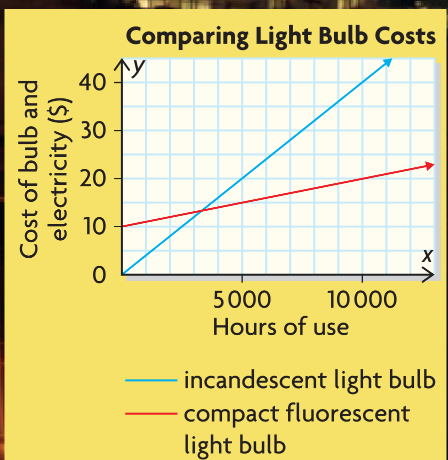
Chapter 1

Systems of Linear Equations

GOALS

You will be able to

- Solve a system of linear equations using a variety of strategies
- Solve problems that are modelled by linear equations or systems of linear equations
- Describe the relationship between the number of solutions to a system of linear equations and the coefficients of the equations



? Why does it make sense to buy energy-efficient compact fluorescent light bulbs, even though they often cost more than incandescent light bulbs?

1 Getting Started

WORDS YOU NEED to Know

1. Complete each sentence using one or more of the given words. Each word can be used only once.

i) *x*-intercept
ii) *y*-intercept
iii) equation
iv) variable
v) coefficient
vi) point of intersection
vii) solution

a) The place where a graph crosses the *x*-axis is called the ______.
b) In the ______ $y = 5x + 2$, 5 is a ______ of the ______ x.
c) Let $x = 0$ to determine the ______ of $y = 4x - 7$.
d) You can determine the ______ to $20 = 3x - 10$ by graphing $y = 3x - 10$.
e) The ordered pair at which two lines cross is called the ______.

SKILLS AND CONCEPTS You Need

Graphing a Linear Relation

Study Aid

- For more help and practice, see Appendix A-6 and A-7.

You can use different tools and strategies to graph a linear relation:

- a table of values
- the slope and *y*-intercept
- the *x*- and *y*-intercepts
- a graphing calculator

EXAMPLE

Graph $3x + 2y = 9$.

Solution

Using the *x*- and *y*-intercepts

Let $y = 0$ to determine the *x*-intercept.

$$3x + 2(0) = 9$$
$$3x = 9$$
$$x = 3$$

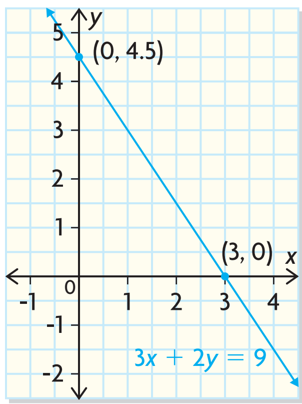
The graph passes through (3, 0).

Let $x = 0$ to determine the *y*-intercept.

$$3(0) + 2y = 9$$
$$2y = 9$$
$$y = 4.5$$

The graph passes through (0, 4.5).

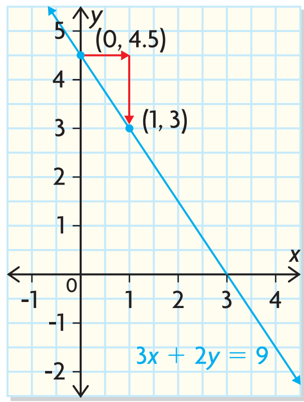
Plot the intercepts, and join them with a straight line.



Using the Slope and y-intercept

$$3x + 2y = 9$$
$$2y = -3x + 9$$
$$\frac{2y}{2} = \frac{-3x}{2} + \frac{9}{2}$$
$$y = -1.5x + 4.5$$

The slope is -1.5. The y-intercept is 4.5, so the line passes through (0, 4.5). Plot (0, 4.5). Use the rise and run to locate a second point on the line, by going right 1 unit and down 1.5 units to (1, 3).



2. Graph each relation using the slope and y-intercept.
a) $y = 4x - 7$ **b)** $x + 2y = 3$

3. Graph each relation using the x- and y-intercepts.
a) $4x - 5y = 10$ **b)** $y = 2 - 3x$

4. Graph each relation using the strategy of your choice.
a) $x - 3y = 6$ **b)** $y = 5 - 2x$

Expanding and Simplifying an Algebraic Expression

You can use an algebra tile model to visualize and simplify an expression. If the expression has brackets, you can use the distributive property to expand it. You can add or subtract like terms.

Study *Aid*

- For more help and practice, see Appendix A-8.

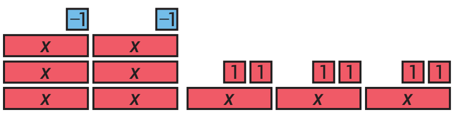
EXAMPLE

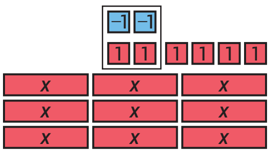
Expand and simplify $2(3x - 1) + 3(x + 2)$.

Solution

Using an Algebra Tile Model

$2(3x - 1) + 3(x + 2)$





$= 9x + 4$

Using Symbols

$$2(3x - 1) + 3(x + 2)$$
$$= 6x - 2 + 3x + 6$$
$$= 6x + 3x - 2 + 6$$
$$= 9x + 4$$

5. Expand and simplify as necessary.
a) $5x + 10 - 3x + 12$
b) $4(3x + 5)$
c) $-2(5x - 2)$
d) $(3x - 6) + (2x + 7)$
e) $6(2x - 4) - 3(2x - 1)$
f) $(8x - 14) + (7x + 6)$

Study Aid

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
6, 7	A-7
10	A-10
11, 12	A-9

PRACTICE

6. Rearrange each equation to complete the table.

	$Ax + By + C = 0$ Form	$y = mx + b$ Form
a)	$3x + 4y - 6 = 0$	
b)		$y = 2x - 5$
c)	$4x - 7y - 3 = 0$	
d)		$y = -\frac{2}{3}x - \frac{5}{6}$

7. State the slope and y-intercept of each relation. Then sketch the graph.

a) $y = 3x - 5$

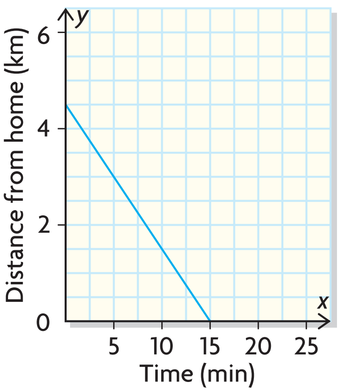
b) $y = -\frac{2}{3}x + 1$

c) $y = 0.5x$

d) $y = 2.6x - 1.2$

8. Which relations in question 7 are **direct variations**? Which are **partial variations**? Explain how you know.

Kyle's Journey Home from School



9. The graph at the left shows Kyle's distance from home as he cycles home from school.

a) How far is the school from Kyle's home?

b) At what speed does Kyle cycle?

10. State whether each relation is linear or nonlinear. Explain how you know.

a) $y = 3x - 6$

b)

x	1	2	3	4	5	6
y	7	9	11	13	15	17

c) $y = 5x^2 + 6x - 4$

d)

x	1	2	3	4	5	6
y	−3	0	5	12	21	32

11. Solve.

a) $x + 5 = 12$

b) $13 = 9 - x$

c) $2x = 18$

d) $-3x = -21$

e) $2x - 5 = 15$

f) $4x - 6 = 8x + 2$

12. a) If $3x - 2y = 14$ and $x = 1.5$, determine the value of y.

b) If $0.36x + 0.54y = 1.1$ and $y = 0.7$, determine the value of x.

13. a) Make a concept map that shows different strategies you could use to graph $2x - 4y = 8$.

b) Which strategy would you use? Explain why.

APPLYING What You Know

Making Change

Barb is withdrawing $100 from her bank account. She asks for the money in $5 bills and $10 bills.

YOU WILL NEED

- grid paper
- ruler

? Which combinations of $5 bills and $10 bills equal $100?

A. If the teller gives Barb four $10 bills, how many $5 bills does he give her?

B. List four more combinations of $100. Record the combinations in a table.

Number of $5 Bills	Number of $10 Bills
	4

C. Let x represent the number of $5 bills, and let y represent the number of $10 bills. Write an equation for combinations of these bills with a total value of $100.

D. Graph your equation for part C. Should you use a solid or broken line? Explain.

E. Describe how the number of $10 bills changes as the number of $5 bills increases.

F. Explain what the x-intercept and y-intercept represent on your graph.

G. Which points on your graph are not possible combinations? Explain why.

H. Determine all the possible combinations of $5 bills and $10 bills that equal $100.

1.1 Representing Linear Relations

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

GOAL

Use tables, graphs, and equations to represent linear relations.

LEARN ABOUT the Math

Aiko's cell-phone plan is shown here. Aiko has a budget of $30 each month for her cell phone.

Services	Cost
calls	20¢/min
text messages	15¢/message

? How can Aiko show how many messages and calls she can make each month for $30?

EXAMPLE 1 **Representing** a linear relation

Show the combinations of messages and calls that are possible each month for $30.

Aiko's Solution: Using a table

Text Messages		Calls		
Number of Messages	**Cost ($)**	**Number of Minutes**	**Cost ($)**	**Total Cost ($)**
0	0	150	30	30
20	3	135	27	30
40	6	120	24	30
:		:		:
200	30	0	0	30

I made a table to show how many messages and calls are possible for $30. I started with 0 messages and let the number of messages increase by 20 each time. I calculated the cost of the messages by multiplying the number in the first column by $0.15. Then I subtracted the cost of the messages from $30 to determine the amount of money that was left for calls. I calculated the number of minutes for calls by dividing this amount by $0.20.

As the number of text messages increases, the number of minutes available for calls decreases. Aiko can make choices based on the numbers in the table. For example, if Aiko sends 40 text messages, she can talk for 120 min.

40 text messages a month is about 1 per day. 120 min a month for calls is about 4 min per day.

Malcolm's Solution: Using an equation and a graph

Let x represent the number of text messages per month. Let y represent the number of minutes of calls per month.

I used letters for the variables.

Aiko has a budget of \$30 for text messages and calls, so $0.15x + 0.20y = 30$.

I wrote an equation based on Aiko's budget of \$30. In my equation, x text messages cost \$0.15x and y minutes of calls cost \$0.20y.

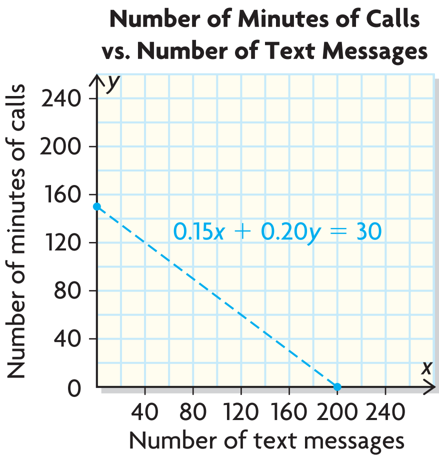
At the x-intercept, $y = 0$.

$$0.15x + 0.20(0) = 30$$
$$x = \frac{30}{0.15}$$
$$x = 200$$

At the y-intercept, $x = 0$.

$$0.15(0) + 0.20y = 30$$
$$y = \frac{30}{0.20}$$
$$y = 150$$

I used my equation to calculate the maximum number of text messages and the maximum time for calls. To do this, I determined the intercepts.



I drew a graph by plotting the x-intercept and y-intercept, and joining them.

I used a broken line because x represents whole numbers only in this equation.

The point (40, 120) shows that if Aiko sends 40 text messages in a month, she has a maximum of 120 min for calls to stay within her budget.

Aiko's options for text messages and calls are displayed as points on the graph. Each point on the graph represents an ordered pair (x, y), where x is the number of text messages per month and y is the number of minutes of calls per month.

Reflecting

A. How does the table show that the relationship between the number of text messages and the number of minutes of calls is linear?

B. How did Malcolm use his equation to draw a graph of Aiko's choices?

C. Which representation do you think Aiko would find more useful: the table or the graph? Why?

APPLY the Math

EXAMPLE 2 | **Representing** a linear relation using graphing technology

Career Connection

Careers as diverse as sales consultants, software developers, and financial analysts have roles in currency exchange.

Patrick has saved \$600 to buy British pounds and euros for a school trip to Europe. On the day that he goes to buy the currency, one pound costs \$2 and one euro costs \$1.50.

a) Create a table, an equation, and a graph to show how many pounds and euros Patrick can buy.

b) Explain why the relationship between pounds and euros is linear.

c) Describe how Patrick can use each representation to decide how much of each currency he can buy.

Brittany's Solution

a) Let x represent the pounds that Patrick buys. Let y represent the euros that he buys.

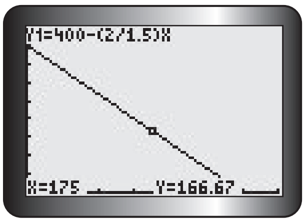
I chose letters for the variables. x pounds cost \$$2x$ and y euros cost \$$1.50y$. Patrick has \$600.

$$2x + 1.50y = 600$$
$$1.50y = 600 - 2x$$
$$\frac{1.50y}{1.50} = \frac{600}{1.50} - \frac{2x}{1.50}$$
$$y = 400 - \left(\frac{2}{1.50}\right)x$$

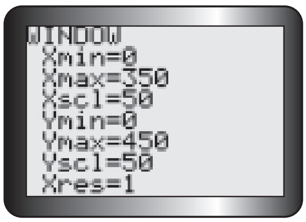
I wrote an equation based on the cost of the currency. I rearranged my equation into the form $y = mx + b$ so I could enter it into a graphing calculator.

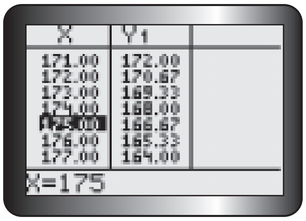
Tech Support

For help using a TI-83/84 graphing calculator to enter then graph relations and use the Table Feature, see Appendix B-1, B-2, and B-6. If you are using a TI-*n*spire, see Appendix B-37, B-38, and B-42.



I graphed the equation using these window settings because I knew that the y-intercept would be at 400 and the x-intercept would be at 300.

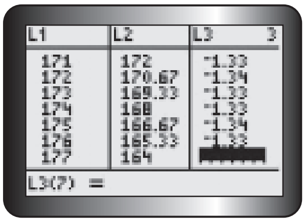




I set the decimal setting to two decimal places because *x* and *y* represent money. Then I created a table of values.

b) Since the **degree** of the equation is one and the graph is a straight line, the relationship is linear. The first differences in the table are constant.

In the table, each increase of 1 in the *x*-values results in a decrease of about 1.33 in the *y*-values.



c) By tracing up and down the line, or by scrolling up and down the table, Patrick can see the combinations of pounds and euros. He can use the equation, in either form, to calculate specific numbers of pounds or euros.

EXAMPLE 3 Selecting a representation for a linear relation

Judy is considering two sales positions. Sam's store offers $1600/month plus 2.5% commission on sales. Carol's store offers $1000/month plus 5% commission on sales. In the past, Judy has had about $15 000 in sales each month.

a) Represent Sam's offer so that Judy can check what her monthly pay would be.

b) Represent the two offers so that Judy can compare them. Which offer pays more?

Justine's Solution

a) Let *x* represent her sales in dollars. Let *y* represent her earnings in dollars.
An equation will help Judy check her pay.

$$y = 1600 + 0.025x$$

I chose letters for the variables. I wrote an equation to describe what Judy's monthly pay would be. Her base salary is $1600. Her earnings for her monthly sales would be $0.025*x*, since $2.5\% = \frac{2.5}{100}$ or 0.025.

Tech Support

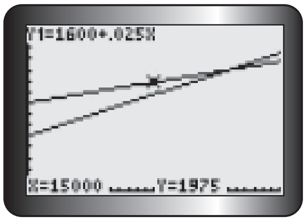
For help creating a difference table with a TI-83/84 graphing calculator, see Appendix B-7. If you are using a TI-*n*spire, see Appendix B-43.

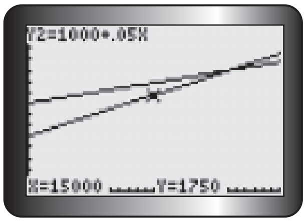
Tech Support

For help changing the window settings and tracing on a graph using a TI-83/84 graphing calculator, see Appendix B-4 and B-2. If you are using a TI-*n*spire, see Appendix B-40 and B-38.

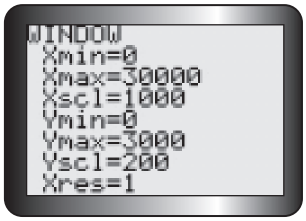
b) The equation for Carol's offer is $y = 1000 + 0.05x$. Judy can use a graph to compare her pay for a typical month.

I wrote an equation for Carol's offer and graphed both relations using a graphing calculator.





I adjusted the settings, as shown, so I could see the point where the graphs crossed.



I used Trace to compare the two offers.

Sam's offer pays more.

In Summary

Key Idea

- Three useful ways to represent a linear relation are
  - a table of values
  - a graph
  - an equation

Need to Know

- A linear relation has the following characteristics:
  - The first differences in a table of values are constant.
  - The graph is a straight line.
  - The equation has a degree of 1.
- The equation of a linear relation can be written in a variety of equivalent forms, such as
  - standard form: $Ax + By + C = 0$
  - slope y-intercept form: $y = mx + b$
- A graph and a table of values display some of the ordered pairs for a relation. You can use the equation of a relation to calculate ordered pairs.

CHECK Your Understanding

1. Which of these ordered pairs are not points on the graph of $2x + 4y = 20$? Justify your decision.

a) (10, 0) **b)** (−3, 7) **c)** (6, 2) **d)** (0, 5) **e)** (12, −1)

2. Jacob has \$15 to buy muffins and doughnuts at the school bake sale, as a treat for the Camera Club. Muffins are 75¢ each and doughnuts are 25¢ each. How many muffins and doughnuts can he buy?
   a) Create a table to show the possible combinations of muffins and doughnuts.
   b) What is the maximum number of muffins that Jacob can buy?
   c) What is the maximum number of doughnuts that he can buy?
   d) Write an equation that describes Jacob's options.
   e) Graph the possible combinations.

3. Refer to question 2. Which representation do you think is more useful for Jacob? Justify your choice.

PRACTISING

4. State two ordered pairs that satisfy each linear relation and one ordered
K pair that does not.
   a) $y = 5x - 1$
   b) $3x - 4y = 24$
   c) $y = -25x + 10$
   d) $5x = 30 - 2y$

5. Define suitable variables for each situation, and write an equation.
   a) Caroline has a day job and an evening job. She works a total of 40 h/week.
   b) Caroline earns \$15/h at her day job and \$11/h at her evening job. Last week, she earned \$540.
   c) Justin earns \$500/week plus 6% commission selling cars.
   d) Justin is offered a new job that would pay \$800/week plus 4% commission.
   e) A piggy bank contains \$5.25 in nickels and dimes.

6. Graph the relations in question 5, parts a) and b).

7. Refer to question 5, parts c) and d). Justin usually has about \$18 000 in weekly sales. Should he take the new job? Justify your decision.

8. Deb pays 10¢/min for cell-phone calls and 6¢/text message. She has a budget of \$25/month for both calls and text messages.
   a) Create a table to show the ways that Deb can spend up to \$25 each month on calls and text messages.
   b) Create a graph to show the information in the table.

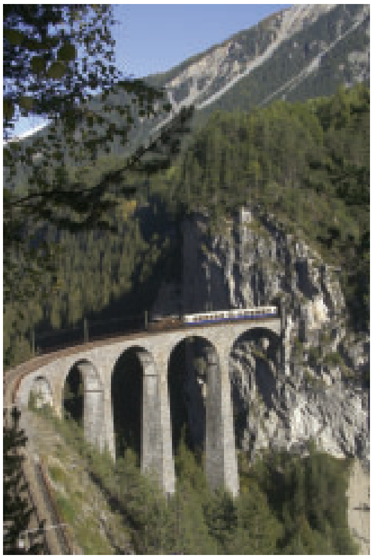
9. Leah earns \$1200/month plus 3.5% commission.
C **a)** Create an equation that she can use to check her paycheque each month.
   b) Last month, Leah had \$96 174 in sales. Her pay before deductions was \$4566.09. Is this amount correct? Explain your answer.

10. Ben's Bikes rents racing bikes for \$25/day and mountain bikes for \$30/day. Yesterday's rental charges were \$3450.

a) Determine the greatest number of racing bikes that could have been rented.

b) Determine the greatest number of mountain bikes that could have been rented.

c) Write an equation and draw a graph to show the possible combinations of racing and mountain bikes rented yesterday.



11. Abigail is planning to fly to Paris and then travel through Switzerland and Austria to Italy by train. On the day that she goes to buy the foreign currencies she needs, one euro costs \$1.40 and one Swiss franc costs \$0.90. What combinations of these currencies can Abigail buy for \$630? Use two different strategies to show the possible combinations.

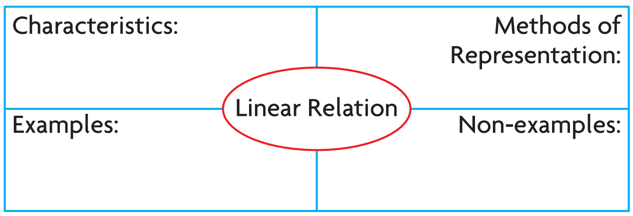
12. A student council invested some of the money from a fundraiser in a savings account that pays 3%/year and the rest of the money in a government bond that pays 4%/year. The investments earned \$150 in the first year.

a) Define two variables for the information, and write an equation.

b) Graph the information.

13. Maureen pays a \$350 registration fee and an \$85 monthly fee to belong to a fitness club. Lia's club has a higher registration fee but a lower monthly fee. After five months, both Maureen and Lia have paid \$775. Determine the possible fees at Lia's club.

14. a) Use the chart to show what you know about linear relations.



b) List the advantages and disadvantages of each of the three ways to represent a linear relation. Describe situations in which each representation might be preferred.

Extending

15. Create a situation that can be represented by each equation.

a) $0.10x + 0.25y = 4.65$ **b)** $y = 900 + 0.025x$

16. Allan plans to create a new coffee blend using Brazilian beans that cost \$12/kg and Ethiopian beans that cost \$17/kg. He is going to make 150 kg of the blend and sell it for \$14/kg. Write and graph two equations for this situation.

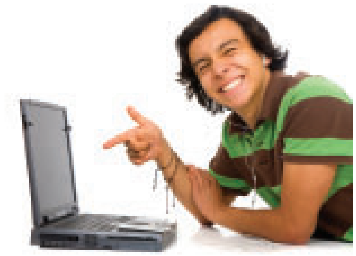
1.2 Solving Linear Equations

GOAL

Connect the solution to a linear equation and the graph of the corresponding relation.

YOU WILL NEED

- grid paper
- ruler
- graphing calculator



LEARN ABOUT the Math

Joe downloads music to his MP3 player from a site that charges \$9.95 per month plus \$0.55 for each song. Joe has budgeted \$40 per month to spend on music downloads.

? How can Joe determine the greatest number of songs that he can download each month?

EXAMPLE 1 **Selecting a strategy** to solve the problem

Determine the maximum number of songs that Joe can download each month.

William's Solution: Solving a problem by reasoning

$\$40.00 - \$9.95 = \$30.05$

I calculated how much of Joe's budget he can spend on the songs he downloads, by subtracting the \$9.95 monthly fee from \$40.

$\$30.05 \div \$0.55 \doteq 54.63$

Each song costs \$0.55, so I divided this into the amount he would have left to spend on songs.

Joe can download a maximum of 54 songs.

I rounded down to 54, since 55 songs would cost more than he can spend.

Tony's Solution: Solving a problem by using an equation

Let n represent the number of songs and let C represent the cost.

$C = 9.95 + 0.55n$
$40 = 9.95 + 0.55n$

I created an equation and substituted the \$40 Joe has budgeted for C.

$$40 - 9.95 = 9.95 + 0.55n - 9.95$$
$$30.05 = 0.55n$$
$$\frac{30.05}{0.55} = n$$
$$54.6 \doteq n$$

I solved for *n* using inverse operations.

Joe can download a maximum of 54 songs.

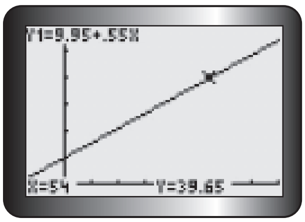
Since *n* has to be a whole number, I used the nearest whole number less than 54.6 for my answer.

Lucy's Solution: Solving a problem using graphing technology

Let X represent the number of songs and Y1 the cost.

Y1 = 9.95 + 0.55X

I entered the equation for the cost of music downloads into a graphing calculator. The number of songs downloaded has to be a whole number, so X represents a whole number. I graphed using Zoom Integer, so the *x*-values would go up by 1 when I traced the graph.



I used Trace to determine which point on the graph is closest to $y = 40$ (but less than \$40). This point is (54, 39.65).

Joe can download 54 songs in a month for \$39.65.

Tech Support

For help graphing and tracing along relations using a TI-83/84 graphing calculator, see Appendix B-2. If you are using a TI-*n*spire, see Appendix B-38.

Reflecting

A. How are William's and Tony's solutions similar? How are they different?

B. How did a single point on Lucy's graph represent a solution to the problem?

C. Which strategy do you prefer? Explain why.

APPLY the Math

EXAMPLE 2 Representing and solving a problem that involves a linear equation

At 9:20 a.m., Adrian left Windsor with 64 L of gas in his car. He drove east at 100 km/h. The low fuel warning light came on when 10 L of gas were left. Adrian's car uses gas at the rate of 8.8 L/100 km. When did the warning light come on?

Stefani's Solution: Solving an equation algebraically

Adrian's car uses 8.8 L of gas every 100 km. Since he drove at 100 km/h, he used 8.8 L/h.

I calculated how much gas the car used each hour.

$$G = 64 - 8.8t$$

I wrote an equation for the amount of gas used. I let t represent the time in hours, and I let G represent the amount of gas in litres.

$$\begin{aligned} 10 &= 64 - 8.8t \\ 10 - 10 &= 64 - 8.8t - 10 \\ 0 &= 54 - 8.8t \\ 8.8t &= 54 \\ t &= \frac{54}{8.8} \\ t &\doteq 6.14 \end{aligned}$$

The warning light came on when $G = 10$, so I let $G = 10$ and solved for t using inverse operations.

The warning light came on after Adrian had been driving about 6.14 h.

$$0.14 \times 60 = 8.4$$

I wrote the time in hours and minutes by multiplying the part of the number to the right of the decimal point by 60.

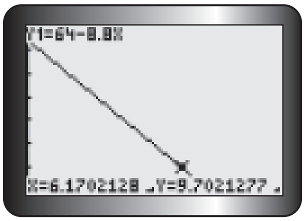
The warning light came on about 6 h 8 min after 9:20 a.m., which is about 3:28 p.m.

Henri's Solution: Solving a problem by using a graph

$$y = 64 - 8.8x$$

I wrote an equation for the amount of gas in the tank at any time. I let x represent the time in hours, and I let y represent the amount of gas in litres.

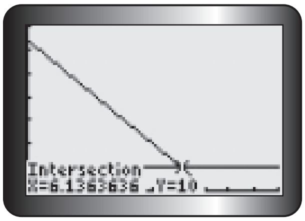
Graph $Y1 = 64 - 8.8X$.



I graphed the equation on a graphing calculator. I knew that the *y*-intercept was 64, and I estimated that the *x*-intercept was about 7, so I used the window settings shown.

I used Trace to locate the point with a *y*-value closest to 10.

After about 6.17 h, there was about 9.7 L of gas in the tank.



To get an exact solution, I entered the line $Y2 = 10$. The *x*-coordinate of the **point of intersection** between the two lines tells the time when 10 L of gas is left in the tank.

Based on the graph, the warning light came on about 6.14 h after Adrian started, at about 3:28 p.m.

Tech | ***Support***

For help determining the point of intersection between two relations on a TI-83/84 graphing calculator, see Appendix B-11. If you are using a TI-*n*spire, see Appendix B-47.

In Summary

Key Idea

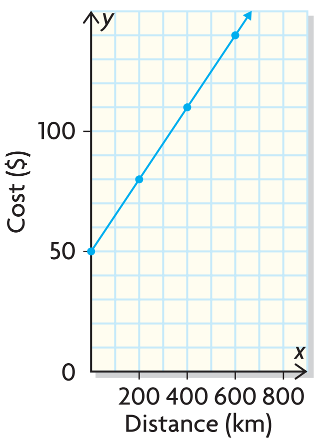
- You can solve a problem that involves a linear relation by solving the associated linear equation.

Need to Know

- You can solve a linear equation in one variable by graphing the associated linear relation and using the appropriate coordinate of an ordered pair on the line. For example, to solve $3x - 2 = 89$, graph $y = 3x - 2$ and look for the value of x at the point where $y = 89$ on the line.

CHECK Your Understanding

Cost of Car Rental vs. Distance



1. Estimate solutions to the following questions using the graph at the left.

a) What is the rental cost to drive 500 km?

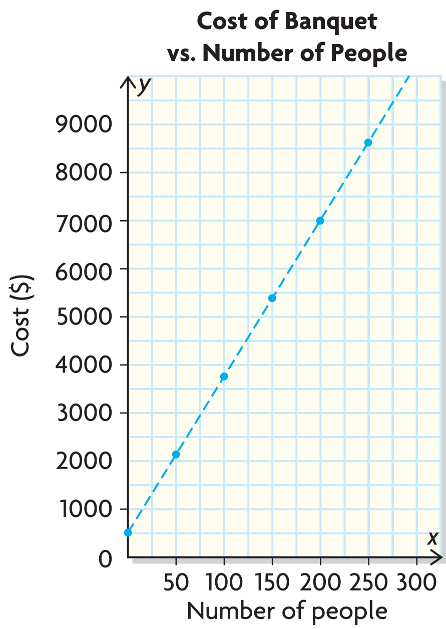
b) How far can you drive for \$80, \$100, and \$75?

2. a) Write an equation for the linear relation in question 1.
   b) Use your equation to answer question 1.
   c) Compare your answers for question 1 with your answers for part b) above. Which strategy gave the more accurate answers?

3. Apple juice is leaking from a carton at the rate of 5 mL/min. There are 1890 mL of juice in the container at 10:00 a.m.
   a) Write an equation for this situation, and draw a graph.
   b) When will 1 L of juice be left in the carton?

PRACTISING

4. The graph at the right shows how the charge for a banquet hall
K relates to the number of people attending a banquet.
   a) Locate the point (160, 5700) on the graph. What do these coordinates tell you about the charge for the banquet hall?
   b) What is the charge for the banquet hall if 200 people attend?
   c) Write an equation for this linear relation.
   d) Use your equation to determine how many people can attend for \$3100, \$4400, and \$5000.
   e) Why is a broken line used for this graph?



5. Max read on the Internet that 1 U.S. gallon is approximately equal to 3.785 L.
   a) Draw a graph that you can use to convert U.S. gallons into litres.
   b) Use your graph to estimate the number of litres in 6 gallons.
   c) Use your graph to estimate the number of gallons in 14 L.

6. Melanie drove at 100 km/h from Ajax to Ottawa. She left Ajax at 2:15 p.m., with 35 L of gas in the tank. The low fuel warning light came on when 9 L was left in the tank. If Melanie's SUV uses gas at the rate of 9.5 L/100 km, estimate when the warning light came on.

7. Hank sells furniture and earns \$280/week plus 4% commission.
   a) Determine the sales that Hank needs to make to meet his weekly budget requirement of \$900.
   b) Write an equation for this situation, and use it to verify your answer for part a).

8. The Perfect Paving Company charges \$10 per square foot to install
A interlocking paving stones, as well as a \$40 delivery fee.
   a) Determine the greatest area that Andrew can pave for \$3500.
   b) Andrew needs to include 5 cubic yards of sand, costing \$15 per cubic yard, to the total cost of the project. How much will this added cost reduce the area that he can pave with his \$3500 budget?

9. A student athletic council raised $4000 for new sports equipment and uniforms, which will be purchased 3 years from now. Until then, the money will be invested in a simple interest savings account that pays 3.5%/year.

a) Write an equation and draw a graph to represent the relationship between time (in years) and the total value of their investment.

b) Use the graph to determine the value of their investment after 2 years.

c) Use the equation to determine when their investment is worth $4385.

10. Maria has budgeted $90 to take her grandmother for a drive. Katey's Kars rents cars for $65 per day plus $0.12/km. Determine how far Maria and her grandmother can travel, including the return trip.

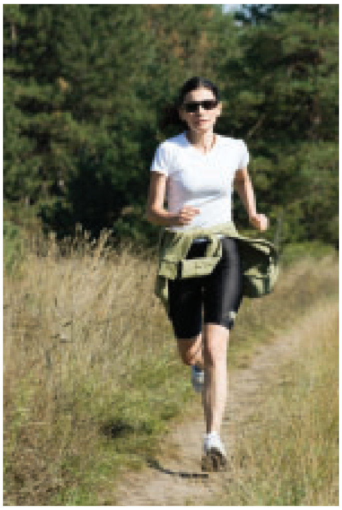
11. **C** Cam earns $400/week plus 2.5% commission. He has been offered another job that pays $700/week but no commission.

a) Describe three strategies that you could use to compare Cam's earnings for the two jobs.

b) Which job should Cam take? Justify your decision.

12. **T** At 9:00 a.m., Chantelle starts jogging north at 6 km/h from the south end of a 21 km trail. At the same time, Amit begins cycling south at 15 km/h from the north end of the same trail. Use a graph to determine when they will meet.

13. Explain how to determine the value of x, both graphically and algebraically, in the linear relation $2x - 3y = 6$ when $y = 5$.



Health *Connection*

Jogging is an exercise that keeps you healthy and can burn about 650 calories per hour.

Extending

14. The owner of a dart-throwing stand at a carnival pays 75¢ every time the bull's-eye is hit, but charges 25¢ every time it is missed. After 25 tries, Luke paid $5.25. How many times did he hit the bull's-eye?

15. Adriana earns 5% commission on her sales up to $25 000, 5.5% on any sales between $25 000 and $35 000, 6% on any sales between $35 000 and $45 000, and 7% for any sales over $45 000. Draw a graph to represent how Adriana's earnings depend on her sales. What sales volume does she need to earn $2000?

Number of Buttons	Cost per Button ($)
1 to 25	1.00
26 to 50	0.80
51 to 100	0.60
101 or more	0.20

16. A fabric store sells fancy buttons for the prices in the table at the left.

a) Make a table of values and draw a graph to show the cost of 0 to 125 buttons.

b) Compare the cost of 100 buttons with the cost of 101 buttons. What advice would you give someone who needed 100 buttons? Comment on this pricing structure.

c) Write equations to describe the relationship between the cost and the number of buttons purchased.

1.3 Graphically Solving Linear Systems

GOAL

Use graphs to solve a pair of linear equations simultaneously.

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

INVESTIGATE the Math

Matt's health-food store sells roasted almonds for \$15/kg and dried cranberries for \$10/kg.

? How can he mix the almonds and the cranberries to create 100 kg of a mixture that he can sell for \$12/kg?

A. Let x represent the mass of the almonds. Let y represent the mass of the cranberries.
  i) Write an equation for the total mass of the mixture.
  ii) Write an equation for the total value of the mixture.

B. Graph your equation of the total mass for part A. What do the points on the line represent?

C. Graph your equation of the total value for part A on the same axes. What do the points on this line represent?

D. Identify the coordinates of the point where the two lines intersect. State what each value represents. How accurately can you estimate these values from your graph?

E. The equations for part A form a **system of linear equations**. Explain why the coordinates for part D give the **solution to a system of linear equations**.

F. Substitute the coordinates into each equation to verify your solution.

system of linear equations

a set of two or more linear equations with two or more variables

For example, $x + y = 10$
$4x - 2y = 22$

solution to a system of linear equations

the values of the variables in the system that satisfy all the equations

For example, (7, 3) is the solution to
$x + y = 10$
$4x - 2y = 22$

Reflecting

G. Explain why you needed two linear relations to describe the problem.

H. Explain how graphing both relations on the same axes helped you solve the problem.

I. Explain why the coordinates of the point of intersection provide an ordered pair that satisfies both relations.

APPLY the Math

EXAMPLE 1 **Selecting a graphing strategy** to solve a linear system

Solve the system $y = 2x + 1$ and $x + 2y = -8$ using a graph.

Leslie's Solution

$y = 2x + 1$

The slope of the line is 2.

$\frac{2}{1} = \frac{\text{rise}}{\text{run}}$

At the y-intercept, $x = 0$.

$y = 2(0) + 1$

$y = 1$

I determined the slope and the y-intercept of the first equation.

$x + 2y = -8$

At the x-intercept, $y = 0$.

$x + 2(0) = -8$

$x = -8$

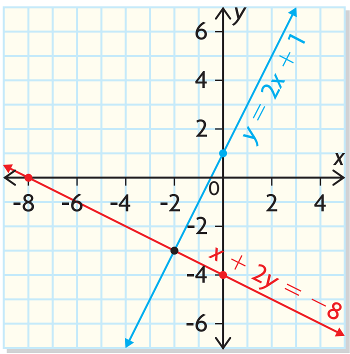
At the y-intercept, $x = 0$.

$0 + 2y = -8$

$\frac{2y}{2} = -\frac{8}{2}$

$y = -4$

I determined the x- and y-intercepts of the second equation.



I graphed the first line (blue) by plotting the y-intercept and using the rise and run to plot another point on the line.

I graphed the second line (red) by plotting points $(-8, 0)$ and $(0, -4)$ and joining them with a straight line.

At the point of intersection, $x = -2$ and $y = -3$.
The solution is $(-2, -3)$.

I located the point of intersection and read its coordinates using the axes of my graph.

$y = 2x + 1$

Left Side	Right Side
y	$2x + 1$
$= -3$	$= 2(-2) + 1$
	$= -3$

$x + 2y = -8$

Left Side	Right Side
$x + 2y$	-8
$= -2 + 2(-3)$	
$= -8$	

I checked my solution by substituting the x- and y-values into each equation.

EXAMPLE 2 Solving a problem using a graphing strategy

Ellen drives 450 km from her university in Kitchener-Waterloo to her home in Smiths Falls. She travels along one highway to Kingston at 100 km/h and then along another highway to Smiths Falls at 80 km/h. The journey takes her 4 h 45 min. What is the distance from Kingston to Smiths Falls?

Bob's Solution

Let x represent the distance that Ellen travels at 100 km/h. Let y represent the distance that she travels at 80 km/h.

I used letters to identify the variables in this situation.

The total trip is 450 km, so $x + y = 450$.

I wrote an equation for the total distance.

$$\frac{x}{100} + \frac{y}{80} = 4\frac{3}{4}$$

Since speed $= \frac{\text{distance}}{\text{time}}$, then time $= \frac{\text{distance}}{\text{speed}}$.
I wrote an equation to describe the total time (in hours) for her trip, where $\frac{x}{100}$ is the time spent driving at 100 km/h and $\frac{y}{80}$ is the time spent driving at 80 km/h.

$$x + y = 450$$

At the x-intercept, $y = 0$.

$$x + 0 = 450$$
$$x = 450$$

At the y-intercept, $x = 0$.

$$0 + y = 450$$
$$y = 450$$

I determined the x- and y-intercepts of the first equation.

$$\frac{x}{100} + \frac{y}{80} = 4\frac{3}{4}$$

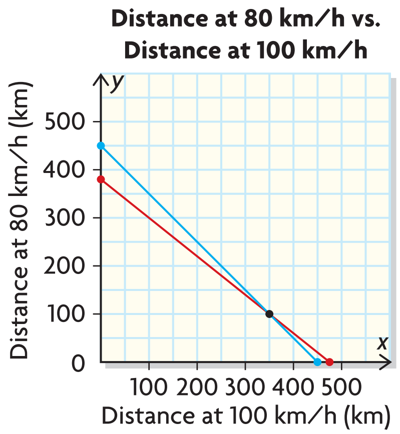
At the x-intercept, $y = 0$.

$$\frac{x}{100} + 0 = 4\frac{3}{4}$$
$$x = 100\left(4\frac{3}{4}\right)$$
$$x = 100\left(\frac{19}{4}\right)$$
$$x = 475$$

At the y-intercept, $x = 0$.

$$0 + \frac{y}{80} = 4\frac{3}{4}$$
$$y = 80\left(4\frac{3}{4}\right)$$
$$y = 80\left(\frac{19}{4}\right)$$
$$y = 380$$

I determined the x- and y-intercepts of the second equation.



I graphed each equation by plotting the *x*- and *y*-intercepts and joining them with a straight line.

The point of intersection is (350, 100), so the distance from Kingston to Smiths Falls is 100 km.

I determined the coordinates of the point of intersection. The *y*-coordinate of the point is the distance.

EXAMPLE 3 **Selecting graphing technology** to solve a system of linear equations

Hayley wants to rent a car for a weekend trip. Kelly's Kars charges $95 for the weekend plus $0.15/km. Rick's Rentals charges $50 for the weekend plus $0.26/km. Which company charges less?

Elly's Solution

Let x represent the distance driven in kilometres. Let y represent the total cost of the car rental in dollars.

I chose x and y for the variables.

The cost to rent a car from Kelly's Kars is $y = 0.15x + 95$.

I wrote an equation for the cost to rent a car from Kelly's Kars. 0.15x represents the distance charge and $95 represents the weekend fee.

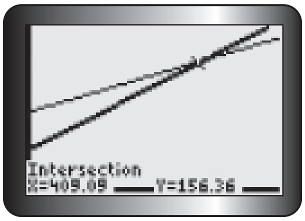
The cost to rent a car from Rick's Rentals is $y = 0.26x + 50$.

I wrote an equation for the cost to rent a car from Rick's Rentals. 0.26x represents the distance charge and $50 represents the weekend fee.

I used a graphing calculator. I entered the equation for Kelly's Kars in Y1 and the equation for Rick's Rentals in Y2. I used a thick line for Rick's Rentals to distinguish between the two lines.

I graphed the lines and adjusted the window settings so that I could see both lines and the point of intersection.

The point of intersection occurred when $0 \leq X \leq 600$ and $0 \leq Y \leq 200$. I used the Intersect operation to determine the point of intersection.



The point of intersection is (409, 156), to the nearest whole numbers.

I looked at my graph to determine which line is lower before and after the point of intersection.

Before the point of intersection, the thick line is lower, so Rick's Rentals charges less. After the point of intersection, the thin line is lower, so Kelly's Kars charges less.

If Hayley drives 409 km, both companies charge about $156.

If she plans to drive less than 409 km, Rick's Rentals charges less.

If she plans to drive more than 409 km, Kelly's Kars charges less.

Tech *Support*

To graph with a thick line using a TI-83/84 graphing calculator, scroll to the left of Y2 and press ENTER.

Tech *Support*

For help using a TI-83/84 graphing calculator to determine the point of intersection, see Appendix B-11. If you are using a TI-*n*spire, see Appendix B-47.

In Summary

Key Idea

- You can solve a system of linear equations by graphing both equations on the same axes. The ordered pair (x, y) at the point of intersection gives the solution to the system.

Need to Know

- You may not be able to determine an accurate solution to a system of equations using a hand-drawn graph.
- To determine an accurate solution to a system of equations, you can use graphing technology. When you use a graphing calculator, express the equations in slope y-intercept form.

CHECK Your Understanding

1. Decide whether each ordered pair is a solution to the given system of equations.

a) $(2, -1)$; $3x + 2y = 4$ and $-x + 3y = -5$

b) $(1, 4)$; $x + y = 5$ and $2x + 2y = 8$

c) $(1, -2)$; $y = 3x - 5$ and $y = 2x - 4$

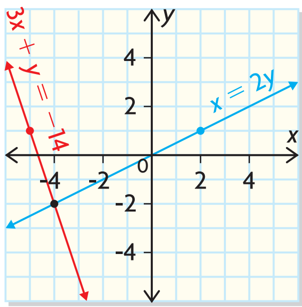
d) $(10, 5)$; $x - y = 5$ and $y = 5x - 40$

2. For each graph:

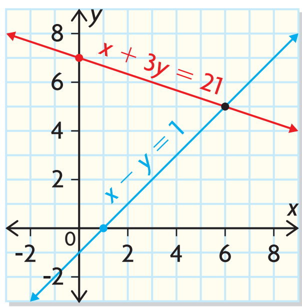
i) Identify the point of intersection.

ii) Verify your answer by substituting into the equations.

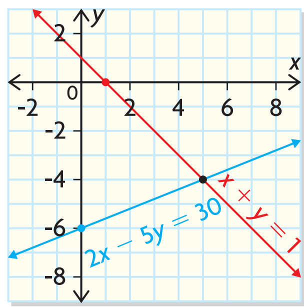
a)



b)



c)



PRACTISING

3. a) Graph the system $x + y = 5$ and $3x + 4y = 12$ by hand.

K **b)** Solve the system using your graph.

c) Verify your solution using graphing technology.

4. Alex needs to rent a minivan for a week to take his band on tour. Easyvans charges \$230 plus \$0.10/km. Cars for All Seasons charges \$150 plus \$0.22/km.

a) Write an equation for each rental company.

b) Graph your equations.

c) Which rental company would you recommend to Alex? Explain.

5. Solve each linear system by graphing.

a) $x + y = 3$
$x - y = 7$

b) $x + y = 8$
$4x - 2y = 8$

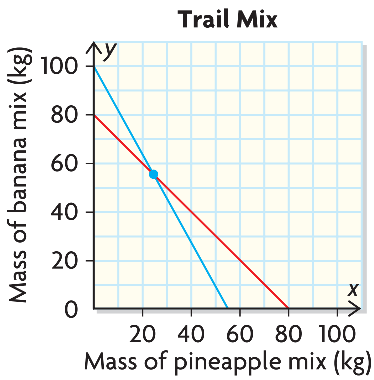
c) $y = 2x - 4$
$3x + y = 6$

d) $2x + y = 10$
$y = x - 2$

e) $y = 3x - 5$
$y = -2x + 5$

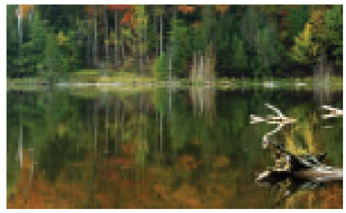
f) $6x - 5y - 12 = 0$
$-2x + 5y + 2 = 0$

6. Austin is creating a new "trail mix" by combining two of his best-selling blends: a pineapple–coconut–macadamia mix that sells for \$18/kg and a banana–papaya–peanut mix that sells for \$10/kg. He is making 80 kg of the new mix and will sell it for \$12.50/kg. Austin uses the graph shown at the right to determine how much of each blend he needs to use.
   a) Write the equations of the linear relations in the graph.
   b) From the graph, how much of each blend will Austin use?



7. At Jessica's Java, a new blend of coffee is featured each week. This week, Jessica is creating a low-caffeine espresso blend from Brazilian and Ethiopian beans. She wants to make 200 kg of this blend and sell it for \$15/kg. The Brazilian beans sell for \$12/kg, and the Ethiopian beans sell for \$17/kg. How many kilograms of each kind of bean must Jessica use to make 200 kg of her new blend of the week?

8. When Arthur goes fishing, he drives 393 km from his home in Ottawa to a lodge near Temagami. He travels at an average speed of 70 km/h along the highway to North Bay and then at 50 km/h on the narrow road from North Bay to Temagami. The journey takes him 6 h.
   a) Write two equations to describe this situation.
   b) Graph your equations.
   c) Use your graph to determine the distance from North Bay to Temagami.



9. Joanna is considering two job offers. Phoenix Fashions offers \$1500/month plus 2.5% commission. Styles by Rebecca offers \$1250/month plus 5.5% commission.
   a) Create a linear system by writing an equation for each salary.
   b) What value of sales would result in the same total salary for both jobs?
   c) Which job should Joanna take? Explain your answer.

10. T Create a situation you can represent by a system of linear equations that has the ordered pair (10, 15) as its solution.

11. Six cups of coffee and a dozen muffins originally cost \$15.35. The price of a cup of coffee increases by 10%. The price of a dozen muffins increases by 12%. The new cost of six cups of coffee and a dozen muffins is \$17.06. Determine the new price of one cup of coffee and a dozen muffins.

12. Willow bought 3 m of denim fabric and 5 m of cotton fabric. The total bill, excluding tax, was \$22. Jared bought 6 m of denim fabric and 2 m of cotton fabric at the same store for \$28.
   a) Write a linear system you can solve to determine the price of denim fabric and the price of cotton fabric.
   b) Solve your system using a graph.
   c) How much will 8 m of denim fabric and 5 m of cotton fabric cost?

13. The drama department of a school sold 679 tickets to the school play, for a total of \$3370. Students paid \$4 for a ticket, and non-students paid \$7.
   a) Write a linear system for this situation.
   b) How many non-students attended the play? Solve the problem by graphing your system.

14. **A** The equations $y = 2$, $y = 4x - 2$, and $y = -2x + 10$ form the sides of a triangle.
   a) Graph the triangle, and determine the coordinates of the vertices.
   b) Calculate the area of the triangle.

Environment Connection

Fluorescent light bulbs decrease energy use and reduce pollution levels.

15. **C** A regular light bulb costs \$0.65 to buy, plus \$0.004/h for the electricity to make it work. A fluorescent light bulb costs \$3.99 to buy, plus \$0.001/h for the electricity.
   a) Write a cost equation for each type of light bulb.
   b) Graph the system of equations using a graphing calculator. Use the window settings $0 \leq X \leq 2000$ and $0 \leq Y \leq 10$.
   c) After how long is the fluorescent light bulb cheaper than the regular light bulb?
   d) Determine the difference in cost after one year of constant use.

16. Rearrange the following sentences to describe the correct sequence of steps for solving a problem by graphing a linear system. Discard any sentences that do not belong in the description. Add any sentences that are needed to make the description clearer.
   - Label the graph.
   - Verify the solution by substituting into both equations.
   - Write two equations that describe the situation in the problem.
   - Determine the slope of each line by calculating the rise over the run.
   - Read the problem, and determine what you need to find.
   - Graph both equations on the same set of axes.
   - Choose the best strategy to graph each equation.
   - Determine the coordinates of the point of intersection.

Extending

17. a) Solve the linear system $3x - y - 11 = 0$ and $x + 2y + 1 = 0$.
   b) Show that the line with the equation $9x + 4y - 19 = 0$ passes through the point where the lines in part a) intersect.
   c) Determine the values of c and d if $9x + 4y - 19 = 0$ is written in the form $c(3x - y - 11) + d(x + 2y + 1) = 0$.

18. Solve the linear system $y = 2x - 1$, $4x - 3y = 7$, and $6x + y + 17 = 0$.

19. Solve each system of equations.
   a) $y = 2x^2$
   $y = -3x + 5$
   b) $y = \sqrt{x}$
   $y = x - 1$

Curious | Math

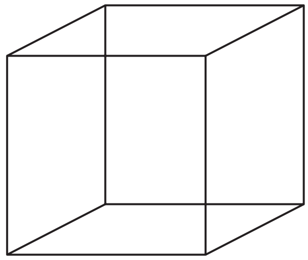
YOU WILL NEED

- ruler

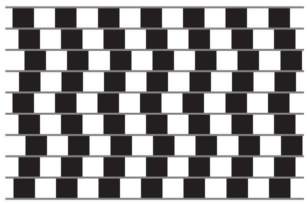
Optical Illusions

An optical illusion occurs when an image you look at does not agree with reality. The image has been deliberately created to deceive the eyes. Many optical illusions are created using intersecting lines.

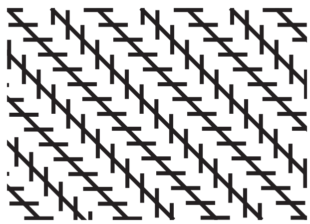
This image is called the Necker Cube. It is created by drawing the frame of a cube. The intersecting lines make the image confusing. When you focus on the cube, it seems to move backward and forward, causing you to question which is the front and which is the back.



This image is called the Café Wall Illusion because it was first noticed on café walls in England. Dark and light "tiles" are arranged alternately in staggered rows. Each row and each tile is bordered by a shade of a colour that is between the shades used for the "tiles." This has the strange effect of making the long parallel lines appear crooked.



This image is called the Zöllner Illusion. The long black lines appear not to be parallel, even though they are. The short lines are drawn at an angle to the long lines, giving the impression of depth and tricking the eyes to perceive that one end of each long line is closer to you than the other end.



1. Do some research to find other examples of optical illusions that have been created using intersecting lines.

2. Create your own optical illusion by drawing a series of intersecting lines.

1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 1.1, Examples 1 to 3.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can you represent the ordered pairs of a linear relation?

A: You can use variables to create an equation and then list the ordered pairs in a table of values or plot them as points to create a graph.

EXAMPLE

Fruit	Price per Kilogram ($)
cherries	10.98
peaches	2.18

Prices for cherries and peaches in July of one year are listed at the left. What amounts of cherries and peaches can you buy for $15?

Solution

Write an equation to describe the relation:

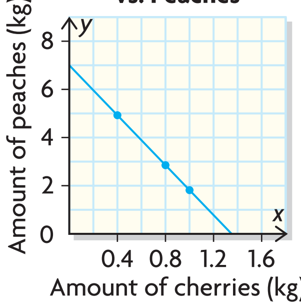
- c kilograms of cherries cost $\$10.98c$.
- p kilograms of peaches cost $\$2.18p$.
- The total amount of money to buy the cherries and peaches is $15.

The equation $10.98c + 2.18p = 15$ describes the linear relation between p and c. Use this equation to calculate the approximate coordinates of ordered pairs.

Amount of Cherries, c (kg)	0	0.40	0.80	1.00
Amount of Peaches, p (kg)	$\frac{15.00}{2.18} = 6.88$	4.87	2.85	1.84

Graph the linear relation by hand, by plotting two or more ordered pairs and drawing a straight line through the points.

Amount of Cherries vs. Peaches



Study *Aid*

- See Lesson 1.2, Examples 1 and 2.
- Try Mid-Chapter Review Questions 3 to 5.

Q: How can you solve a linear equation in one variable using a linear relation?

A: Each ordered pair, or point, on the graph of a linear relation represents the solution to the related linear equation.

EXAMPLE

Consider $0.03x + 0.04y = 120$, where x represents the amount of money invested at 3%/year and y represents the amount invested at 4%/year. The total interest earned for one year was $120. If $1500 was invested at 3%/year, how much was invested at 4%/year?

Solution

Solve $0.03(1500) + 0.04y = 120$.

Solving the Equation Algebraically

$$0.03(1500) + 0.04y = 120$$
$$45 + 0.04y = 120$$
$$45 + 0.04y - 45 = 120 - 45$$
$$0.04y = 75$$
$$\frac{0.04y}{0.04} = \frac{75}{0.04}$$
$$y = 1875$$

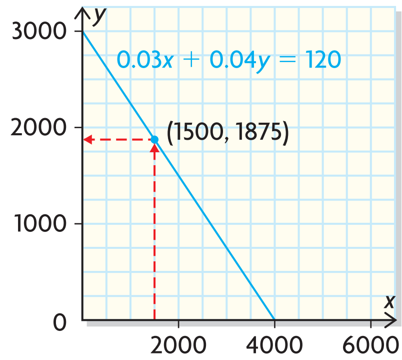
The amount invested at 4%/year was $1875.

Solving the Equation Graphically

Determine the point with an x-coordinate of 1500 on the graph of the relation. The y-coordinate of this point is the solution to the equation.

$y = 1875$

The amount invested at 4%/year was $1875.



Q: How can you solve a linear system of equations using graphs?

A: Graph the equations on the same axes by hand or using graphing technology. The coordinates of the point where the lines intersect give the solution to the system.

Study Aid

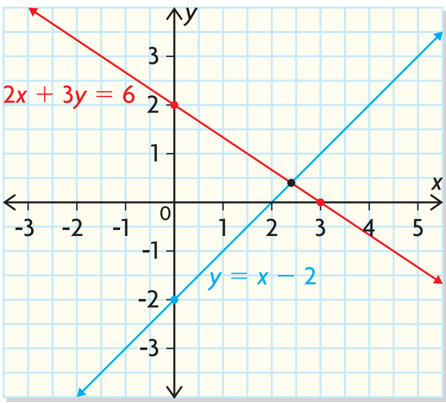
- See Lesson 1.3, Examples 1 to 3.
- Try Mid-Chapter Review Questions 6 to 10.

EXAMPLE

Solve $y = x - 2$ and $2x + 3y = 6$.

Solution

Graphing by Hand



From the graph, the solution is $x \doteq 2.5$ and $y \doteq 0.5$.

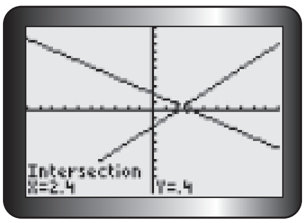
Graphing with Technology

Write the second equation in the form $y = mx + b$:

$3y = 6 - 2x$

$y = 2 - \frac{2}{3}x$ or $y = -\frac{2}{3}x + 2$

Enter both equations, and use the Intersect operation.



From the graph on the calculator, the solution is $x = 2.4$ and $y = 0.4$.

PRACTICE Questions

Lesson 1.1

1. Doreen has $10 to buy apples and pears.

Fruit	Price per Kilogram ($)
apples	2.84
pears	2.18

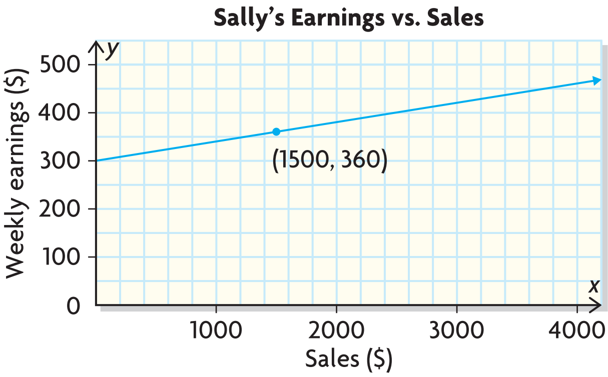
Use each representation below to determine the possible amounts of each type of fruit she can buy.

a) a table **b)** a graph **c)** an equation

2. Jon downloads music to his MP3 player from a site that charges $12.95 per month and $0.45 per song. Another site charges $8.99 per month and $0.95 per song. Compare the cost of the two sites using a table and a graph.

Lesson 1.2

3. The graph shows how Sally's weekly earnings vary with the dollar value of the sales she makes at a clothing store.



a) What do the coordinates (1500, 360) mean?
b) Use the graph to determine what Sally earns when her sales are $3200.
c) Use the graph to determine what sales Sally needs to make if she wants to earn $450.
d) Check your answers for parts b) and c) algebraically.

4. a) Determine an equation for the perimeter of any rectangle whose width is 8 cm less than its length.
b) Determine the length of the rectangle whose width is 72 cm.

5. Len plans to invest money he has saved so that he can earn $100 interest in one year. He will deposit some of his money in an account that pays 4%/year. He will use the rest of his money to buy a one-year GIC that pays 5%/year.
a) Write an equation for Len's situation, and draw a graph.
b) Suppose that Len buys a GIC for $1500. Use your graph to determine how much he would need to put in the account.
c) Suppose that Len deposits $2200 in the account. Determine how much he would need for the GIC.

Lesson 1.3

6. Solve each linear system.
a) $y = x + 4$, $y = -2x + 1$
b) $y = 4x - 7$, $2x - 3y = 6$
c) $3x - y = 3$, $2x + y = 2$
d) $5x - 2y = 10$, $2x + 4y = 4$

7. Solve each linear system.
a) $y = 5x - 8$, $10x - 5y = 7$
b) $2x + y = 2$, $x - \frac{1}{2}y = 4$
c) $x - 4y = -1$, $-3x + 8y = -2$
d) $x + y = 0.7$, $5x - 4y = -1$

8. The art department at a school sold 323 tickets to an art show, for a total of $790. Students paid $2 for tickets, and non-students paid $3.50. The principal asked how many non-students attended the art show.
a) Write a system of two linear equations for this situation.
b) Solve the problem by graphing the system.

9. Suppose you are solving the system $y = 2x + m$ and $3x - y = n$, where m and n are integers. Could this system have solutions in all four quadrants? Justify your answer.

10. Create a situation that can be represented by a system of linear equations that has the ordered pair (5, 12) as its solution.

1.4 Solving Linear Systems: Substitution

GOAL

Solve a system of linear equations using an algebraic strategy.

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

LEARN ABOUT the Math

Marla and Nancy played in a volleyball marathon for charity. They played for 38 h and raised \$412. Marla was sponsored for \$10/h. Nancy was sponsored for \$12/h.

? How many hours did each student play?

EXAMPLE 1 — Selecting an algebraic strategy to solve a linear system

Determine the length of time that each student played algebraically.

Isabel's Solution

Let m represent the hours Marla played. Let n represent the hours Nancy played.

I used variables for the number of hours each student played.

$$m + n = 38 \quad ①$$
$$10m + 12n = 412 \quad ②$$

I wrote one equation for the hours played and another equation for the money raised. I am looking for an ordered pair (m, n) that satisfies both equations.

$$m = 38 - n$$

I decided to solve for a variable in equation ① and then substitute into equation ②. I solved for m in equation ① since this was easier than solving for n.

$$10(38 - n) + 12n = 412$$

I used a **substitution strategy** by substituting the expression for m into equation ②. This gave me an equation in one variable, which included both pieces of information I had.

substitution strategy

a method in which a variable in one expression is replaced with an equivalent expression from another expression, when the value of the variable is the same in both

$$10(38) - 10n + 12n = 412$$
$$380 + 2n = 412$$
$$2n = 412 - 380$$
$$2n = 32$$
$$n = \frac{32}{2}$$
$$n = 16$$

I used the distributive property to multiply.

I solved for n using inverse operations.

$$m = 38 - n$$
$$m = 38 - 16$$
$$m = 22$$

I solved for m by substituting the value of n into the expression for m.

Marla played for 22 h and Nancy played for 16 h.

22 h + 16 h = 38h

22 h @ \$10/h = \$220

16 h @ \$12/h = \$192

\$220 + \$192 = \$412

They raised \$412.

I verified my solution. The total number of hours played by both girls and the total amount they raised matches the information in the problem.

Reflecting

A. When you substitute to solve a linear system, does it matter which equation or which variable you start with? Explain.

B. Why did Isabel need to do the second substitution after solving for n?

C. What would you do differently if you substituted for n instead of m?

APPLY the Math

EXAMPLE 2 Solving a problem modelled by a linear system using substitution

Most gold jewellery is actually a mixture of gold and copper. A jeweller is reworking a few pieces of old gold jewellery into a new necklace. Some of the jewellery is 84% gold by mass, and the rest is 75% gold by mass. The jeweller needs 15.00 g of 80% gold for the necklace. How much of each alloy should he use?

Wesley's Solution

Let x represent the mass of the 84% alloy in grams. Let y represent the mass of the 75% alloy in grams.

I used variables for the mass of each alloy.

$x + y = 15.00$ ①

I wrote an equation to represent the total mass of both alloys.

$0.84x + 0.75y = 0.80(15)$
$0.84x + 0.75y = 12.00$ ②

I wrote another equation to represent the amount of pure gold in the necklace, with the percents as decimals. I calculated 80% of 15.00 as 12.00. This means that the final 15.00 g of the 80% alloy must contain 12.00 g of pure gold.

$y = 15.00 - x$

I solved for *y* in equation ①.

$0.84x + 0.75(15.00 - x) = 12.00$
$0.84x + 11.25 - 0.75x = 12.00$

I substituted 15.00 − *x* for *y* in equation ②. Then I used the distributive property to multiply.

$0.84x - 0.75x = 12.00 - 11.25$
$0.09x = 0.75$
$x = \frac{0.75}{0.09}$
$x \doteq 8.33$

I solved for *x*, the mass of the 84% alloy, to the nearest hundredth of a gram.

$y = 15.00 - x$
$y = 15.00 - 8.33$
$y = 6.67$

I substituted the value of *x* into *y* = 15.00 − *x* and calculated the mass of the 75% alloy to the nearest hundredth of a gram.

The jeweller should use about 8.33 g of the 84% alloy and about 6.67 g of the 75% alloy.

EXAMPLE 3 Connecting the solution to a linear system to the break-even point

Sarah is starting a business in which she will hem pants. Her start-up cost, to buy a sewing machine, is $1045. She will use about $0.50 in materials to hem each pair of pants. She will charge $10 for each pair of pants. How many pairs of pants does Sarah need to hem to break even?

Robin's Solution

Let *x* represent the number of pairs of pants that Sarah hems, let *C* represent her total costs, and let *R* represent her revenue.

I chose letters for the variables in this problem.

$C = 1045 + 0.50x$

The cost of materials to hem *x* pairs of pants is $0.50*x*. I added the start-up cost to get the total cost.

$R = 10x$

Sarah charges $10 for each pair of pants. When she hems *x* pairs of pants, her revenue is $10*x*.

Communication *Tip*

A company makes a profit when it has earned enough revenue from sales to pay its costs. The point at which revenue and costs are equal is the break-even point.

At the break-even point, total costs and total revenue are the same.

$y = 1045 + 0.50x$ ①
$y = 10x$ ②

Because costs and revenue are the same at the break-even point, I used y to represent both dollar amounts.

$10x = 1045 + 0.50x$

I substituted the expression for y in equation ② into equation ①.

$$10x - 0.50x = 1045$$
$$9.50x = 1045$$
$$x = \frac{1045}{9.50}$$
$$x = 110$$

I used inverse operations to solve for x.

$$y = 10(110)$$
$$y = 1100$$

I determined y by substituting the value of x into equation ②.

Check:

$C = 1045 + 0.50(110)$
$C = 1045 + 55$
$C = 1100$

$R = 10(110)$
$R = 1100$

I verified my solution. The break-even point is 110 pairs of pants since the cost and revenue are both $1100.

Sarah will break even when she has hemmed 110 pairs of pants.

EXAMPLE 4 Selecting a substitution strategy to solve a linear system

Determine, without graphing, where the lines with equations $5x + 2y = -2$ and $2x - 3y = -16$ intersect.

Carmen's Solution

$5x + 2y = -2$ ①
$2x - 3y = -16$ ②

$$2y = -2 - 5x$$
$$\frac{2y}{2} = \frac{-2 - 5x}{2}$$
$$y = -1 - \frac{5}{2}x \quad ③$$

I decided to isolate the variable y in equation ①. This resulted in an equivalent form of the equation, which I called equation ③.

$$2x - 3\left(-1 - \frac{5}{2}x\right) = -16$$

The values of *x* and *y* must satisfy both equations at the point of intersection, so I substituted the expression for *y* in equation ③ into equation ②.

$$2x + 3 + \frac{15}{2}x = -16$$
$$2x + \frac{15}{2}x = -16 - 3$$
$$2x + \frac{15}{2}x = -19$$

I used the distributive property to multiply, and then I simplified.

$$2(2x) + 2\left(\frac{15}{2}x\right) = 2(-19)$$
$$4x + 15x = -38$$
$$19x = -38$$
$$\frac{19x}{19} = \frac{-38}{19}$$
$$x = -2$$

I multiplied all the terms in the equation by the lowest common denominator of 2 to eliminate the fractions. Then I used inverse operations to solve for *x*.

$$y = -1 - \frac{5}{2}(-2)$$
$$y = -1 + \frac{10}{2}$$
$$y = -1 + 5$$
$$y = 4$$

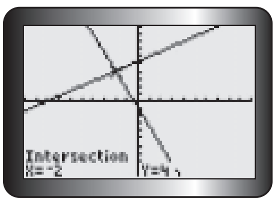
I substituted the value of *x* into equation ③. Then I determined the value of *y*.

The lines intersect at the point (–2, 4).

Check by graphing:

$$2x - 3y = -16 \quad ②$$
$$-3y = -2x - 16$$
$$y = \frac{2}{3}x + \frac{16}{3}$$

I solved for *y* in equation ② to get the equation in the form $y = mx + b$.



I graphed equations ① and ② using a graphing calculator. I used the Intersect operation to verify the point of intersection.

The graph confirms that the lines intersect at (−2, 4).

Tech Support

For help using a TI-83/84 graphing calculator to determine the point of intersection, see Appendix B-11. If you are using a TI-*n*spire, see Appendix B-47.

In Summary

Key Idea

- To determine the solution to a system of linear equations algebraically:
  - Isolate one variable in one of the equations.
  - Substitute the expression for this variable into the other equation.
  - Solve the resulting linear equation.
  - Substitute the solved value into one of the equations to determine the value of the other variable.

Need to Know

- Substitution is a convenient strategy when one of the equations can easily be rearranged to isolate a variable.
- Solving the equation created by substituting usually involves the distributive property. It may involve operations with fractional expressions.
- To verify a solution, you can use either of these strategies:
  - Substitute the solved values into the equation that you did not use when you substituted.
  - Graph both linear relations on a graphing calculator, and determine the point of intersection.

CHECK Your Understanding

1. For each equation, isolate the indicated variable.

a) $10x - y = 1, y$ **c)** $\frac{1}{2}x + y = 10, x$

b) $4x - y + 3 = 0, x$ **d)** $2x - y = 12, y$

2. To raise money for a local shelter, some Grade 10 students held a car wash and charged the prices at the left. They washed 53 vehicles and raised $382.

a) Write an equation to describe the number of vehicles washed.

b) Write an equation to describe the amount of money raised in terms of the number of each type of vehicle.

c) Solve for one of the variables in your equation for part a).

d) Substitute your expression for part c) into the equation for part b). Solve the new equation.

e) Substitute your answer for part d) into your equation for part a). Solve for the other variable. How many of each type of vehicle did the students wash?

PRACTISING

3. Decide which variable to isolate in one of the equations in each system. Then substitute for this variable in the other equation, and solve the system.

a) $x + 3y = 5$
$2x - 3y = -17$

b) $2x + y = 4$
$3x - 16y = 6$

4. Solve for the indicated variable.

a) $8a = 4 - b$, b

b) $6r + 3s = 9$, r

c) $3u + 7v = 21$, v

d) $0.3x - 0.3y = 1.8$, x

e) $0.12x - 0.06y = 0.24$, y

f) $\frac{1}{3}x + \frac{1}{2}y = 5$, x

5. Decide which variable to isolate. Then substitute for this variable, and solve the system.
K

a) $y = x - 5, x + y = 9$

b) $x = y + 4, 3x + y = 16$

c) $x + 4y = 21, 4x - 16 = y$

d) $3x - 2y = 10, x + 3y = 7$

e) $2x + y = 5, x - 3y = 13$

f) $x + 2y = 0, x - y = -4$

6. Tom pays a one-time registration charge and regular monthly fees to belong to a fitness club. After four months, he had paid \$420. After nine months, he had paid \$795. Determine the registration charge and the monthly fee.

7. A health-food company packs almond butter in jars. Some jars hold 250 g. Other jars hold 500 g. On Tuesday, the company packed 186.5 kg of almond butter in 511 jars. How many jars of each size did they pack?

8. The difference between two angles in a triangle is 11°. The sum of the same two angles is 77°. Determine the measures of all three angles in the triangle.

9. Solve each system. Check your answers.

a) $x + 3y = 7$ and $3x - 2y = -12$

b) $3 = 2a - b$ and $4a - 3b = 5$

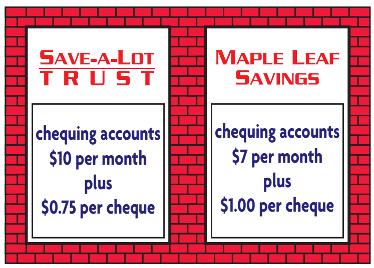
c) $7m + 2n = 21$ and $10m + 4n = -10$

d) $6x - 2y + 1 = 0$ and $3x - 5y + 7 = 0$

e) $3c + 2d = -24$ and $2c + 5d = -38$

f) $\frac{1}{4}x - 3y = \frac{1}{2}$ and $\frac{1}{3}x - 9y = 5$

10. Dan has saved \$500. He wants to open a chequing account at Save-A-Lot Trust or Maple Leaf Savings. Using the information at the right, which financial institution charges less per month?



11. Wayne wants to use a few pieces of silver to make a bracelet. Some of the jewellery is 80% silver, and the rest is 66% silver. Wayne needs 30.00 g of 70% silver for the bracelet. How much of each alloy should he use?

12. Sue is starting a lawn-cutting business. Her start-up cost to buy two
A lawn mowers and an edge trimmer is \$665. She has figured out that she will use about \$1 in gas for each lawn. If she charges \$20 per lawn, what will her break-even point be?

Safety Connection

Eye protection must be worn when operating a lathe.

13. A woodworking shop makes tables and chairs. To make a chair, 8 min is needed on the lathe and 8 min is needed on the sander. To make a table, 8 min is needed on the lathe and 20 min is needed on the sander. The lathe operator works 6 h/day, and the sander operator works 7 h/day. How many chairs and tables can they make in one day working at this capacity?

14. James researched these nutrition facts:
   - 1 g of soy milk has 0.005 g of carbohydrates and 0.030 g of protein.
   - 1 g of vegetables has 0.14 g of carbohydrates and 0.030 g of protein.

   James wants his lunch to have 50.000 g of carbohydrates and 20.000 g of protein. How many grams of soy milk and vegetables does he need?

15. Nicole has been offered a sales job at High Tech and a sales job at Best Computers. Which offer should Nicole accept? Explain.
   - High Tech: \$500 per week plus 5% commission
   - Best Computers: \$400 per week plus 7.5% commission

$$2x - (4x - 10) = 4$$
$$2x - 4x - 10 = 4$$
$$-2x - 10 = 4$$
$$-2x = 14$$
$$x = -7$$
$$y = 4(-7) - 10$$
$$y = -28$$

16. Monique solved the system of equations $2x - y = 4$ and $y = 4x - 10$
C by substitution. Her solution is at the left.
   a) What did she do incorrectly?
   b) Write a correct solution. Explain your steps.

17. Jennifer has nickels, dimes, and quarters in her piggy bank. In total,
T she has 49 coins, with a value of \$5.20. If she has five more dimes than all the nickels and quarters combined, how many of each type of coin does she have?

18. Explain why you think the strategy that was presented in this lesson is called substitution. Use the linear system $x + 4y = 8$ and $3x - 16y = 3$ in your explanation.

Extending

19. Marko invested \$300 000 in stocks, bonds, and a savings account at the rates shown at the left. He invested four times as much in stocks as he invested in the savings account. After one year, he earned \$35 600 in interest. How much money did he put into each type of investment?

1.5 Equivalent Linear Systems

GOAL

Compare solutions for equivalent systems of linear equations.

YOU WILL NEED

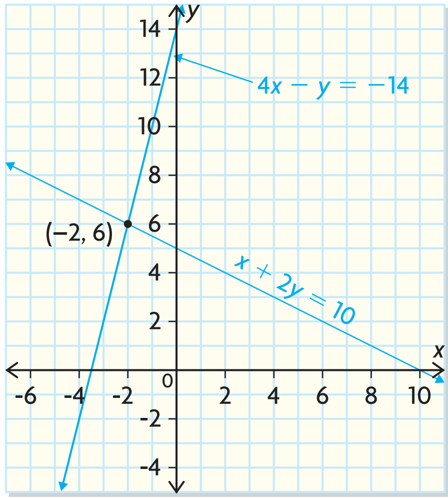
- grid paper
- ruler

LEARN ABOUT the Math

Cody solved this system of linear equations by graphing.

$x + 2y = 10$
$4x - y = -14$

He concluded that the solution to this linear system is the ordered pair $(-2, 6)$.



❓ What other systems of linear equations have the same solution?

EXAMPLE 1 **Connecting addition and subtraction with the equations of a linear system**

Add and subtract the two equations in the system that Cody solved. Graph all four equations on the same axes.

Sean's Solution

Add the equations.

$$\begin{aligned} x + 2y &= 10 \\ 4x - y &= -14 \\ \hline 5x + y &= -4 \end{aligned}$$

If $x + 2y = 10$ and $4x - y = -14$, $(x + 2y) + (4x - y) = 10 + (-14)$. This is another way to add the equations. You can collect like terms by adding the x terms, y terms, and constants.

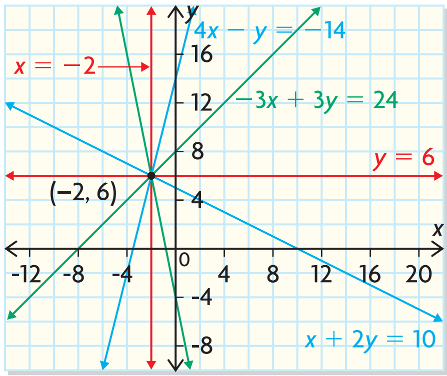
Subtract the equations.

$$\begin{aligned} x + 2y &= 10 \\ 4x - y &= -14 \\ \hline -3x + 3y &= 24 \end{aligned}$$

If $x + 2y = 10$ and $4x - y = -14$, $(x + 2y) - (4x - y) = 10 - (-14)$. This is another way to subtract the equations. You can collect like terms by subtracting the x terms, y terms, and constants.

The x-intercept for $5x + y = -4$ is at $(-0.8, 0)$, and the y-intercept is at $(0, -4)$. The x-intercept for $-3x + 3y = 24$ is at $(-8, 0)$, and the y-intercept is at $(0, 8)$.

I determined the intercepts for each new equation.



I graphed the new equations in green on the same axes. Both of the new lines pass through $(-2, 6)$. I graphed the lines $x = -2$ and $y = 6$ in red, as well. These two lines also intersect at $(-2, 6)$. All four new lines pass through the point of intersection for the original two lines. The solution did not change when I added or subtracted the equations in the original system of equations.

$$\begin{aligned} x + 2y &= 10 \\ 4x - y &= -14 \end{aligned} \qquad \begin{aligned} 4x - y &= -14 \\ -3x + 3y &= 24 \end{aligned}$$

$$\begin{aligned} 4x - y &= -14 \\ 5x + y &= -4 \end{aligned} \qquad \begin{aligned} x + 2y &= 10 \\ -3x + 3y &= 24 \end{aligned}$$

$$\begin{aligned} x + 2y &= 10 \\ 5x + y &= -4 \end{aligned} \qquad \begin{aligned} 5x + y &= -4 \\ -3x + 3y &= 24 \end{aligned}$$

Any pair of the two original and two new equations can be used to form a linear system that has the same solution, $(-2, 6)$.

These are all **equivalent systems of linear equations**.

equivalent systems of linear equations

two or more systems of linear equations that have the same solution

The system of linear equations $x = -2$ and $y = 6$ is equivalent to the other systems, but written in a simpler form that directly shows the solution.

EXAMPLE 2 Connecting multiplication by a constant with the equations of a linear system

Multiply $x + 2y = 10$ by 4 and $4x - y = -14$ by -2. Graph the original and two new equations on the same axes.

Donovan's Solution

$$\begin{aligned} 4(x + 2y) &= 4(10) \\ 4x + 8y &= 40 \end{aligned}$$

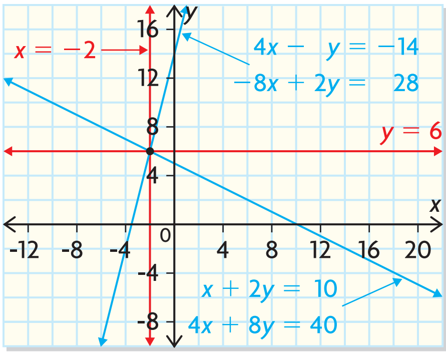
I multiplied both sides of the first equation by 4.

$$-2(4x - y) = -2(-14)$$
$$-8x + 2y = 28$$

I multiplied both sides of the second equation by -2.

$4x + 8y = 40$ has intercepts at (10, 0) and (0, 5). $-8x + 2y = 28$ has intercepts at (–3.5, 0) and (0, 14).

I determined the intercepts for the new equations.



I graphed the equations on the same axes.

The new lines were identical to the original lines, and the point of intersection was unchanged at $(-2, 6)$. I graphed the lines $x = -2$ and $y = 6$, as well. These two lines also intersect at $(-2, 6)$. When I multiplied each equation by a constant, their graphs did not change.

$$\begin{aligned} x + 2y &= 10 \\ 4x - y &= -14 \end{aligned} \qquad \begin{aligned} 4x + 8y &= 40 \\ 4x - y &= -14 \end{aligned}$$

$$\begin{aligned} x + 2y &= 10 \\ -8x + 2y &= 28 \end{aligned} \qquad \begin{aligned} 4x + 8y &= 40 \\ -8x + 2y &= 28 \end{aligned}$$

Multiples of either original equation, combined together, form a linear system that has the same solution, $(-2, 6)$.

These are all equivalent systems of linear equations.

The system of linear equations $x = -2$ and $y = 6$ is equivalent to the other systems, but it is written in a simpler form that directly shows the solution.

Reflecting

A. Suppose that Sean had doubled one equation first before he added. Would the new equation go through the same point of intersection as the original equations did?

B. What would have happened to the graphs if the equations in Donovan's solution had been multiplied by constants other than 4 and -2?

C. Why does solving a system of linear equations result in an equivalent system, with a horizontal line and a vertical line?

APPLY the Math

EXAMPLE 3 **Reasoning about equivalent linear systems**

a) Solve the system $2x + 5y = -4$ and $x - 2y = 7$.

b) Show that an equivalent system is formed by multiplying the second equation by 2 and then adding and subtracting the equations.

Emma's Solution

a) $x - 2y = 7$

$x = 7 + 2y$

I decided to solve the system using substitution. I used the second equation to isolate *x*. Since *x* has a coefficient of 1 in this equation, I was able to avoid working with fractions.

$2(7 + 2y) + 5y = -4$

I substituted $7 + 2y$ for *x* in the first equation.

$$\begin{aligned} 14 + 4y + 5y &= -4 \\ 14 + 9y - 14 &= -4 - 14 \\ 9y &= -18 \\ y &= -2 \end{aligned}$$

I solved the new equation for *y*.

$$\begin{aligned} x &= 7 + 2y \\ x &= 7 + 2(-2) \\ x &= 7 - 4 \\ x &= 3 \end{aligned}$$

Then I substituted -2 for *y* and solved for *x*.

The solution is the ordered pair (3, –2).

b) $$\begin{aligned} 2(x - 2y) &= 2(7) \\ 2x - 2(2y) &= 2(7) \\ 2x - 4y &= 14 \end{aligned}$$

I multiplied both sides of the second equation by 2.

$$\begin{aligned} 2x + 5y &= -4 \\ 2x - 4y &= 14 \\ \hline 4x + y &= 10 \end{aligned}$$

I added the new equation to the first equation in the system.

$$\begin{aligned} 2x + 5y &= -4 \\ 2x - 4y &= 14 \\ \hline 0x + 9y &= -18 \end{aligned}$$

Then I subtracted the new equation from the first equation.

$$\begin{aligned} 4x + y &= 10 \\ 9y &= -18 \end{aligned}$$

I wrote the new system that was formed by adding and subtracting. The coefficient for x was 0.

$$y = -2$$

I solved for y in $9y = -18$.

$$\begin{aligned} 4x + y &= 10 \\ 4x - 2 &= 10 \\ 4x &= 10 + 2 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

I substituted the value of y into $4x + y = 10$ and determined x.

The solution to the new system is the ordered pair $(3, -2)$.

Since the solutions are the same, the new system $4x + y = 10$ and $9y = -18$ is equivalent to the original system.

In Summary

Key Ideas

- When you add and subtract the equations in a linear system, you create an equivalent linear system of equations that has the same solution as the original system.
- When you multiply one or both equations of a system by a constant other than 0, you create an equivalent linear system of equations that has the same solution as the original system.

Need to Know

- When you add and subtract the equations of a linear system, the graphs of the new equations are different from the graphs of the original equations. However, they pass through the same point of intersection.
- Multiplying an equation by a constant other than 0 does not change the graph of the equation.
- Adding or subtracting the equations of a linear system may result in a simpler equation, with only one variable.

CHECK Your Understanding

1. Add and subtract the equations in each linear system to create a new linear system.

a) $x - 3y = 2$
$2x + y = -5$

b) $x - y = 2$
$2x + 3y = 19$

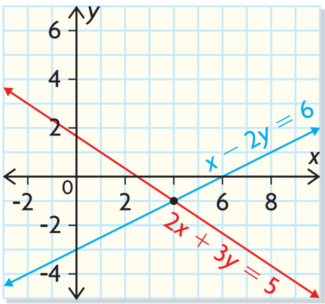
c) $3x + y = 3$
$x - 2y = 8$

d) $4x + 2y = 8$
$-x - 2y = -4$

2. a) Solve each original system in question 1.

b) Verify that the original system and the new system for question 1 have the same solution.

PRACTISING



3. a) State the solution to the linear system that is shown in the graph at the left.

b) Create a new linear system by adding and then subtracting the equations of the original system.

c) Substitute the x- and y-values for part a) into each of the new equations for part b) to verify that the new system and the original system are equivalent.

4. a) Multiply one of the equations in the graph in question 3 by any integer other than zero.

b) Determine the x- and y-intercepts of the new equation to verify that the graph of the new equation is the same as the graph of the original equation in question 3.

c) Repeat parts a) and b) for the other equation in question 3.

5. a) Use substitution to solve the linear system $4x + y = 1$ and $x - 2y = -11$.

b) Create another linear system by adding and subtracting the equations in part a).

c) Verify that the systems in parts a) and b) have the same solutions.

6. A linear system is defined by $x + 2y = 2$ and $-2x - y = 5$.

K **a)** Multiply the first equation by 3 and the second equation by 2.

b) Create another linear system by adding and subtracting the equations you formed for part a).

c) Use a graph to show that the systems in parts a) and b) are equivalent.

7. a) Use substitution to solve the linear system $-2x + 5y = 2$ and $x - 3y = -2$.

b) Multiply the first equation by -3 and the second equation by -5.

c) Add and subtract the equations for part b) to create a new linear system.

d) Use substitution to verify that the systems in parts a) and c) are equivalent.

8. **a)** You have seen that multiplying an equation by a non-zero constant does not change the graph of the equation. How do you think dividing an equation by a non-zero constant would affect its graph? Explain your reasoning.

C

b) Use a graph to solve the linear system:
$3x - 3y = 6$
$8x + 4y = 4$

c) Divide the first equation by 3 and the second equation by 4. Then graph the new system you created. What effect, if any, did dividing the equations have on the graph?

d) Add and subtract the equations for part c) to create a new linear system.

e) Determine whether the systems for parts b) and d) are equivalent.

9. **a)** Solve this linear system:

T

$4x + y = 4$
$2x - 3y = -5$

b) Show that your solution also satisfies the equation $2x + 11y = 23$.

c) Determine constants a and b so that a times the first equation in the system added to b times the second equation results in the equation in part b).

10. **a)** Create two linear systems that are equivalent to this system:
$3x - 4y = 3$
$-x - y = 6$

b) Verify that all three systems have the same solution.

11. **a)** Why might you solve this system of equations by adding or subtracting?

A

$2x + 3y = 4$
$2x - 3y = 8$

b) Create a system of linear equations that you might solve by adding.

c) Create a system of linear equations that you might solve by subtracting.

12. Add and subtract the equations in this system. Use the new equations to solve it.
$2x + 3y = 4$
$2x - 3y = 8$

13. The sum of two numbers is 33, and their difference is 57.

a) Create a system of linear equations for this situation.

b) Create an equivalent system by adding and subtracting your equations.

c) Solve the equivalent system to determine the two numbers.

Career *Connection*

Banquet halls offer careers in financial management, cooking, catering, and hospitality.

14. As the owner of a banquet hall, you are in charge of catering a reception. You are serving two dinners: a chicken dinner that costs $20 and a fish dinner that costs $18. Two hundred guests have ordered their dinners in advance, and the total bill is $3880.

a) Create a system of linear equations for this situation.

b) Create an equivalent system by multiplying the guest equation by 20 and then subtracting the cost equation from this new equation.

c) Simplify and then solve the equivalent system to determine the number of each type of dinner ordered.

15. a) Candice claims that these systems of linear equations are equivalent. Is she correct? Justify your decision.

System A	System B	System C
$3x - 2y = 2$	$-7x + y = 10$	$x = -2$
$-10x + 3y = 8$	$13x - 5y = -6$	$y = -4$

b) Create another system of linear equations that is equivalent to the systems in part a).

16. a) What are equivalent systems of linear equations?

b) How can you use the equations for a linear system to create an equivalent system?

c) How can this help you to solve the original system?

Extending

17. If you create equivalent linear systems in which there is only one variable in one or more of the new equations, you can solve the original system without graphing it. Use this strategy to solve the following linear systems.

a) $x - 4y = -22$
$2x + y = 1$

b) $3x - 4y = 30$
$2x + 5y = -26$

18. Consider this system of linear equations:
$2x + y = 7$
$8x + 4y = 28$

a) Can you create an equivalent system that contains only one variable?

b) What does your result for part a) suggest about the solution to the original system?

c) What does your result for part a) suggest about the graphs of both lines?

19. Repeat question 18 for the system $2x + y = 7$ and $8x + 4y = 10$.

1.6 Solving Linear Systems: Elimination

GOAL

Solve a linear system of equations using equivalent equations to remove a variable.

LEARN ABOUT the Math

Every day, Brenna bakes chocolate chip and low-fat oatmeal cookies in her bakery. She uses different amounts of butter and oatmeal in each recipe. Brenna has 47 kg of butter and 140 kg of oatmeal.

Chocolate Chip	Low-Fat Oatmeal
• 13 kg butter	• 2 kg butter
• 8 kg oatmeal	• 29 kg oatmeal

? How many batches of chocolate chip and low-fat oatmeal cookies can Brenna bake?

EXAMPLE 1 **Selecting an algebraic strategy** to eliminate a variable

Determine the number of batches of each type of cookie that Brenna can bake using all the butter and oatmeal she has.

Chantal's Solution: Selecting an algebraic strategy to eliminate a variable

Let r represent the number of batches of chocolate chip cookies. Let s represent the number of batches of low-fat cookies.

I used variables for the numbers of batches.

$13r + 2s = 47$ ① butter
$8r + 29s = 140$ ② oatmeal

I wrote two equations, ① to represent the amount of butter and ② the amount of oatmeal. I decided to use an **elimination strategy** to eliminate the r terms by subtracting two equations.

$8(13r + 2s) = 8(47)$ ① × 8
$8(13r) + 8(2s) = 8(47)$
$104r + 16s = 376$

$13(8r + 29s) = 13(140)$ ② × 13
$13(8r) + 13(29s) = 13(140)$
$104r + 377s = 1820$

To eliminate the r terms by subtracting, I had to make the coefficients of the r terms the same in both equations. I multiplied equation ① by 8 and equation ② by 13.

elimination strategy
a method of removing a variable from a system of linear equations by creating an equivalent system in which the coefficients of one of the variables are the same or opposites

Communication *Tip*
The steps that are required to eliminate a variable can be described by showing the operation and the equation number. For example, "① × 8" means "equation ① multiplied by 8."

$$\begin{aligned} 104r + 16s &= 376 \\ 104r + 377s &= 1820 \\ \hline -361s &= -1444 \end{aligned}$$

I subtracted the equations to eliminate *r*. ① × 8 − ② × 13

$$s = \frac{-1444}{-361}$$
$$s = 4$$

I solved for *s*.

$$\begin{aligned} 13r + 2(4) &= 47 \\ 13r + 8 &= 47 \\ 13r &= 47 - 8 \\ 13r &= 39 \\ r &= \frac{39}{13} \\ r &= 3 \end{aligned}$$

I substituted the value of *s* into equation ①. (I could have used equation ② instead, if I had wanted.) I solved for *r*.

Brenna can make three batches of chocolate chip cookies and four batches of low-fat cookies.

Check:

I verified my answers.

Type of Cookie	Number of Batches	Butter (kg)	Oatmeal (kg)
chocolate chip	3	$3 \times 13 = 39$	$3 \times 8 = 24$
low-fat	4	$4 \times 2 = 8$	$4 \times 29 = 116$
Total		$39 + 8 = 47$	$24 + 116 = 140$

Leif's Solution: Selecting an algebraic strategy to eliminate a different variable

$$\begin{aligned} 13r + 2s &= 47 \quad ① \\ 8r + 29s &= 140 \quad ② \end{aligned}$$

I started with the same linear system as Chantal, but I decided to eliminate the *s* terms by adding the two equations.

$$\begin{aligned} 29(13r + 2s) &= 29(47) \quad ① \times 29 \\ 29(13r) + 29(2s) &= 29(47) \\ 377r + 58s &= 1363 \end{aligned}$$

$$\begin{aligned} -2(8r + 29s) &= -2(140) \quad ② \times -2 \\ -2(8r) - 2(29s) &= -2(140) \\ -16r - 58s &= -280 \end{aligned}$$

To eliminate the *s* terms by adding, I had to make the coefficients of the *s* terms opposites. To do this, I multiplied equation ① by 29 and equation ② by −2.

$$\begin{array}{rcl} 377r + 58s &=& 1363 \\ -16r - 58s &=& -280 \\ \hline 361r &=& 1083 \end{array}$$

I added the equations to eliminate *s*. ① × 29 − ② × −2

$$r = \frac{1083}{361}$$
$$r = 3$$

I solved for *r*.

$$13(3) + 2s = 47$$
$$39 + 2s = 47$$
$$2s = 47 - 39$$
$$2s = 8$$
$$s = \frac{8}{2}$$
$$s = 4$$

I substituted the value of *r* into equation ①.

Brenna can make three batches of chocolate chip cookies and four batches of low-fat cookies.

Reflecting

A. How did Chantal and Leif use elimination strategies to change a system of two equations into a single equation?

B. Why did Chantal and Leif need to multiply both equations to eliminate a variable?

C. Explain when you would add and when you would subtract to eliminate a variable.

D. Whose strategy would you choose: Chantal's or Leif's? Why?

APPLY the Math

EXAMPLE 2 Selecting an elimination strategy to solve a linear system

Use elimination to solve this linear system:

$7x - 12y = 42$

$17x + 8y = -2$

John's Solution

$$7x - 12y = 42 \quad ①$$
$$17x + 8y = -2 \quad ②$$

I decided to eliminate the *y* terms because I prefer to add. Since their signs were different, I could make the coefficients of the *y* terms opposites.

$$14x - 24y = 84 \quad ① \times 2$$
$$51x + 24y = -6 \quad ② \times 3$$
$$65x = 78$$
$$x = \frac{78}{65}$$
$$x = 1.2$$

The coefficients of *y* are factors of 24. I multiplied equation ① by 2 and equation ② by 3 to make the coefficients of the *y* terms opposites. Then I added the new equations to eliminate *y*.
① × 2 + ② × 3

$$7(1.2) - 12y = 42 \quad ①$$
$$8.4 - 12y = 42$$
$$-12y = 42 - 8.4$$
$$-12y = 33.6$$
$$y = \frac{33.6}{-12}$$
$$y = -2.8$$

I substituted the value of *x* into equation ① and solved for *y*.

Verify by substituting $x = 1.2$ and $y = -2.8$ into both original equations.

$7x - 12y = 42$

Left Side	Right Side
$7x - 12y$	42
$= 7(1.2) - 12(-2.8)$	
$= 8.4 + 33.6$	
$= 42$	

$17x + 8y = -2$

Left Side	Right Side
$17x + 8y$	-2
$= 17(1.2) + 8(-2.8)$	
$= 20.4 - 22.4$	
$= -2$	

The solution is $(1.2, -2.8)$.

EXAMPLE 3 **Selecting an elimination strategy** to solve a system with rational coefficients

During a training exercise, a submarine travelled 20 km/h on the surface and 10 km/h underwater. The submarine travelled 200 km in 12 h. How far did the submarine travel underwater?

Tanner's Solution

Let x represent the distance that the submarine travelled on the surface. Let y represent the distance that it travelled underwater.

I used variables for the distances that the submarine travelled.

$$x + y = 200 \quad ①$$

I wrote an equation for the total distance travelled during the training exercise.

The time spent on the surface is $\frac{x}{20}$.

The time spent underwater is $\frac{y}{10}$.

$$\frac{x}{20} + \frac{y}{10} = 12 \quad ②$$

I used the formula time $= \frac{\text{distance}}{\text{speed}}$ to write expressions for the time spent on the surface and the time spent underwater. Then I wrote an equation for the total time.

$$20\left(\frac{x}{20} + \frac{y}{10}\right) = 20(12) \quad ② \times 20$$

$$20\left(\frac{x}{20}\right) + 20\left(\frac{y}{10}\right) = 20(12)$$

$$x + 2y = 240 \quad ② \times 20$$

I created an equivalent system with no fractional coefficients by multiplying equation ② by 20, since 20 is a common multiple of 20 and 10.

$$\begin{aligned} x + y &= 200 \quad ① \\ x + 2y &= 240 \quad ② \times 20 \\ \hline -y &= -40 \\ y &= 40 \end{aligned}$$

Since the coefficients of x were now the same, I decided to eliminate x by subtracting the equations.

$$\begin{aligned} x + 40 &= 200 \\ x &= 200 - 40 \\ x &= 160 \end{aligned}$$

To determine x, I substituted 40 for y in the equation $x + y = 200$.

The submarine travelled 40 km underwater.

In Summary

Key Idea

- To eliminate a variable from a system of linear equations, you can
  - add two equations when the coefficients of the variable are opposite integers
  - subtract two equations when the coefficients of the variable are the same

Need to Know

- Elimination is a convenient strategy when the variable you want to eliminate is on the same side in both equations.
- If there are fractional coefficients in a system of equations, you can form equivalent equations without fractional coefficients by choosing a multiplier that is a common multiple of the denominators.
- Adding, subtracting, multiplying, or dividing both sides of a linear equation in the same way produces an equation that is equivalent to the original equation.

CHECK Your Understanding

1. For each linear system, state whether you would add or subtract to eliminate one of the variables without using multiplication.

a) $4x + y = 5$, $3x + y = 7$ **b)** $3x - 2y = 8$, $5x - 2y = 9$ **c)** $4x - 3y = 6$, $4x + 7y = 9$ **d)** $4x - 5y = 4$, $3x + 5y = 10$

2. a) Describe how you would eliminate the variable x from the system of equations in question 1, part a).

b) Describe how you would eliminate the variable x from the system of equations in question 1, part c).

3. When a welder works for 3 h and an apprentice works for 5 h, they earn a total of \$175. When the welder works for 7 h and the apprentice works for 8 h, they earn a total of \$346. Determine the hourly rate for each worker.

Safety *Connection*

Welders must wear a helmet with a mask that has a darkened lens, as well as gloves and clothing that are flame- and heat-resistant.

PRACTISING

4. To eliminate y from each linear system, by what numbers would you multiply equations ① and ②?

a) $4x + 2y = 5$ ① $3x - 4y = 7$ ②

b) $3x - 7y = 11$ ① $5x + 8y = 9$ ②

c) $4x + 3y = 12$ ① $-2x + 5y = 7$ ②

d) $9x - 4y = 10$ ① $3x + 2y = 10$ ②

5. To eliminate x from each linear system in question 4, by what numbers would you multiply equations ① and ②?

6. Solve each system by using elimination.

K **a)** $3x + y = -2$
$x - y = -6$

b) $x + 5y = 1$
$2x + 3y = 9$

c) $4x - y = 5$
$-5x + 2y = -1$

d) $2x - 3y = -2$
$3x - y = 0.5$

e) $3x - 2y = -39$
$x + 3y = 31$

f) $5x - y = -3.8$
$4x + 3y = 7.6$

7. Determine, without graphing, the point of intersection for the lines with equations $x + 3y = -1$ and $4x - y = 22$. Verify your solution.

8. In a charity walkathon, Lori and Nicholas walked 72.7 km. Lori walked 8.9 km farther than Nicholas.

a) Create a linear system to model this situation.

b) Solve the system to determine how far each person walked.

9. The perimeter of a beach volleyball court is 54 m. The difference between its length and its width is 9 m.

a) Create a linear system to model this situation.

b) Solve the system to determine the dimensions of the court.

10. A Rolf needs 500 g of chocolate that is 86% cocoa for a truffle recipe. He has one kind of chocolate that is 99% cocoa and another kind that is 70% cocoa. How much of each kind of chocolate does he need to make the 86% cocoa blend? Round your answer to the nearest gram.

11. Determine the point of intersection for each pair of lines. Verify your solution.

a) $4x + 7y = 23$
$6x - 5y = -12$

b) $\frac{x}{11} - \frac{y}{8} = -2$
$\frac{x}{2} - \frac{y}{4} = 3$

c) $0.5x - 0.3y = 1.5$
$0.2x - 0.1y = 0.7$

d) $\frac{x}{2} - 5y = 7$
$3x + \frac{y}{2} = \frac{23}{2}$

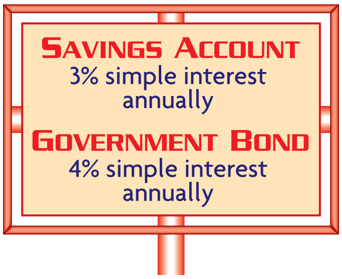
e) $5x - 12y = 1$
$13x + 9y = 16$

f) $\frac{x}{9} + \frac{y - 3}{3} = 1$
$\frac{x}{2} - (y + 9) = 0$

12. Each gram of a mandarin orange has 0.26 mg of vitamin C and 0.13 mg of vitamin A. Each gram of a tomato has 0.13 mg of vitamin C and 0.42 mg of vitamin A. How many grams of mandarin oranges and tomatoes have 13 mg of vitamin C and 20.7 mg of vitamin A?

13. C On weekends, as part of his exercise routine, Carl goes for a run, partly on paved trails and partly across rough terrain. He runs at 10 km/h on the trails, but his speed is reduced to 5 km/h on the rough terrain. One day, he ran 12 km in 1.5 h. How far did he run on the rough terrain?

14. Two fractions have denominators 3 and 4. Their sum is $\frac{17}{12}$. If the numerators are switched, the sum is $\frac{3}{2}$. Determine the two fractions.



15. A student athletic council raised \$6500 in a volleyball marathon. The students put some of the money in a savings account and the rest in a government bond. The rates are shown at the left. After one year, the students earned \$235. How much did they invest at each rate?

16. The caterers for a Grade 10 semi-formal dinner and dance are preparing two different meals: chicken at \$12 or pasta at \$8. The total cost of the dinners for 240 students is \$2100.
   a) How many chicken dinners did the students order?
   b) How many pasta dinners did they order?

17. A magic square is an array of numbers with the same sum across any row, column, or main diagonal.

16	2	B
A		14
8		

24	$\frac{A}{2}$	18
9		
B		

   a) Determine a system of linear equations you can use to determine the values of A and B in both squares.
   b) What are the values of A and B?

18. Explain what it means to eliminate a variable from a linear system. Use the linear system $3x + 7y = 31$ and $5x - 8y = 91$ to compare different strategies for eliminating a variable.

Extending

19. The sum of the squares of two negative numbers is 74. The difference of their squares is 24. Determine the two numbers.

20. Solve the system $2xy + 3 = 4y$ and $3xy + 2 = 5y$.

21. A general system of linear equations is

$$ax + by = e$$
$$cx + dy = f$$

where $a, b, c, d, e,$ and f are constant values.
   a) Use elimination to solve for x and y in terms of $a, b, c, d, e,$ and f.
   b) Are there any values that $a, b, c, d, e,$ and f cannot have?

1.7 Exploring Linear Systems

GOAL

Connect the number of solutions to a linear system with its equations and graphs.

YOU WILL NEED

- graphing calculator, or grid paper and ruler

EXPLORE the Math

Three different linear systems are given below.

A	B	C
$2x + 3y = -4$ $-4x - 3y = -1$	$2y = 6 - 3x$ $6x - 5 = -4y$	$x - y = 5$ $3x = 15 + 3y$

? How many solutions can a linear system have, and how can you predict the number of solutions without solving the system?

A. Solve each system of linear equations algebraically. Record the number of solutions you determine.

	A	B	C
Linear System	$2x + 3y = -4$ $-4x - 3y = -1$	$3x + 2y = 6$ $6x + 4y = 5$	$x - y = 5$ $3x - 3y = 15$
Number of Solutions			

B. Examine your algebraic solution for system A. How do you think the lines that represent the equations in this system intersect? Explain. Graph the system to check your conjecture.

C. Repeat part B for each of the other two systems.

D. For each system, explain how the graphical solution is related to the algebraic solution, and vice versa.

E. Examine the equations in each system. Are there clues that tell you how the lines will intersect? Explain.

F. Can a linear system of two equations have exactly two solutions? Can it have exactly three solutions? Explain.

G. Discuss the different cases you have identified and how each case relates to the equations and their corresponding graphs.

Reflecting

H. Both equations in a linear system that has no solution are written in the form $Ax + By = C$. Describe the relationship between the coefficients and the constants. What does this tell you about the graphs of both lines?

I. Both equations in a linear system that has an infinite number of solutions are written in the form $Ax + By = C$. Describe the relationship between the coefficients and the constants. What does this tell you about the graphs of both lines?

J. How can you tell, by looking at the equations, that a linear system has exactly one solution?

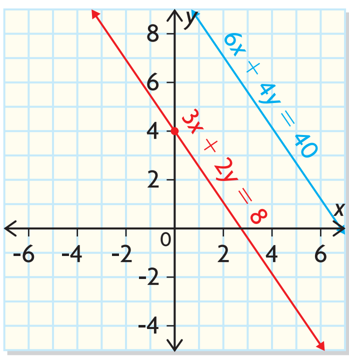
In Summary

Key Idea

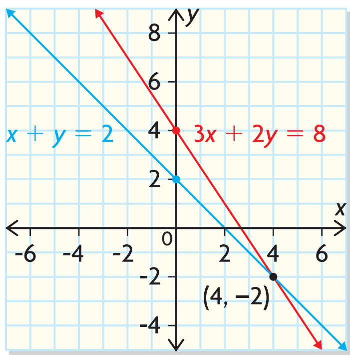
- A linear system can have no solution, one solution, or an infinite number of solutions.

Need to Know

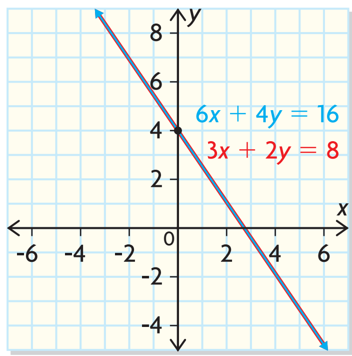
- When a linear system has no solution, the graphs of both lines are parallel and never intersect. For example, the system $3x + 2y = 8$ and $6x + 4y = 40$ does not have a solution. The coefficients in the equations are multiplied by the same amount, but the constants are not.



- When a linear system has one solution, the graphs of the two lines intersect at a single point. For example, the system $3x + 2y = 8$ and $x + y = 2$ has one solution. The coefficients and constants in the equations are not multiplied by the same amount.



- When a linear system has an infinite number of solutions, the graphs of both equations are identical and intersect at every point. For example, the system $3x + 2y = 8$ and $6x + 4y = 16$ has an infinite number of solutions. The coefficients and constants in the equations are multiplied by the same amount.



FURTHER Your Understanding

1. Graph a linear system of equations that has each number of solutions.
a) none **b)** one **c)** infinitely many

2. Use the equation $3x + 4y = 2$.
a) Write another equation that will create a linear system with each number of solutions.
i) none **ii)** one **iii)** infinitely many
b) Verify your answers for part a) algebraically and graphically.

3. Predict the number of solutions for each linear system. Then test your predictions by solving each system algebraically and verify with graphing technology.
a) $y = 3x - 5$
$y = 4x + 6$
b) $y = 4x - 3$
$y = 4x - 7$
c) $y = 5x - \frac{3}{2}$
$y = 5x - 1.5$
d) $x + 2y = 10$
$y = 8 - 0.5x$
e) $2x + 3y = 10$
$10x + 15y = 50$
f) $3x - 5y - 2 = 0$
$4x + 5y + 2 = 0$
g) $y = 1.25x - 0.375$
$5y = 4x$
h) $2x - 5 = 4y$
$0.01x - 0.02y = 0.25$

4. Create a system of linear equations that has each number of solutions. Then verify the number of solutions algebraically and graphically.
a) none **b)** one **c)** infinitely many

5. Both equations in a linear system are written in the form $Ax + By = C$. Explain how you could predict the number of solutions using the coefficients and constants of the two equations.

6. An air traffic controller is plotting the course of two jets scheduled to land in 15 min. One aircraft is following a path defined by the equation $3x - 5y = 20$ and the other by the equation $18x = 30y + 72$. Should the controller alter the paths of either aircraft? Justify your decision.

1 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 1.4, Examples 1 to 4.
- Try Chapter Review Questions 7 to 9.

Q: How can you use algebra to solve a linear system?

A1: You can use a substitution strategy. Express one variable in one of the equations in terms of the other variable. Substitute this expression into the other equation, and solve for the remaining variable. Finally, substitute the solved value into the expression to determine the value of the other variable.

EXAMPLE

Solve the system.

$2x + y = 29$ ①

$4x - 3y = 18$ ②

Solution

From ①, $y = 29 - 2x$.

Substitute this expression for y into ② and solve for x.

$$\begin{aligned} 4x - 3(29 - 2x) &= 18 \\ 4x - 87 + 6x &= 18 \\ 10x - 87 &= 18 \\ 10x &= 18 + 87 \\ 10x &= 105 \\ x &= 10.5 \end{aligned}$$

Determine y by substituting this value of x into the expression for y.

$y = 29 - 2(10.5)$

$y = 8$

The solution is $x = 10.5$ and $y = 8$.

Study *Aid*

- See Lesson 1.6, Examples 1 and 3.
- Try Chapter Review Questions 12 to 16.

A2: You can use an elimination strategy. Eliminate x or y from the system by multiplying one or both equations by a constant other than zero, and then adding or subtracting the equations. Solve the resulting equation for the remaining variable. Substitute the solved value into one of the original equations, and determine the value of the other variable.

EXAMPLE

Solve the system.

$2x + y = 29$ ①

$4x - 3y = 18$ ②

Solution

Eliminating *x*

Multiply ① by 2 and subtract.

$$\begin{aligned} 4x + 2y &= 58 \quad ① \times 2 \\ 4x - 3y &= 18 \\ \hline 5y &= 40 \\ y &= 8 \end{aligned}$$

Substitute $y = 8$ into ①.

$$\begin{aligned} 2x + 8 &= 29 \\ 2x &= 29 - 8 \\ 2x &= 21 \\ x &= 10.5 \end{aligned}$$

Eliminating *y*

Multiply ① by 3 and add.

$$\begin{aligned} 6x + 3y &= 87 \quad ① \times 3 \\ 4x - 3y &= 18 \\ \hline 10x &= 105 \\ x &= 10.5 \end{aligned}$$

Substitute $x = 10.5$ into ①.

$$\begin{aligned} 2(10.5) + y &= 29 \\ 21 + y &= 29 \\ y &= 29 - 21 \\ y &= 8 \end{aligned}$$

The solution is $x = 10.5$ and $y = 8$.

Q: When you use elimination, how do you decide whether to add or subtract the equations?

A: If the coefficients of the variable you want to eliminate are the same, subtract. If the coefficients are opposites, add.

Study Aid

- See Lesson 1.6, Examples 1 to 3.
- Try Chapter Review Questions 12 and 15.

Q: How do you decide whether to use graphs, substitution, or elimination to solve a linear system?

A: The strategy you use will depend on what degree of accuracy is required, what form the equations are written in, and whether more than just the solution is required. Algebraic solutions give exact answers, whereas hand-drawn graphs often do not.

Use graphs if

- you do not need an exact answer
- you need to look for trends or compare the graphs before and after the point of intersection
- you have a graphing calculator and both equations are in the form $y = mx + b$

Use substitution if

- you need exact answers
- one of the variables in the equation is already isolated, ready to make the substitution (that is, in the form $y = mx + b$)
- you can easily rearrange one equation to isolate a variable

Use elimination if

- you need exact answers
- both equations are in the form $Ax + By = C$ or $Ax + By + C = 0$

PRACTICE Questions

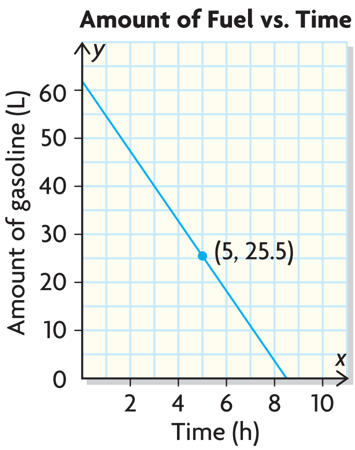
Lesson 1.1

1. Sheila is planning to visit relatives in England and Spain. On the day that she wants to buy the currencies for her trip, one euro costs \$1.50 and one British pound costs \$2.00. What combinations of these currencies can Sheila buy for \$700? Use three different strategies to show the possible combinations.

2. After a fundraiser, the treasurer for a minor soccer league invested some of the money in a savings account that paid 2.5%/year and the rest in a government bond that paid 3.5%/year. After one year, the money earned \$140 in interest. Define two variables, write an equation, and draw a graph for this information.

Lesson 1.2

3. Gary drove his pickup truck from Cornwall to Chatham. He left Cornwall at 8:15 a.m. and drove at a steady 100 km/h along Highway 401. The graph below shows how the fuel in the tank varied over time.



a) What do the coordinates of the point (5, 25.5) tell you about the amount of fuel?
b) How much fuel was in the tank at 11:45 a.m.?
c) The low fuel warning light came on when 6 L of fuel remained. At what time did this light come on?

4. Readycars charges \$59/day plus \$0.14/km to rent a car. Bestcars charges \$69/day plus \$0.11/km. Describe three different strategies you could use to compare these two rental rates. What advice would you give someone who wants to rent a car from one of these companies?

Lesson 1.3

5. Solve each linear system graphically.
a) $x + y = 2$
$x = 2y + 2$
b) $y - x = 1$
$2x - y = 1$

6. Tools-R-Us rents snowblowers for a base fee of \$20 plus \$8/h. Randy's Rentals rents snowblowers for a base fee of \$12 plus \$10/h.
a) Create an equation that represents the cost of renting a snowblower from Tools-R-Us.
b) Create the corresponding equation for Randy's Rentals.
c) Solve the system of equations graphically.
d) What does the point of intersection mean in this situation?

Lesson 1.4

7. Use substitution to solve each system.
a) $2x + 3y = 7$
$-2x - 1 = y$
b) $3x - 4y = 5$
$x - y = 5$
c) $5x + 2y = 18$
$2x + 3y = 16$
d) $9 = 6x - 3y$
$4x - 3y = 5$

8. Courtney paid a one-time registration fee to join a fitness club. She also pays a monthly fee. After three months, she had paid \$315. After seven months, she had paid \$535. Determine the registration fee and the monthly fee.

9. A rectangle has a perimeter of 40 m. Its length is 2 m greater than its width.

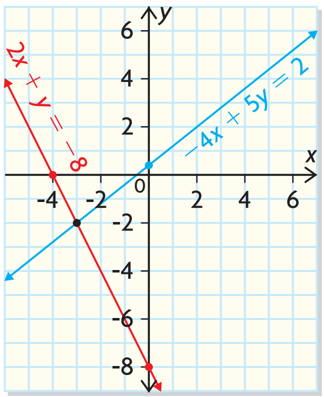
a) Represent this situation with a linear system.

b) Solve the linear system using substitution.

c) What do the numbers in the solution represent? Explain.

Lesson 1.5

10. a) Which linear system below is equivalent to the system that is shown in the graph?



A. $2x - 5y = 4$
$-x + y = 1$

B. $x - 3y = -1$
$2x + y = 4$

b) Use addition and subtraction to create another linear system that is equivalent to the system in the graph.

c) Use multiplication to create another linear system that is equivalent to the system in the graph.

11. a) Create two linear systems that are equivalent to the following system.

$-2x - 3y = 5$
$3x - y = 9$

b) Verify that all three systems have the same solution.

Lesson 1.6

12. Use elimination to solve each linear system.

a) $2x - 3y = 13$
$5x - y = 13$

b) $x - 3y = 0$
$3x - 2y = -7$

c) $3x + 21 = 5y$
$4y + 6 = -9x$

d) $x - \frac{1}{3}y = -1$
$\frac{2}{3}x - \frac{1}{4}y = -1$

13. Lyle needs 200 g of chocolate that is 86% cocoa for a cake recipe. He has one kind of chocolate that is 99% cocoa and another kind that is 70% cocoa. How much of each kind of chocolate does he need to make the cake? Round your answer to the nearest gram.

14. A Grade 10 class is raising money for a school-building project in Uganda. To buy 35 desks and 3 chalkboards, the students need to raise \$2082. To buy 40 desks and 2 chalkboards, they need to raise \$2238. Determine the cost of a desk and the cost of a chalkboard.

15. Solve the linear system.
$2(2x - 1) - (y - 4) = 11$
$3(1 - x) - 2(y - 3) = -7$

16. Juan is a cashier at a variety store. He has a total of \$580 in bills. He has 76 bills, consisting of \$5 bills and \$10 bills. How many of each type does he have?

17. a) Sketch a linear system that has no solution.

b) Determine two possible equations that could represent both lines in your sketch.

c) Explain how the slopes of these lines are related.

18. The linear system $6x + 5y = 10$ and $ax + 2y = b$ has an infinite number of solutions. Determine a and b.

1 Chapter Self-Test

Number of 500 g Cartons	Number of 750 g Bags
50	1150
100	1117
150	1083
200	1050
:	:
:	:
1000	517
1500	183
:	:

Process Checklist

✔ Questions 1 and 2: Did you **connect** and compare the **representations**?

✔ Questions 5 and 9: Did you **communicate** your thinking clearly?

✔ Questions 7 and 8: Did you **problem solve** by selecting and applying appropriate strategies?

1. The Rainin' Raisins company packs raisins in 500 g cartons and 750 g bags. One day, 887.5 kg of raisins were packed into full cartons and bags. Ben wondered how many cartons and how many bags could have been packed. He began to chart some possible combinations shown at the left. Use a graph and an equation to show all the possible combinations.

2. Milk is leaking from a carton at a rate of 4 mL/min. There is 1500 mL of milk in the carton at 8:30 a.m.

a) Write an equation and draw a graph for this situation.

b) Determine graphically when 1 L of milk will be left in the carton.

c) Use your equation to determine algebraically when 1 L of milk will be left in the carton.

3. Solve each system of equations. Use a different strategy for each system.

a) $3x + y = -2$
$5x - y = -10$

b) $2x + 7 = y$
$-3x - 2y = 10$

c) $3x + 5y = -9$
$2x - y = 7$

4. Shannon needs 20 g of 80% gold to make a pendant. She has some 85% gold and some 70% gold, from broken jewellery, and wants to know how much of each she should use. Determine the quantity of each alloy that she should use.

5. How would you explain to someone why it makes sense that you can add two equations and subtract them to create an equivalent system of linear equations?

6. a) Create two linear systems that are equivalent to the system $3x + 4y = -8$ and $x - 2y = 9$.

b) Verify that all three systems are equivalent.

7. In his spare time, Kim likes to go cycling. He cycles partly on paved surfaces and partly off-road, through hilly and wooded areas. He cycles at 25 km/h on paved surfaces and at 10 km/h off-road. One day, he cycled 41 km in 2 h. How far did he cycle off-road?

8. Last summer, Betty earned $4200 by painting houses. She invested some of the money in a savings account that paid 3.5%/year and the rest in a government bond that paid 4.5%/year. After one year, she has earned $174 in interest. How much did she invest at each rate?

9. Explain how you know that this system of equations has no solution.

$$15 - 6y = 9x$$
$$3x + 2y = 8$$

1

Chapter Task

Coefficient Clues

Each morning, Stuart adds 250 g of fruit to his yogurt as a source of vitamin C. Today, he wants to use one of the following combinations of two fruits:

- pear and pineapple
- banana and blueberry
- apple and cranberry

Fruit	Amount of Vitamin C in 1 g of Fruit (mg)
apple	0.15
banana	0.10
blueberry	0.10
cherry	0.10
cranberry	0.15
grapefruit	0.40
kiwi	0.70
mango	0.53
orange	0.49
pear	0.04
pineapple	0.25
strawberry	0.60

Health Connection

Researchers claim that vitamin C can help prevent colds, heart disease, and cancer.

? How can Stuart determine which 250 g fruit combinations will provide 25 mg of vitamin C?

A. Write a system of linear equations to model each of today's possible fruit combinations.

B. Solve each system.

C. How many solutions does each system have?

D. Examine at least three other fruit combinations using the data in the table. Repeat parts A to C to determine if any of these combinations will provide 25 mg of vitamin C.

E. Describe the 250 g fruit combinations that will provide 25 mg of vitamin C.

Task Checklist

- ✔ Did you label all your graphs?
- ✔ Did you answer all the questions completely?
- ✔ Did you check your answers?
- ✔ Did you explain your thinking clearly?