9 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: If you are given the graphs of two functions, *f* and *g*, how can you determine the location of a point that would appear on the graphs of $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: For any particular x-value, determine the y-value on each graph, separately. For $f + g$, add these two y-values together. For $f - g$, subtract the y-value of g from the y-value of f. For $f \times g$, multiply these two y-values together. For $f \div g$, divide the y-value of f by the y-value of g. Each of these points has, as its coordinates, the same x-value and the new y-value.

Study *Aid*

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Question 2.

Q: If you are given the equations of two functions, *f* and *g*, how can you determine the equations of the functions $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: Every time you combine two functions in one of these ways, you are simply performing a different arithmetic operation on every pair of y-values, one from each of the functions being combined, provided that the x-values are the same. Since the equation of each function defines the y-values of each function, the new equation can be determined by adding, subtracting, multiplying, or dividing the y-value expressions as required.

For example, if $f(x) = x^2 + 8$ and $g(x) = 5^x$, then

$$(f + g)(x) = f(x) + g(x) = x^2 + 8 + 5^x$$

$$(f \times g)(x) = f(x) \times g(x) = (x^2 + 8)(5^x)$$

$$(f - g)(x) = f(x) - g(x) = x^2 + 8 - 5^x$$

$$(f \div g)(x) = f(x) \div g(x) = \frac{x^2 + 8}{5^x}$$

Study *Aid*

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Questions 5 and 7.

Q: How can you determine the domain of the combined functions $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: Since you can only combine points from two functions when they share the same x-value, the domain of the combined function must consist of the set of x-values where the domains of the two given functions intersect. The only exception occurs when you are dividing two functions. The function $f \div g$ is not defined when its denominator is equal to zero, since division by zero is undefined. As a result, x-values that cause $g(x)$ to equal zero must be excluded from the domain.

Study *Aid*

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Questions 5 and 7.

PRACTICE Questions

Lesson 9.1

1. Given the functions $f(x) = \cos x$ and $g(x) = \sin x$, which operations can be used to combine the two functions to create a new function with an amplitude that is less than 1?

Lesson 9.2

2. Let $f(x) = \{(-9, -2), (-6, -3), (-3, 0), (0, 2), (3, 7)\}$ and $g(x) = \{(-12, 9), (-9, 4), (-8, 1), (-7, 10), (-6, -6), (0, 12)\}$. Determine

a) $(f + g)(x)$
b) $(g + f)(x)$
c) $(f - g)(x)$
d) $(g - f)(x)$

3. The cost, in thousands of dollars, for a company to produce x thousand of its product is given by the function $C(x) = 10x + 30$. The revenue from the sales of the product is given by the function $R(x) = -5x^2 + 150x$.

a) Write the function that represents the company's profit on sales of x thousand of its product.
b) Graph the cost, revenue, and profit functions on the same coordinate grid, where $0 \leq x \leq 40$.
c) What is the company's profit on the sale of 7500 of its product?

4. Steve earns \$24.39/h operating an industrial plasma torch at a rail-car manufacturing plant. He receives \$0.58/h more for working the night shift, as well as \$0.39/h more for working weekends.

a) Write a function that describes Steve's daily earnings under regular pay.
b) What function shows his daily earnings under the night-shift premium?
c) What function shows his daily earnings under the weekend premium?
d) What function represents his earnings for the night shift on Saturday?
e) How much does Steve earn for working 11 h on Saturday night, if he earns time and a half on that day's rate for more than 8 h of work?

Lesson 9.3

5. Determine $(f \times g)(x)$ for each of the following pairs of functions, and state its domain.

a) $f(x) = x + \frac{1}{2}, g(x) = x + \frac{1}{2}$
b) $f(x) = \sqrt{x - 10}, g(x) = \sin(3x)$
c) $f(x) = 11x^3, g(x) = \frac{2}{x + 5}$
d) $f(x) = 90x - 1, g(x) = 90x + 1$

6. A diner is open from 6 a.m. to 6 p.m., and the average number of customers in the diner at any time can be modelled by the function $C(h) = -30 \cos\left(\frac{\pi}{6}h\right) + 34$, where h is the number of hours after the 6 a.m. opening time. The average amount of money, in dollars, that each customer in the diner will spend can be modelled by the function $D(h) = -3 \sin\left(\frac{\pi}{6}h\right) + 7$.

a) Write the function that represents the diner's average revenue from the customers.
b) Graph the function you wrote in part a).
c) What is the average revenue from the customers in the diner at 2 p.m.?

Lesson 9.4

7. Calculate $(f \div g)(x)$ for each of the following pairs of functions, and state its domain.

a) $f(x) = 240, g(x) = 3x$
b) $f(x) = 10x^2, g(x) = x^3 - 3x$
c) $f(x) = x + 8, g(x) = \sqrt{x - 8}$
d) $f(x) = 14x^2, g(x) = 2 \log x$

8. Recall that $y = \tan x$ can be written as the quotient of two functions: $f(x) = \sin x$ and $g(x) = \cos x$. List as many other trigonometric functions as possible that could be written as the quotient of two functions.