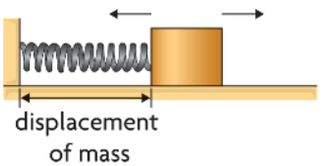
9 Chapter Task

Modelling a Situation Using a Combination of Functions

A mass is attached to a spring at one end and secured to a wall at the other end. When the mass is pulled away from the wall and released, it moves back and forth (oscillates) along the floor.



If there is no friction between the mass and the floor, and no drag from the air, then the displacement of the mass versus time could be modelled by a sinusoidal function. Because of friction, however, the speed of the mass is reduced, which causes the displacement to decrease exponentially with each oscillation.

The displacement function $d(t)$ is a combination of functions: $d(t) = f(t)g(t) + r$.

Consider the following situation:

- The mass is at a resting position of $r = 30$ cm.
- The spring provides a period of 2 s for the oscillations.
- The mass is pulled to $d = 50$ cm and released.
- After 10 s, the spring is at $d = 33$ cm.

? How would the displacement and speed of the mass at time $t = 7.7$ s differ if there were no friction between the mass and the floor?

A. Make a sketch of the displacement versus time graph to ensure that you understand this situation.

B. Write the general equation of the function that models this situation, with the necessary parameters.

C. Use the information provided to determine the values of the parameters, and write the equation of the model.

D. Graph the function you determined in part C using graphing technology. Check that it models the motion of the mass correctly.

E. Write the function for displacement that would be correct if there were no damping of the motion due to friction.

F. Calculate the displacement at 7.7 s for each model you determined in parts C and E, and compare your results.

G. Estimate the instantaneous speed of the mass at 7.7 s for each model, and compare your results.

Task Checklist

- ✔ Did you draw and label your displacement versus time graph accurately?
- ✔ Did you show all your steps when determining both models?
- ✔ Did you show all your steps when determining the displacements and speeds?
- ✔ Did you discuss the difference between the displacements and speeds?