9 Chapter Review

FREQUENTLY ASKED Questions

Q: How can you determine the composition of two functions, *f* and *g*?

A1: The composition of f with g can be determined numerically by evaluating g for some input value, x, and then evaluating f using $g(x)$ as the input value.

A2: The composition of f with g can be determined graphically by interpolating on the graph of g to determine its output for some input value, x, and then interpolating on the graph of f using the input value $g(x)$.

A3: The composition of f with g can be determined algebraically by taking the expression for g and then substituting this into the function f.

Study *Aid*

- See Lesson 9.5, Examples 1 and 2.
- Try Chapter Review Questions 8, 9, and 10.

Q: How do you solve an equation or inequality when an algebraic strategy is difficult or not possible?

A1: If you have access to graphing technology, there are two different strategies you can use to solve an equation:

- Represent the two sides of the equation/inequality as separate functions. Then graph the functions together using a graphing calculator or graphing software, and apply the intersection operation to determine the solution(s).
- Rewrite the equation/inequality so that one side is zero. Graph the nonzero side as a function. Use the zero operation to determine each of the zeros of the function.

A2: If you do not have access to graphing technology, you can use a guess and improvement strategy to solve an equation. Estimate where the intersection of $f(x)$ and $g(x)$ will occur, and substitute this value into both sides of the equation. Based on the outcome, adjust your estimate. Repeat this process until the desired degree of accuracy is found.

A3: Solving an inequality requires using either of the three previous strategies to find solutions to either $f(x) - g(x) = 0$ or $f(x) = g(x)$. Use these values to construct intervals. Test each interval to see whether it satisfies the inequality.

Study *Aid*

- See Lesson 9.6, Example 3.
- Try Chapter Review Question 12.

PRACTICE Questions

Lesson 9.1

1. Given the functions $f(x) = x + 5$ and $g(x) = x^2 - 6x - 55$, determine which of the following operations can be used to combine the two functions into one function that has both a vertical asymptote and a horizontal asymptote: addition, subtraction, multiplication, division.

Lesson 9.2

2. A franchise owner operates two coffee shops. The sales, S_1, in thousands of dollars, for shop 1 are represented by $S_1(t) = 700 - 1.4t^2$, where $t = 0$ corresponds to the year 2000. Similarly, the sales for shop 2 are represented by $S_2(t) = t^3 + 3t^2 + 500$.
   - **a)** Which shop is showing an increase in sales after the year 2000?
   - **b)** Determine a function that represents the total sales for the two coffee shops.
   - **c)** What are the expected total sales for the year 2006?
   - **d)** If sales continue according to the individual functions, what would you recommend that the owner do? Explain.

3. A company produces a product for \$9.45 per unit, plus a fixed operating cost of \$52 000. The company sells the product for \$15.80 per unit.
   - **a)** Determine a function, $C(x)$, to represent the cost of producing x units.
   - **b)** Determine a function, $I(x)$, to represent income from sales of x units.
   - **c)** Determine a function that represents profit.

Lesson 9.3

4. Calculate $(f \times g)(x)$ for each of the following pairs of functions.
   - **a)** $f(x) = 3 \tan (7x)$, $g(x) = 4 \cos (7x)$
   - **b)** $f(x) = \sqrt{3x^2}$, $g(x) = 3\sqrt{3x^2}$
   - **c)** $f(x) = 11x - 7$, $g(x) = 11x + 7$
   - **d)** $f(x) = ab^x$, $g(x) = 2ab^{2x}$

5. A country projects that the average amount of money, in dollars, that it will collect in taxes from each taxpayer over the next 50 years can be modelled by the function $A(t) = 2850 + 200t$, where t is the number of years from now. It also projects that the number of taxpayers over the next 50 years can be modelled by the function $C(t) = 15\,000\,000(1.01)^t$.
   - **a)** Write the function that represents the amount of money, in dollars, that the country expects to collect in taxes over the next 50 years.
   - **b)** Graph the function you wrote in part a).
   - **c)** How much does the country expect to collect in taxes 26 years from now?

Lesson 9.4

6. Calculate $(f \div g)(x)$ for each of the following pairs of functions.
   - **a)** $f(x) = 105x^3$, $g(x) = 5x^4$
   - **b)** $f(x) = x - 4$, $g(x) = 2x^2 + x - 36$
   - **c)** $f(x) = \sqrt{x + 15}$, $g(x) = x + 15$
   - **d)** $f(x) = 11x^5$, $g(x) = 22x^2 \log x$

7. State the domain of $(f \div g)(x)$ for each of your answers in the previous question.

Lesson 9.5

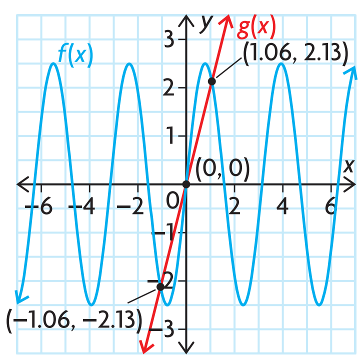
8. Let $f(x) = \dfrac{1}{\sqrt{x + 1}}$ and $g(x) = x^2 + 3$.
   - **a)** What are the domain and range of $f(x)$ and $g(x)$?
   - **b)** Find $f(g(x))$.
   - **c)** Find $g(f(x))$.
   - **d)** Find $f(g(0))$.
   - **e)** Find $g(f(0))$.
   - **f)** State the domain of each of the functions you found in parts b) and c).

9. Let $f(x) = x - 3$. Determine each of the following functions:
   a) $(f \circ f)(x)$
   b) $(f \circ f \circ f)(x)$
   c) $(f \circ f \circ f \circ f)(x)$
   d) f composed with itself n times

10. A circle has radius r.
   a) Write a function for the circle's area in terms of r.
   b) Write a function for the radius in terms of the circumference, C.
   c) Determine $A(r(C))$.
   d) A tree's circumference is 3.6 m. What is the area of the cross-section?

Lesson 9.6

11. In the graph shown below, $f(x) = 5 \sin x \cos x$ and $g(x) = 2x$. State the values of x in which $f(x) < g(x)$, $f(x) = g(x)$, and $f(x) > g(x)$. Express the values to the nearest tenth.



12. Solve each of the following equations for x in the given interval, using a guess and improvement strategy. Express your answers to the nearest tenth, and verify them using graphing technology.
   a) $-3 \csc x = x, \pi \leq x \leq \frac{3\pi}{2}$
   b) $\cos^2 x = 3 - 2\sqrt{x}, 0 \leq x \leq \pi$
   c) $8^x = x^8, -1 \leq x \leq 1$
   d) $7 \sin x = \frac{3}{x}, 0 \leq x \leq 2$

Lesson 9.7

13. Let P represent the size of the frog population in a marsh at time t, in years. At $t = 0$, a species of frog is released into a marsh. When $t = 5$, biologists estimate that there are 2000 frogs in the marsh. Two years later, the biologists estimate that there are 3200 frogs.
   a) Find a formula for $P = f(t)$, assuming linear growth. Interpret the slope and the P-intercept of your formula in terms of the frog population.
   b) Find a formula for $P = g(t)$, assuming exponential growth. Interpret the parameters of your formula in terms of the frog population.

14. The population of the world from 1950 to 2000 is shown. Create a scatter plot of the data, and determine an algebraic model for this situation. Use your model to estimate the world's population in 1963, 1983, and 2040.

Year	Population (millions)
1950	2555
1955	2780
1960	3039
1965	3346
1970	3708
1975	4088
1980	4457
1985	4855
1990	5284
1995	5691
2000	6080

Source: U.S. Census Bureau