

9.1

Exploring Combinations of Functions

YOU WILL NEED

- graphing calculator or graphing software

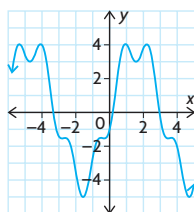
GOAL

Explore the characteristics of new functions created by combining functions.

Explore the Math

Ahmad was given the graphs pictured below. They were created by combining two familiar functions.

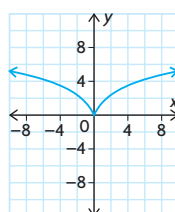
Graph 1



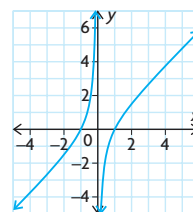
Graph 2



Graph 3



Graph 4



Ahmad does not recognize these new functions and wonders which type of functions have been combined to create them. He also wonders whether any of these graphs could model a real-life situation.

? How can two functions be combined to create a new function?

A. Compare each of the graphs above with the function equations in the table below.

$y = x\sqrt{x-1}$	$y = 4 \sin x - \cos 4x$	$y = x - \frac{1}{x}$	$y = 5 \log(x + 1)$
$y = (x^2)(\sin(x))$	$y = \begin{cases} -0.5(x-2)^2 + 2, & x < 0 \\ 0.5(x-2)^2 - 2, & x \geq 0 \end{cases}$	$y = (0.5^x)(4 \sin(2\pi x))$	$y = x^3 \div (x+1)$

Predict which equations will match each graph. Copy the table on the next page, and record your predictions and your rationale for each.

Graph	Equation of Function	Rationale
1		
2		
3		
4		

- B.** Compare your predictions with a partner's predictions. Explain to each other why you made each prediction.
- C.** Using graphing technology in radian mode, graph the equation that you predicted would match graph 1. Use a domain and range in the window settings that match the scale given on each of the given graphs.
- D.** Does the graph of your equation match graph 1? If it does not, choose another equation from the table and try again.
- E.** Once you have correctly matched the equation with graph 1, repeat parts C and D until all the graphs have been correctly matched.
- F.** Examine the equation that matches each graph.
- List the parent functions in each equation.
 - State the transformations that were applied to each parent function.
 - Explain how the parent functions were combined.

Reflecting

- G.** Which of the four given graphs is periodic? How does it differ from other periodic functions you have seen before? What type of combination produced this effect?
- H.** Do any of the graphs represent an even function? Do any represent an odd function? Explain how you know.
- I.** Which graph contains an asymptote? Describe the functions that were combined to produce this graph. Explain how you can tell from the equation where the vertical asymptote occurs.
- J.** Which graph could be used to model the motion of a swaying building moments after an earthquake? Explain why.

In Summary

Key Idea

- Many interesting functions can be created by combining two or more simpler functions. This can be done by adding, subtracting, multiplying, or dividing functions to create more complex functions.

Need to Know

- The characteristics of the functions that are combined affect the properties and characteristics of the resulting function.

FURTHER Your Understanding

- Using graphing technology (in radian mode) and the functions given in the chart below, experiment to create new functions by combining different types of functions. Each time, use different operations and different types of functions. You may need to experiment with the window settings to get a clear picture of what the graph looks like. Include a sketch of your new graphs and the equations that were used for the models.

$y = 2 - 0.5x$	$y = 2^x$	$y = \sin 2\pi x$	$y = \cos 2\pi x$
$y = \log x$	$y = \left(\frac{1}{2}\right)^x$	$y = x^4 - x^2$	$y = 2x$

- Using the functions in the chart above, create a new function that has each of the characteristics given below. Include a sketch of your new graphs and the equations that were used for the models.
 - a function that has a vertical asymptote and a horizontal asymptote
 - a function that is even
 - a function that is odd
 - a function that is periodic
 - a function that resembles a periodic function with decreasing maximum values and increasing minimum values
 - a function that resembles a periodic function with increasing maximum values and decreasing minimum values
- Select any two functions that you have studied in this course. Experiment by combining these functions in various ways and graphing them on a graphing calculator. Include a sketch of your new graphs and the equations of the functions you selected. Challenge your classmates to see who can produce the most interesting graph.