Chapters 7–9 Cumulative Review

Multiple Choice

1. Which of these is an equivalent trigonometric ratio for $\sin \frac{2\pi}{5}$?

a) $\cos \frac{\pi}{10}$ **c)** $-\cos \frac{9\pi}{10}$

b) $\sin \frac{3\pi}{5}$ **d)** all of these

2. What is the exact value of $\cos \frac{\pi}{12}$?

a) $\frac{\sqrt{3}}{4}$ **c)** $\frac{\sqrt{6}}{4}$

b) $\frac{\sqrt{2}+\sqrt{6}}{4}$ **d)** $\frac{\sqrt{6}-\sqrt{2}}{4}$

3. If α and β are acute angles with $\sin \alpha = \frac{12}{13}$ and $\sin \beta = \frac{8}{17}$, what is the value of $\tan (\alpha + \beta)$?

a) $-\frac{220}{21}$ **c)** $\frac{220}{221}$

b) $\frac{220}{21}$ **d)** $\frac{220}{123}$

4. Given that $\sin \theta = \frac{3}{8}$ and θ is obtuse, what is the value of $\tan 2\theta$?

a) $-\frac{3\sqrt{55}}{23}$ **c)** $-\frac{3\sqrt{55}}{55}$

b) $\frac{3\sqrt{55}}{46}$ **d)** $\frac{3\sqrt{55}}{55}$

5. What is the exact value of $\cos \frac{\pi}{8}$?

a) $\frac{2+\sqrt{2}}{2}$ **c)** $\frac{\sqrt{2-\sqrt{2}}}{4}$

b) $\frac{\sqrt{2-\sqrt{2}}}{2}$ **d)** $\frac{\sqrt{2+\sqrt{2}}}{2}$

6. Which expression is equivalent to $\cos x$?

a) $\frac{2\cos^2\left(\frac{1}{2}x\right)-1}{\cos^2\left(\frac{1}{2}x\right)}$ **c)** $\frac{2-\sec^2\left(\frac{1}{2}x\right)}{\sec^2\left(\frac{1}{2}x\right)}$

b) $2\cos^2(2x)-1$ **d)** $1-2\sin^2(2x)$

7. Which of the following identities could you use to help you prove that $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$?

a) $1+\tan^2 x = \sec^2 x$

b) $\sin 2x = 2\sin x \cos x$

c) $\tan x = \frac{\sin x}{\cos x}$

d) all of these

8. Which set of value(s), in radians, is the solution of $5 + 7\sin\theta = 0$, where $-\pi \le \theta \le \pi$?

a) $\theta = -0.80$

b) $\theta = -0.80, -2.35$

c) $\theta = 0.80, 2.35$

d) $\theta = -0.80, 0.80$

9. The height of the tip of one blade of a wind turbine above the ground, $h(t)$, can be modelled by $h(t) = 18\cos\left(\pi t + \frac{\pi}{4}\right) + 23$, where t is the time passed in seconds. Which time interval describes a period when the blade tip is at least 30 m above the ground?

a) $5.24 \le t \le 7.33$ **c)** $1.37 \le t \le 2.12$

b) $0.42 \le t \le 1.08$ **d)** $0.08 \le t \le 1.42$

10. Which set of values is the solution of $(2\sin x + 1)(\cos x - 1) = 0$, where $0° \le x \le 360°$?

a) $x = 180°, 210°, 330°$

b) $x = 30°, 180°, 150°$

c) $x = 0°, 150°, 210°, 360°$

d) $x = 0°, 210°, 330°, 360°$

11. The equation $\cos 2\theta + d\cos\theta + e = 0$ has solutions $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ in the interval $0 \le \theta \le 2\pi$. What are the values of d and e?

a) $d = -3, e = 2$ **c)** $d = 1, e = 3$
b) $d = 2, e = 3$ **d)** $d = -3, e = 1$

12. What is the exponential form of $y = \log_7 x$?

a) $x = \log_7 y$ **c)** $y = 7^x$
b) $x = 7^y$ **d)** $y = x^7$

13. The function $f(x) = \log_{10} x$ is reflected in the x-axis, stretched horizontally by a factor of 3, and translated up 2 units. Which of these functions is the result?

a) $g(x) = -\log_{10}(3x) - 2$

b) $g(x) = \log_{10}\left(-\frac{1}{3}x\right) + 2$

c) $g(x) = -\log_{10}\left(\frac{1}{3}(x - 2)\right) + 2$

d) $g(x) = -\log_{10}\left(\frac{1}{3}x\right) + 2$

14. What is the value of $7^{\log_7 49}$?

a) 7 **b)** 2 **c)** 14 **d)** 49

15. The equation $\log_{10} T = 1.5\log_{10} d - 0.45$ describes the orbit of a planet around the star Gliese 581. In this equation, T is the length of the planet's year in days, and d is its average distance in millions of kilometres from Gliese 581. The earth-like planet Gliese 581c is 11 000 000 km from Gliese 581. How long is its year?

a) 16.1 days **c)** 12.9 days
b) 1.1 days **d)** 3.9 days

16. What is the solution of the equation $\log_4 x + 3 = \log_4 1024$?

a) 16 **b)** 4 **c)** 128 **d)** $\frac{16}{3}$

17. A transformation that takes the graph of $f(x) = \log_5 x$ to that of $g(x) = \log_5 25x$ is

a) horizontal translation 2 units left
b) vertical translation 2 units up
c) vertical stretch by a factor of 25
d) horizontal stretch by a factor of 25

18. Solve $x = \log_3 27\sqrt{3}$.

a) $2\frac{1}{2}$ **b)** $3\frac{1}{2}$ **c)** $3\frac{1}{3}$ **d)** $9\frac{1}{2}$

19. An investment of \$1600 grows at a rate of 1% per month, compounded monthly. How long will it take for the investment to be worth more than \$6400? Recall that the formula for compound interest is $A = P(1 + i)^n$.

a) 11 years 7 months **c)** 11 years 8 months
b) 33 years 3 months **d)** 33 years 4 months

20. The loudness of a sound in decibels, L, is $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound in watts per square metre (W/m^2) and $I_0 = 10^{-12}\ \text{W/m}^2$. If the loudness of a jet taking off is 133 dB, what is the intensity of this sound?

a) $2.00 \times 10^{13}\ \text{W/m}^2$ **c)** $10^{-1}\ \text{W/m}^2$
b) $10.0\ \text{W/m}^2$ **d)** $20.0\ \text{W/m}^2$

21. Solve the following:
$\log_a(x - 3) + \log_a(x - 2) = \log_a(5x - 15)$

a) $x = 3$ **c)** $x = -3$ or 7
b) $x = 7$ **d)** $x = 2$

22. Carbon-14 has a half-life of 5730 years. A fossil human jawbone that contains 0.017 g of carbon-14 is estimated to have contained 3.9 g when the person was alive. How old is the fossil?

a) 45 000 years **c)** 1 300 000 years
b) 13 500 years **d)** 12 000 years

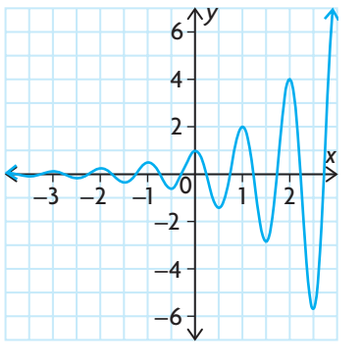
23. Assume that the annual rate of inflation will average 3.1% over the next 5 years. For a product that currently costs P dollars, which is the best model for the approximate cost, C, of goods and services during any year in the next 5 years?

a) $C = P(1 + 0.031^t)$
b) $C = (1.031)^t$
c) $C = P(1.031)^t$
d) $C = P(1 + 3.1^t)$

24. The population of a city is currently 150 000 and is increasing at a rate of 2.3%/a. Predict the instantaneous rate of growth in the population 7 years from now.

a) 175 900 people/a **c)** 4000 people/a
b) 25 900 people/a **d)** 3700 people/a

25. Which combination of functions could result in this graph?



a) $y = x^2 \cos(2\pi x)$
b) $y = \sin(2\pi x) + \log x$
c) $y = 2^x \cos(2\pi x)$
d) $y = \sin(2\pi|x|)0.5^x$

26. If $f(x) = \log x$ and $g(x) = \frac{1}{x-3}$, which set is the domain of $f - g$?

a) $\{x \in \mathbf{R} \mid x > 3\}$
b) $\{x \in \mathbf{R} \mid x > 0, x \neq 3\}$
c) $\{x \in \mathbf{R} \mid x > 0, x \neq -3\}$
d) $\{x \in \mathbf{R} \mid x < 3\}$

27. Which combination is always an odd function?

a) the sum of two odd functions
b) the difference of an odd function and an even function
c) the sum of an odd function and an even function
d) the difference of two even functions

28. For which pair of functions, $f(x)$ and $g(x)$, is the range of $f \times g$ equal to $\{y \in \mathbf{R} \mid y \geq 1\}$?

a) $f(x) = g(x) = \sec x$
b) $f(x) = \sec x, g(x) = \csc x$
c) $f(x) = 2^x, g(x) = |x| + 1$
d) $f(x) = 2^x, g(x) = x^2 + 1$

29. Given $f(x) = ax^2 + 3$ and $g(x) = bx - 1$, the graph of the product $f \times g$ passes through the points $(-1, -3)$ and $(1, 9)$. What are the values of a and b?

a) $a = -6, b = 10$ **c)** $a = -8, b = 2$
b) $a = 6, b = 2$ **d)** $a = -6, b = -2$

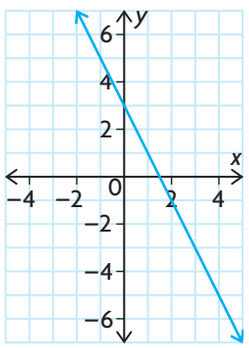
30. What is the domain of $f \div g$, where $f(x) = \log x$ and $g(x) = |x - 2|$?

a) $\{x \in \mathbf{R} \mid x \neq 0, 2\}$
b) $\{x \in \mathbf{R} \mid x > 2\}$
c) $\{x \in \mathbf{R} \mid 0 < x < 2\}$
d) $\{x \in \mathbf{R} \mid x > 0, x \neq 2\}$

31. If $f(x) = \sqrt{3 - x}$ and $g(x) = 3x^2$, what is the domain of $f \circ g$?

a) $\{x \in \mathbf{R} \mid -3 \leq x \leq 3\}$
b) $\{x \in \mathbf{R} \mid x \leq 3\}$
c) $\{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$
d) $\{x \in \mathbf{R} \mid x \geq 0\}$

32. Which combination of the functions $f(x) = 2x$, $g(x) = x + 5$, and $h(x) = 3 - x$ has this graph?



a) $f \circ g$ **c)** $h \circ g$
b) $f \circ h$ **d)** $h \circ f$

33. Which values are solutions of the equation $x^3 = \sqrt[3]{\tan x}$?

a) $x = 0$ **c)** $x = 1.07$
b) $x = -1.07$ **d)** all of these

34. Given $f(x) = 4 - x^2$, for which function $g(x)$ is $f(x) < g(x)$ when $x \in (-\infty, -1)$ or $(4, \infty)$?

a) $g(x) = 4x$ **c)** $g(x) = 4x - 8$
b) $g(x) = -3x$ **d)** $g(x) = -4x$

Investigations

Touchdown Pass

35. The horizontal distance, d, in metres, that a football can be thrown from its release point to the point where it hits the ground can be modelled by the equation $d = \frac{v^2}{9.8} \sin 2\theta + 1.8$, where v is the initial speed of the football in metres per second and θ is the angle relative to the horizontal at which the football leaves the quarterback's hand. If the football is thrown at 20 m/s and travels 35 m, determine the possible angles at which the football could be thrown. Give your answer to the nearest degree.

Projecting Populations

36. The data below were collected by the Ontario Ministry of Finance and released in July 2000. It shows the projected populations (in thousands) of the Regional Municipalities of Niagara and Waterloo.

	Historical		Projections						
Regional Municipality	**1996**	**1999**	**2001**	**2006**	**2011**	**2016**	**2021**	**2026**	**2028**
Niagara	414.8	421.7	426.4	435.9	445.3	455.1	464.9	473.8	476.8
Waterloo	418.3	438.4	452.1	483.6	512.6	541.4	569.8	596.3	606.1

a) Determine suitable models that the Ministry of Finance might have used to make these projections.

b) Use your models to estimate the doubling time of the population in each region.

c) Use your models to predict which region's population will be increasing the fastest in 2025. Support your answer with the necessary calculations.

It's Rocket Science

37. The mass of a rocket just before launch is 30 000 kg. During its ascent, the rocket burns 100 kg of fuel every second, and therefore decreases in mass at a rate of 100 kg/s. The mass m, acceleration a, and thrust T are related by the equation $T - 10m = ma$. The velocity v is related to the mass by the equation $m = 30\,000(2.72)^{-v-gt}$. Determine the functions $m(t)$, $a(t)$, and $v(t)$, in terms of the variable t (time measured in seconds) and the constants T and g. Use the fact that $a(0) > 0$ for the rocket to lift off, to determine the constraint on T.