6 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How are the roots of a quadratic equation and the zeros of a quadratic relation related?

A: The roots of the equation $ax^2 + bx + c = 0$ are the zeros, or x-intercepts, of the relation $y = ax^2 + bx + c$.

Q: What strategies can you use to solve a quadratic equation?

A1: If the equation is in the form $ax^2 + bx + c = 0$, you can graph the relation $y = ax^2 + bx + c$ and locate the zeros on your graph. If the trinomial is factorable, you can factor it, set each factor equal to zero, and solve the equations.

Study *Aid*

- See Lesson 6.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 to 5.

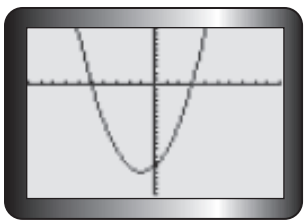
EXAMPLE

Solve $x^2 + 2x - 15 = 0$.

Solution

By Graphing Technology

Graph using a scale $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



$x = -5$ and $x = 3$

or

By Factoring

$x^2 + 2x - 15 = 0$
$(x - 3)(x + 5) = 0$
$x - 3 = 0$ or $x + 5 = 0$
$x = 3$ $x = -5$

$x = 3$ and $x = -5$

A2: If the equation is in the form $ax^2 + bx + c = d$, you can graph $y = ax^2 + bx + c$ and $y = d$ and determine the points of intersection. Alternatively, you can rearrange the equation so that one side is equal to zero. Then you can graph or factor the resulting equation to solve it.

EXAMPLE

Solve $x^2 + 6x + 5 = -3$.

Solution

$y = x^2 + 6x + 5$ or $x^2 + 6x + 5 = -3$ or $x^2 + 6x + 5 = -3$

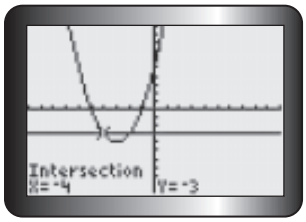
$y = -3$ $x^2 + 6x + 8 = 0$ $x^2 + 6x + 8 = 0$

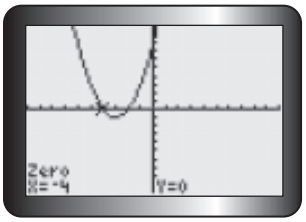
$(x + 4)(x + 2) = 0$

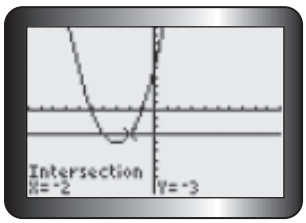
$x + 4 = 0$ or $x + 2 = 0$

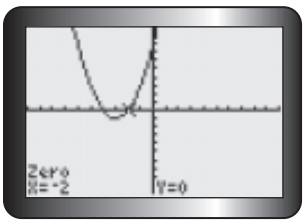
$x = -4$ $x = -2$

$x = -4$ and $x = -2$









$x = -4$ and $x = -2$

Study Aid

- See Lesson 6.3, Examples 1 to 4.
- Try Mid-Chapter Review Questions 7 to 10.

Q: How can you change a quadratic relation from standard form to vertex form?

A: To write a quadratic relation in vertex from, complete the square as shown below.

EXAMPLE

Write the equation in vertex form.

$y = 2x^2 - 9x + 2.5$

Solution

- When the coefficient of x^2 is a number other than 1, factor it from the x^2 and x terms. This will leave a binomial inside the brackets.
- To complete the square for the binomial, add and subtract the square of half the coefficient of the x term.
- Group together the three terms that form the perfect square, and factor it.
- Use the distributive property to multiply. Then combine the constants.

$y = 2x^2 - 9x + 2.5$

$y = 2(x^2 - 4.5x) + 2.5$

$y = 2(x^2 - 4.5x + 2.25^2 - 2.25^2) + 2.5$

$y = 2(x^2 - 4.5x + 5.0625 - 5.0625) + 2.5$

$y = 2[(x^2 - 4.5x + 5.0625) - 5.0625] + 2.5$

$y = 2[(x - 2.25)^2 - 5.0625] + 2.5$

$y = 2[(x - 2.25)^2 - (5.0625)] + 2.5$

$y = 2(x - 2.25)^2 - 2(5.0625) + 2.5$

$y = 2(x - 2.25)^2 - 10.125 + 2.5$

$y = 2(x - 2.25)^2 - 7.625$

PRACTICE Questions

Lesson 6.1

1. Solve each quadratic equation.

a) $x^2 + 4x = 12$

b) $x^2 + 8x + 9 = 0$

c) $x^2 - 9x = -4$

d) $-3x^2 - 2x + 3 = 0$

e) $2x^2 - 5x + 10 = 15$

f) $\frac{1}{2}x^2 + 10x - 2 = -10$

2. Determine the roots.

a) $x^2 + 6x - 16 = 0$

b) $2x^2 + x - 3 = 0$

c) $x^2 + 3x - 10 = 0$

d) $6x^2 + 7x - 5 = 0$

e) $-3x^2 - 9x + 12 = 0$

f) $\frac{1}{2}x^2 + 6x + 16 = 0$

3. Solve using any strategy.

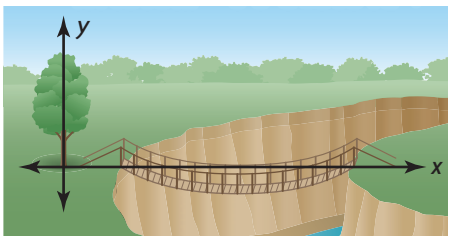
a) $x^2 + 12x + 45 = 10$

b) $2x^2 + 7x + 5 = 9$

c) $x(6x - 1) = 12$

d) $x(x + 3) - 20 = 5(x + 3)$

4. Kari drew this sketch of a small suspension bridge over a gorge near her home.



She determined that the bridge can be modelled by the relation $y = 0.1x^2 - 1.2x + 2$. How wide is the gorge, if 1 unit on her graph represents 1 m?

5. If a ball were thrown on Mars, its height, h, in metres, might be modelled by the relation $h = -1.9t^2 + 18t + 1$, where t is the time in seconds since the ball was thrown.

a) Determine when the ball would be 20 m or higher above Mars' surface.

b) Determine when the ball would hit the surface.

Lesson 6.2

6. Determine the value of c needed to create a perfect-square trinomial.

a) $x^2 + 8x + c$

b) $x^2 - 10x + c$

c) $x^2 + 5x + c$

d) $x^2 - 7x + c$

e) $-4x^2 + 24x + c$

f) $2x^2 - 18x + c$

Lesson 6.3

7. Write each relation in vertex form by completing the square.

a) $y = x^2 + 6x - 3$

b) $y = x^2 - 4x + 5$

c) $y = 2x^2 + 16x + 30$

d) $y = -3x^2 - 18x - 17$

e) $y = 2x^2 + 10x + 8$

f) $y = -3x^2 + 9x - 2$

8. Consider the relation $y = -4x^2 + 40x - 91$.

a) Complete the square to write the equation in vertex form.

b) Determine the vertex and the equation of the axis of symmetry.

c) Graph the relation.

9. Martha bakes and sells her own organic dog treats for \$15/kg. For every \$1 price increase, she will lose sales. Her revenue, R, in dollars, can be modelled by $R = -10x^2 + 100x + 3750$, where x is the number of \$1 increases. What selling price will maximize her revenue?

10. For his costume party, Byron hung a spider from a spring that was attached to the ceiling at one end. Fern hit the spider so that it began to bounce up and down. The height of the spider above the ground, h, in centimetres, during one bounce can be modelled by $h = 10t^2 - 40t + 240$, where t seconds is the time since the spider was hit. When was the spider closest to the ground during this bounce?