{2} \log 5x$
d) $f(x) = \log \left(-\frac{1}{3}x\right) - 3$

3. For each sequence of transformations of the parent function $y = \log x$, write the equation of the resulting function.

a) vertical compression by a factor of $\frac{2}{5}$, followed by a vertical translation 3 units down
b) reflection in the x-axis, followed by a horizontal stretch by a factor of 2, and a horizontal translation 3 units to the right
c) vertical stretch by a factor of 5, followed by a horizontal compression by a factor of $\frac{1}{2}$, and a reflection in the y-axis
d) a reflection of the y-axis, a horizontal translation 4 units to the left, followed by a vertical translation 2 units down

4. Describe how the graphs of $f(x) = \log x$ and $g(x) = 3 \log (x - 1) + 2$ are similar yet different.

Lesson 8.3

5. Evaluate.

a) $\log_7 343$
b) $\log_{\frac{1}{5}} 25$
c) $\log_{19} 1$
d) $\log_4 \left(\frac{1}{256}\right)$

6. Estimate the value to three decimal places.

a) $\log_3 53$
b) $\log_4 \frac{1}{10}$
c) $\log_6 159$
d) $\log_{15} 1456$

Lesson 8.4

7. Express as a single logarithm.

a) $\log 5 + \log 11$
b) $\log 20 - \log 4$
c) $\log_5 6 + \log_5 8 - \log_5 12$
d) $2 \log 3 + 4 \log 2$

8. Use the laws of logarithms to evaluate.

a) $\log_6 42 - \log_6 7$
b) $\log_3 5 + \log_3 18 - \log_3 10$
c) $\log_7 \sqrt[3]{49}$
d) $2 \log_4 8$

9. Describe how the graph of $y = \log (10\,000x)$ is related to the graph of $y = \log x$.

Lesson 8.5

10. Solve.

a) $5^x = 3125$
b) $4^x = 16\sqrt{128}$
c) $4^{5x} = 16^{2x-1}$
d) $3^{5x}9^{x^2} = 27$

11. Solve. Express each answer to three decimal places.

a) $6^x = 78$
b) $(5.4)^x = 234$
c) $8(3^x) = 132$
d) $200(1.23)^x = 540$

12. Solve.

a) $4^x + 6(4^{-x}) = 5$
b) $8(5^{2x}) + 8(5^x) = 6$

13. The half-life of a certain substance is 3.6 days. How long will it take for 20 g of the substance to decay to 7 g?

Lesson 8.6

14. Solve.

a) $\log_5(2x - 1) = 3$
b) $\log 3x = 4$
c) $\log_4(3x - 5) = \log_4 11 + \log_4 2$
d) $\log(4x - 1) = \log(x + 1) + \log 2$

15. Solve.

a) $\log(x + 9) - \log x = 1$
b) $\log x + \log(x - 3) = 1$
c) $\log(x - 1) + \log(x + 2) = 1$
d) $\log\sqrt{x^2 - 1} = 2$

16. Recall that $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of sound in watts per square metre (W/m^2) and $I_0 = 10^{-12}\ \text{W/m}^2$. Determine the intensity of a baby screaming if the noise level is 100 dB.

Lesson 8.7

17. What is the sound intensity in watts per square metre (W/m^2) of an engine that is rated at 82 dB?

18. How many times more intense is an earthquake of magnitude 6.2 than an earthquake of magnitude 5.5?

19. Pure water has a pH value of 7.0. How many times more acidic is milk, with a pH value of 6.4, than pure water?

20. Does an increase in acidity from pH 4.7 to pH 2.3 result in the same change in hydrogen ion concentration as a decrease in alkalinity from 12.5 to 10.1? Explain.

21. Is an exponential model appropriate for the data in the following table? If it is, determine the equation that models the data.

x	0	2	4	6	8	10
y	3.0	15.2	76.9	389.2	1975.5	9975.8

22. The population of a town is decreasing at the rate of 1.6%/a. If the population today is 20 000, how long will it take for the population to decline to 15 000?

Lesson 8.8

23. The following table gives the population of a city over time.

Year	1950	1970	1980	1990	1994
Population	132 459	253 539	345 890	465 648	514 013

a) Calculate the average rate of growth over the entire time period.
b) Calculate the average rate of growth for the first 30 years. How does it compare with the rate of growth for the entire time period?
c) Determine an exponential model for the data.
d) Estimate the instantaneous rate of growth in
i) 1970 ii) 1990

24. The following data show the number of people (in thousands) who own a DVD player in a large city or linear is best for over a period of years.

Year	1998	1999	2000	2001	2002
Number of DVD Owners (thousands)	23	27	31	37	43

a) Determine if an exponential or linear model is best for this data.
b) Use your model to predict how many people will own a DVD player in the year 2015.
c) What assumptions did you make to make your prediction in part b)? Do you think this is reasonable? Explain.
d) Determine the average rate of change in the number of DVD players in this city between 1999 and 2002.
e) Estimate the instantaneous rate of change in the number of DVD players in this city in 2000.
f) Explain why using an exponential model to answer part b) does not make sense.