Curious Math

Does 65 Equal 64?

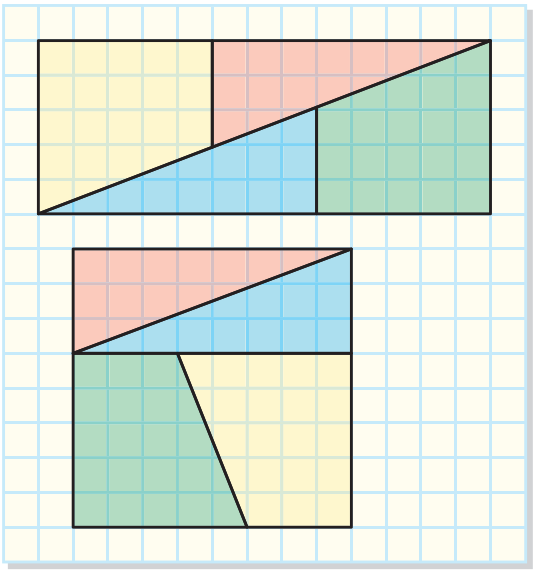
The steps at the right seem to prove that 65 equals 64.

1. Copy the steps. Explain how each step is obtained from the step above it.
2. Can you find any problems with any of the steps?

Let $a = 1$ and $b = 1$.
So $a = b$.

$$\begin{aligned} a \times a &= a \times b \\ a^2 &= ab \\ a^2 - b^2 &= ab - b^2 \\ (a + b)(a - b) &= b(a - b) \\ a + b &= b \\ 2 &= 1 \\ 2 + 63 &= 1 + 63 \\ 65 &= 64 \end{aligned}$$

The two diagrams below also seem to prove that 65 equals 64.



3. How do the colours make the rectangle and the square appear to have the same area?
4. Determine the area of each figure.
5. Use your answers for steps 3 and 4 to explain why these two figures appear to prove that 65 equals 64.
6. These two proofs are called fallacious proofs because they contain an error. How would mathematics and our daily lives be affected if either of these proofs were true?
7. Some fallacious proofs are very complex. Try to create or research another fallacious proof that you can explain to a classmate.