Chapter

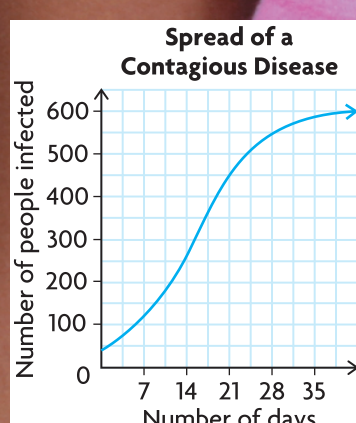
9

Combinations of Functions

GOALS

You will be able to

- Consolidate your understanding of the characteristics of functions
- Create new functions by adding, subtracting, multiplying, and dividing functions
- Investigate the creation of composite functions numerically, graphically, and algebraically
- Determine key characteristics of these new functions
- Solve problems using a variety of function models



? *Epidemiology* is the scientific study of contagious diseases. A combination of functions is often used to model the way that a contagious disease spreads through a population. What types of functions could be combined to create an algebraic model that represents the graph shown?

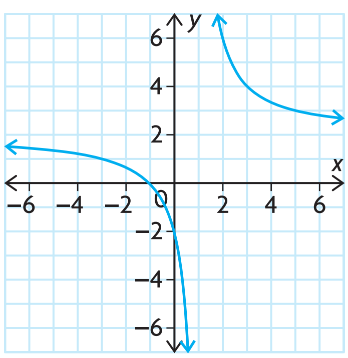
9 Getting Started

SKILLS AND CONCEPTS You Need

Study Aid

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
3	R-8



1. Evaluate each of the following functions for $f(-1)$ and $f(4)$. Round your answers to two decimal places, if necessary.

a) $f(x) = x^3 - 3x^2 - 10x + 24$

b) $f(x) = \dfrac{4x}{1 - x}$

c) $f(x) = 3\log_{10}(x)$

d) $f(x) = -5(0.5^{(x-1)})$

2. Identify the following characteristics of functions for the graph displayed.

- domain and range
- maximum or minimum values
- interval(s) where the function is increasing
- interval(s) where the function is decreasing
- end behaviour
- equations of asymptotes

3. For each parent function, apply the given transformation(s) and write the equation of the new function.

a) $y = |x|$; vertical stretch by a factor of 2, shift 3 units to the right

b) $y = \cos(x)$; reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$

c) $y = \log_3 x$; reflection in the y-axis, shift 4 units left and 1 unit down

d) $y = \frac{1}{x}$; vertical stretch by a factor of 4, reflection in the x-axis, shift 5 units down

4. Solve each equation for x, $x \in \mathbf{R}$. State any restrictions on x, as required.

a) $2x^3 - 7x^2 - 5x + 4 = 0$

b) $\dfrac{2x + 3}{x + 3} + \dfrac{1}{2} = \dfrac{x + 1}{x - 1}$

c) $\log x + \log(x - 3) = 1$

d) $10^{-4x} - 22 = 978$

e) $5^{x+3} - 5^x = 0.992$

f) $2\cos^2 x = \sin x + 1, 0 \le x \le 2\pi$

5. Solve each inequality for x, $x \in \mathbf{R}$.

a) $x^3 - x^2 - 14x + 24 < 0$

b) $\dfrac{(2x - 3)(x - 4)}{(x + 2)} \ge 0$

6. Identify each function as even, odd, or neither.

a) $f(x) = 2\sin(x - \pi)$

b) $f(x) = \dfrac{3}{4 - x}$

c) $f(x) = 4x^4 - 3x^2$

d) $f(x) = 2^{3x-1}$

7. Classify the types of functions you have studied (polynomial, rational, exponential, logarithmic, and trigonometric) as continuous or not.

APPLYING What You Know

Building a Sandbox

Duncan is planning to build a rectangular sandbox in his backyard for his son to play in during the summer. He has designed the sandbox so that it will have an open top and a volume of 2 m^3. The length of the base will measure four times the height of the sandbox. The wood for the base will cost $\$5/m^2$, and the wood for the sides will cost $\$4/m^2$.

? What dimensions should Duncan use to minimize the cost of the sandbox he has designed?

A. Let h represent the height (in metres) and let w represent the width of the sandbox. Determine an expression for the width of the sandbox in terms of its height.

B. Write an expression for the cost of the wood for the base of the sandbox in terms of its height.

C. Express the cost of the wood for the two longer sides in terms of the height. Is the cost for the two shorter sides the same?

D. Let $C(h)$ represent the total cost of the wood for the sandbox as a function of its height. Determine the equation for $C(h)$.

E. What types of functions are added in your equation for $C(h)$?

F. What would be a reasonable domain and range for this cost function? Explain.

G. Using graphing technology, graph the cost function using window settings that correspond to its domain and range.

H. Determine the height of the sandbox that will minimize the total cost.

I. What dimensions would you recommend that Duncan use to build the sandbox? Justify your answer.