6.2 Exploring the Creation of Perfect Squares

YOU WILL NEED

- algebra tiles

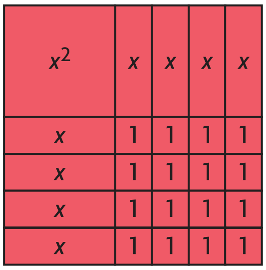
GOAL

Recognize the relationship between the coefficients and constants of perfect-square trinomials.

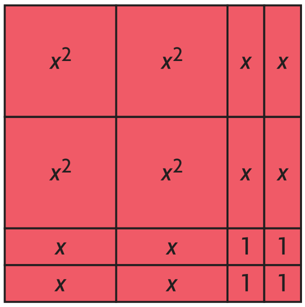
EXPLORE the Math

Quadratic expressions like $x^2 + 8x + 16$ and $4x^2 + 8x + 4$ are perfect-square trinomials. Quadratic expressions like $4x^2 + 8x + 3$ are not.

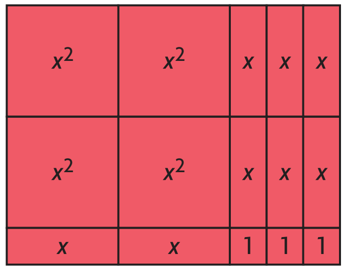
$x^2 + 8x + 16$



$4x^2 + 8x + 4$



$4x^2 + 8x + 3$



? How can you decide what value for *c* makes expressions of the form $ax^2 + abx + c$, $a \neq 0$, perfect-square trinomials?

A. Factor $x^2 + 8x + 16$ and $4x^2 + 8x + 4$ completely. Explain why these expressions are called perfect-square trinomials.

B. Using algebra tiles, create an arrangement that helps you determine the constant term c that must be added to create perfect-square trinomials. Verify by factoring each new trinomial you created.

i) $x^2 + 2x + c$	**iii)** $x^2 + 6x + c$	**v)** $x^2 + 10x + c$
ii) $x^2 + 4x + c$	**iv)** $x^2 + 8x + c$	**vi)** $x^2 + 12x + c$

C. For each trinomial you created in part B, compare the coefficient of x and the constant term you added. Explain how these numbers are related.

D. How are the expressions below different from those in part B?

i) $x^2 - 4x + c$	**iii)** $x^2 - 6x + c$	**v)** $x^2 - 2x + c$
ii) $x^2 - 8x + c$	**iv)** $x^2 - 12x + c$	**vi)** $x^2 - 10x + c$

E. Using algebra tiles or an area diagram, determine the constant term c that must be added to each of the expressions in part D to create perfect-square trinomials. Verify by factoring each new trinomial.

F. For each trinomial you created for part E, compare the coefficient of x and the constant term you added. Does the relationship you discovered in part C still apply?

G. Each expression below contains a common factor. Factor the expression and then determine the constant term c that must be added to each expression to make it a multiple of a perfect-square trinomial. Verify by factoring each new trinomial.

i) $2x^2 + 4x + c$ **iii)** $3x^2 - 6x + c$ **v)** $5x^2 + 25x + c$
ii) $3x^2 - 12x + c$ **iv)** $-x^2 + 4x + c$ **vi)** $6x^2 + 54x + c$

Reflecting

H. How can you predict the value of c that will make $x^2 + bx + c$ a perfect-square trinomial?

I. How can you predict the value of c that will make $ax^2 + abx + c$ a perfect-square trinomial?

In Summary

Key Idea

- If $(x + b)^2 = x^2 + 2bx + b^2$, then $\left(x + \frac{b}{2}\right)^2 = x^2 + bx + \left(\frac{b}{2}\right)^2$. So, in all perfect-square trinomials, the constant term is half the coefficient of the x term squared.

Need to Know

- To create a perfect square that includes $x^2 + bx$ and no other terms with a variable, add $\left(\frac{b}{2}\right)^2$.
- To create a perfect square that includes $ax^2 + abx$ and no other terms with a variable, factor out a and then create a perfect square that includes $x^2 + bx$. This results in adding $a\left(\frac{b}{2}\right)^2$.

FURTHER *Your Understanding*

1. Determine the value of c that will create a perfect-square trinomial. Verify by factoring the trinomial you created.

a) $x^2 + 8x + c$ **c)** $x^2 + 40x + c$ **e)** $x^2 - 5x + c$
b) $x^2 - 14x + c$ **d)** $x^2 + 20x + c$ **f)** $x^2 + x + c$

2. Each expression is a multiple of a perfect-square trinomial. Determine the value of c.

a) $3x^2 + 30x + c$ **c)** $-4x^2 - 8x + c$ **e)** $5x^2 - 10x + c$
b) $2x^2 - 12x + c$ **d)** $6x^2 - 60x + c$ **f)** $7x^2 + 42x + c$