Chapter 7

Similar Triangles and Trigonometry

GOALS

You will be able to

- Determine and compare properties of congruent and similar triangles
- Solve problems using similar triangles
- Determine side lengths and angle measures in right triangles using the primary trigonometric ratios
- Solve problems using right triangle models and trigonometry

? The distance from Earth to the Sun can only be measured indirectly. What are some ways that you can measure indirectly?

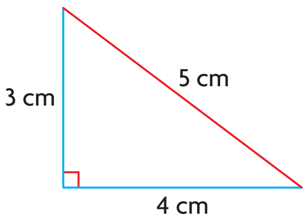
7 Getting Started

WORDS YOU NEED to Know

1. Match each term with the example or diagram that best represents it.

a) ratio **b)** proportion **c)** congruent triangles **d)** Pythagorean theorem **e)** hypotenuse **f)** acute angle

i)



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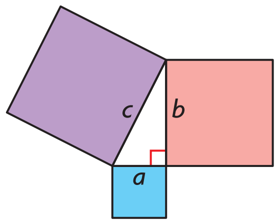

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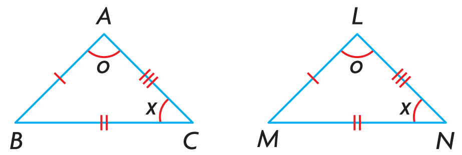
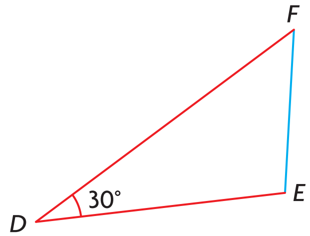


ii) $\frac{3}{5}$ or 3:5 or 3 to 5

iv) $\frac{x}{5} = \frac{3}{15}$

vi)



SKILLS AND CONCEPTS You Need

Solving a Proportion

Study Aid

- For more help and practice, see Appendix A-14.

To solve a proportion, you need to determine the missing value that will result in an equivalent ratio.

EXAMPLE

Solve each proportion.

a) $\frac{x}{3} = \frac{7}{2}$

b) $\frac{2}{c} = \frac{5}{9.5}$

Solution

a)

$$\frac{x}{3} = \frac{7}{2}$$

$$3\left(\frac{x}{3}\right) = 3\left(\frac{7}{2}\right)$$ ← *x* is divided by 3, so multiply both sides by 3.

$$x = \frac{21}{2}$$

$$x = 10.5$$

b)

$$\frac{2}{c} = \frac{5}{9.5}$$

$$c\left(\frac{2}{c}\right) = c\left(\frac{5}{9.5}\right)$$ ← 2 is divided by c, so multiply both sides by c.

$$2 = \frac{5c}{9.5}$$

$$9.5(2) = 9.5\left(\frac{5c}{9.5}\right)$$ ← Multiply both sides by 9.5.

$$19 = 5c$$ ← Divide both sides by 5.

$$3.8 = c$$

2. Solve each proportion.

a) $\frac{x}{4} = \frac{9}{36}$ **c)** $\frac{2}{7} = \frac{5}{c}$ **e)** $\frac{a}{5.2} = \frac{7.8}{12.0}$

b) $\frac{2}{5} = \frac{b}{20}$ **d)** $\frac{2d}{9} = \frac{12}{4}$ **f)** $\frac{4.5}{y} = \frac{5.4}{2.4}$

Applying the Pythagorean Theorem

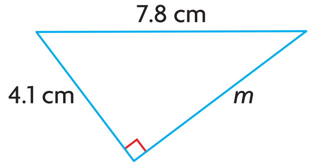
The Pythagorean theorem can be used to calculate an unknown side length in a right triangle if the other two side lengths are known.

Study Aid

- For more help and practice, see Appendix A-4.

EXAMPLE

Determine the length of side m.



Solution

$c = 7.8$ — Identify the hypotenuse, c.

$4.1^2 + m^2 = 7.8^2$ — Substitute the given information into $a^2 + b^2 = c^2$. Square the numbers.

$16.81 + m^2 = 60.84$

$m^2 = 60.84 - 16.81$ — Solve for m^2. Take the square root of both sides.

$m^2 = 44.03$

$m = \sqrt{44.03}$

$m \doteq 6.6$ — Round the value of m to one decimal place.

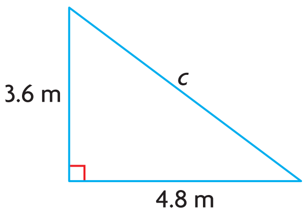
The length of side m is about 6.6 cm.

Communication Tip

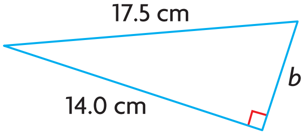
When calculating side lengths and angle measures, round your answers to the same number of decimal places as the given information when the required degree of accuracy is not stated.

3. Determine each unknown side length.

a)



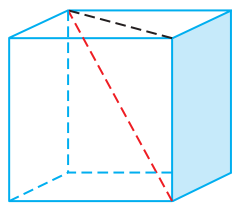
b)



4. The side length of a sugar cube is 2.0 cm. Calculate each distance to the nearest tenth.

a) from corner to corner across the top

b) from the top corner to the bottom corner



Study **Aid**

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
6	A-15
7, 8	A-16

PRACTICE

5. Express each ratio in simplest terms.

a) $10:14$

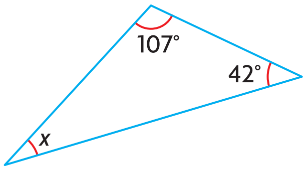
b) $\frac{7.5}{15}$

c) $-8:2$

d) $\frac{27}{36}$

6. In each diagram, determine the measure of x.

a)



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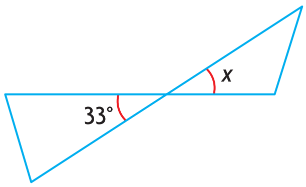

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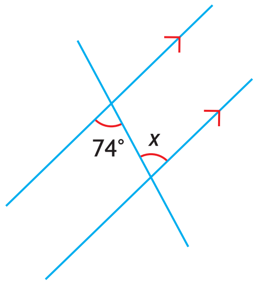

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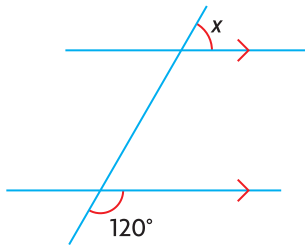
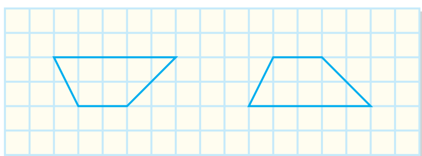
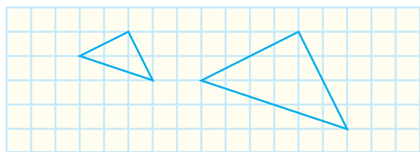


7. State whether the two figures in each pair appear **congruent** or **similar**. Explain how you know.

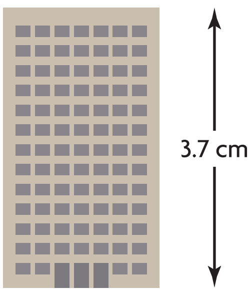
a)



b)



8. $\triangle ABC$ has two 50° angles. What other piece of information do you need to construct a triangle that is congruent to $\triangle ABC$?



9. The scale of the building in the diagram at the left is 1:1100. Calculate the actual height of the building.

10. Complete each sentence as many ways as you can.

a) You find equal angles when . . .

b) Angles add up to 180° when . . .

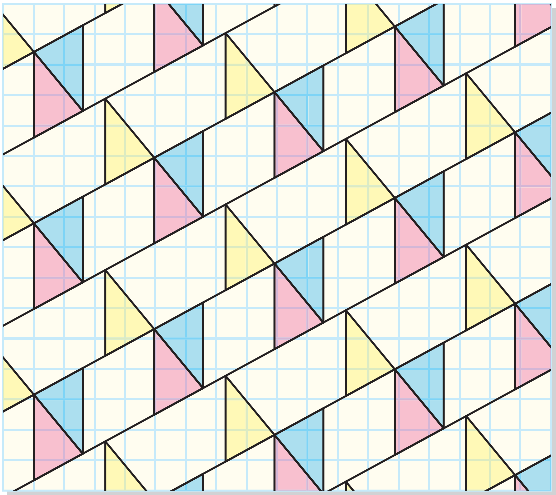
APPLYING What You Know

Which Triangles Are the Same?

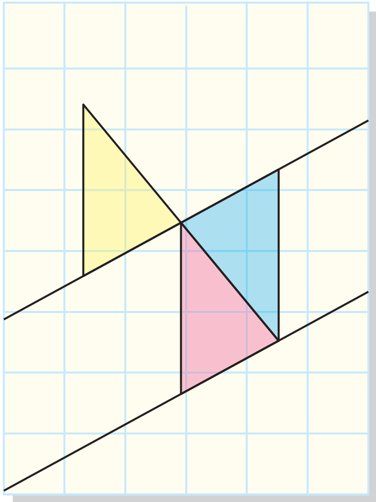
YOU WILL NEED

- grid paper
- protractor
- ruler
- coloured pencils

Clara has part of a quilt pattern. She wants to know if the different-coloured triangles are identical.



quilt pattern



enlarged section

? Which triangles in the pattern are congruent?

A. Draw the enlarged section. Label the vertices of the three triangles.

B. Measure the lengths of all the sides in the enlarged section shown above. Record these lengths on your sketch.

C. Are any lines in the quilt pattern parallel? Explain how you know, and indicate these lines on your sketch.

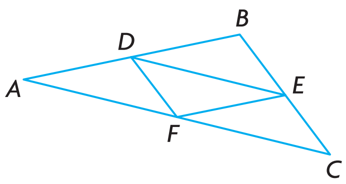
D. Determine pairs of equal angles in each pair of triangles. Mark the equal angles on your sketch.
- **i)** yellow triangle and blue triangle
- **ii)** yellow triangle and pink triangle
- **iii)** blue triangle and pink triangle

E. Which triangles are congruent? Explain how you know.

7.1 Congruence and Similarity in Triangles

YOU WILL NEED

- dynamic geometry software, or ruler and protractor



GOAL

Investigate the relationships between corresponding sides and angles in pairs of congruent and similar triangles.

INVESTIGATE the Math

Colin is a graphic artist. He is creating a logo for a client. He knows that four new triangles are created when the midpoints of the sides in a triangle are joined.

? What are the relationships among the four new triangles in Colin's design?

A. Construct a triangle, and mark the midpoint of each side.

B. Join the midpoints with line segments and label all the vertices, as shown in Colin's design.

C. Measure all the angles in each of the four small triangles. Measure all the sides by determining the distances between vertices. What do you notice?

D. Are the four small triangles congruent? Explain how you know.

E. Measure all the angles and sides in the large triangle. Compare these measurements with those for the small triangles. What do you notice?

F. Are there **similar triangles** in Colin's design? If so, identify them. Explain how you know they are similar.

G. Explain why it makes sense that the **scale factor** that relates each small triangle to the large triangle is $\frac{1}{2}$.

H. Drag one of the vertices in the large triangle.
  i) Are the four small triangles still congruent?
  ii) Are the small and large triangles still similar? If they are, does the scale factor change?

I. Repeat part H several times by choosing different vertices to drag.

Tech | *Support*

For help constructing triangles, determining midpoints, measuring angles and sides, and calculating using dynamic geometry software, see Appendix B-25, B-30, B-26, B-29, and B-28.

similar triangles

triangles in which corresponding sides are proportional; similar triangles are enlargements or reductions of each other

scale factor

the value of the ratio of corresponding side lengths in a pair of similar figures

Reflecting

J. Are all congruent triangles similar? Are all similar triangles congruent? Explain.

K. Suppose that you started with $\triangle XYZ$ and used a scale factor of 2 to create a similar triangle, $\triangle X'Y'Z'$. What would the scale factor be if you started with $\triangle X'Y'Z'$ and created $\triangle XYZ$? Explain your thinking.

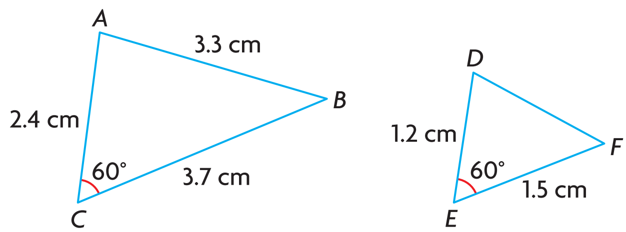
L. Suppose that you measured two pairs of corresponding angles in two triangles and discovered that they were equal. Which of these conclusions could you make? Explain.

i) All the corresponding angles are equal.
ii) The triangles are similar.
iii) The triangles are congruent.

APPLY the Math

EXAMPLE 1 | Reasoning about congruence and similarity

a) Create a triangle that is similar but not congruent to $\triangle ABC$.
b) Is $\triangle DEF$ similar to $\triangle ABC$?

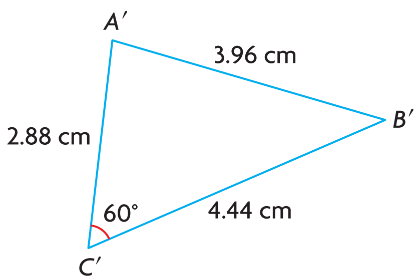


Gary's Solution

a) $A'C' \rightarrow 1.2 \times 2.4 = 2.88$ cm
$A'B' \rightarrow 1.2 \times 3.3 = 3.96$ cm
$B'C' \rightarrow 1.2 \times 3.7 = 4.44$ cm

I had to create either a larger triangle or a smaller triangle with the same angles.

I named the vertices in my new triangle *A*′, *B*′, and *C*′. I multiplied each side length in $\triangle ABC$ by 1.2 to determine the corresponding side lengths in the new larger triangle.



Since *C*′ corresponds to *C*, $\angle A'C'B'$ must measure 60°. I constructed *A′C′* and measured a 60° angle. Then I constructed *B′C′* and joined *A*′ to *B*′. I measured *A′B′* to check that it matched my calculated value.

b) $\dfrac{AC}{DE} = \dfrac{2.4}{1.2} = 2$

$\dfrac{CB}{EF} = \dfrac{3.7}{1.5} \doteq 2.47$

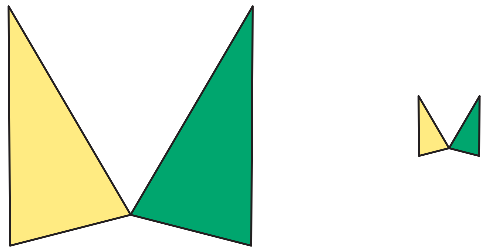
$\angle C$ and $\angle E$ are corresponding angles in the two triangles. If the triangles are similar, then their corresponding sides are proportional.

The sides are not proportional, so the triangles are not similar. $\angle F$ looks a little greater than $\angle B$, and $\angle D$ looks a little less than $\angle A$.

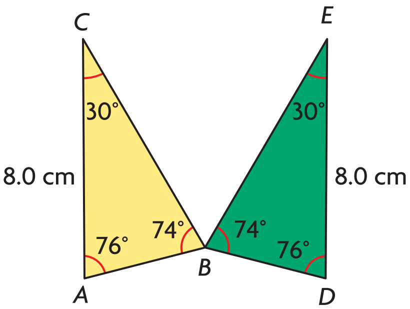
If the angles are different, it makes sense that one triangle cannot be an enlargement of the other.

EXAMPLE 2 Connecting similar triangles with the scale factor

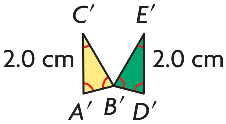
Calculate the scale factor that relates the side lengths of the large logo to the side lengths of the small one.



Monica's Solution



I named the vertices in each triangle. Then I measured all the angles and the long sides in the triangles in both logos.



$\angle A' = \angle D' = 76°$
$\angle C' = \angle E' = 30°$
$\angle A'B'C' = \angle D'B'E' = 74°$

Corresponding angles in the yellow triangles are equal. Corresponding angles in the green triangles are equal. Therefore, the yellow triangles are similar, and so are the green triangles.

Showing that there are two pairs of equal angles is enough to conclude that the triangles are similar, since the third pair of angles must also be equal. Since there are two pairs of equal angles, the triangles are similar.

$\triangle ABC \sim \triangle A'B'C'$ and $\triangle DBE \sim \triangle D'B'E'$

As well as equal pairs of corresponding angles, the yellow and green triangles contain a pair of equal corresponding sides. The large yellow and green triangles are congruent. The small yellow and green triangles are also congruent.

In similar triangles, when two corresponding sides are equal, the ratio of all the corresponding sides is 1. When the scale factor is 1, the triangles are congruent.

$\triangle ABC \cong \triangle DBE$ and $\triangle A'B'C' \cong \triangle D'B'E'$

$$\text{Scale factor} = \frac{A'C'}{AC} = \frac{2.0}{8.0} \text{ or } \frac{1}{4}$$

I calculated the ratio of the corresponding sides in the similar triangles. This calculation is the same for both the yellow triangles and the green triangles.

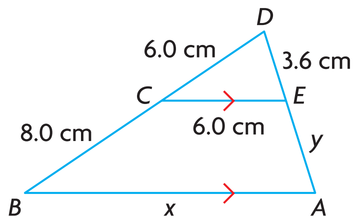
The sides of the large logo have been multiplied by a scale factor of $\frac{1}{4}$ to reduce it to the small one.

Communication | ***Tip***

The symbol $\sim$ is used to indicate similarity. The symbol $\cong$ is used to indicate congruence. When naming triangles that are congruent or similar, the corresponding vertices must be listed in the same order. For example, if $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$, then $\triangle ABC \sim \triangle DEF$.

EXAMPLE 3 | **Reasoning** about similar triangles to determine side lengths

Show that the two triangles in this diagram are similar. Then determine the values of x and y.



Jake's Solution

$\angle BAD = \angle CED$
$\angle ABD = \angle ECD$
So, $\triangle ABD \sim \triangle ECD$.

Since AB and EC are parallel, the corresponding angles in the small and large triangles are equal.

$$\frac{BD}{CD} = \frac{AB}{EC}$$
$$\frac{14.0}{6.0} = \frac{x}{6.0}$$
$$6.0\left(\frac{14.0}{6.0}\right) = 6.0\left(\frac{x}{6.0}\right)$$
$$14.0 = x$$

To calculate x, I set up a proportion using the corresponding sides I knew and the side I needed to determine in the similar triangles. Then I solved for x.

$$\frac{AD}{ED} = \frac{BD}{CD}$$
$$\frac{y + 3.6}{3.6} = \frac{14.0}{6.0}$$
$$3.6\left(\frac{y + 3.6}{3.6}\right) = 3.6\left(\frac{7.0}{3.0}\right)$$
$$y + 3.6 = 8.4$$
$$y = 4.8$$

To calculate y, I set up another proportion using the corresponding sides I knew and a side that contains y in the similar triangles. Then I solved for y.

The value of x is 14.0 cm, and the value of y is 4.8 cm.

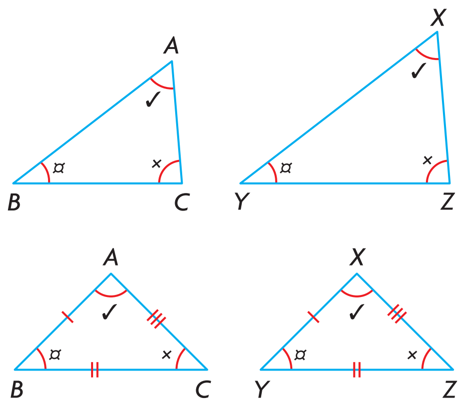
In Summary

Key Ideas

- If two triangles are congruent, then they are also similar. If two triangles are similar, however, they are not always congruent.
- If two pairs of corresponding angles in two triangles are equal, then the triangles are similar. In addition, if two corresponding sides are equal, then the triangles are congruent.

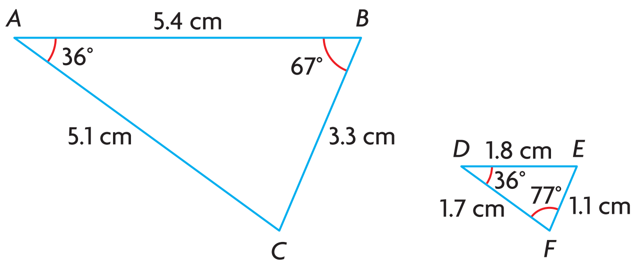
Need to Know

- If $\angle A = \angle X$, $\angle B = \angle Y$, and $\angle C = \angle Z$, then $\triangle ABC \sim \triangle XYZ$ and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$.
- If $\angle A = \angle X$, $\angle B = \angle Y$, and $\angle C = \angle Z$, and if $AB = XY$ or $BC = YZ$ or $AC = XZ$, then $\triangle ABC \cong \triangle XYZ$.
- When comparing similar triangles, if the scale factor is
  - greater than 1, the larger triangle is an enlargement of the smaller triangle
  - between 0 and 1, the smaller triangle is a reduction of the larger triangle
  - 1, the triangles are congruent



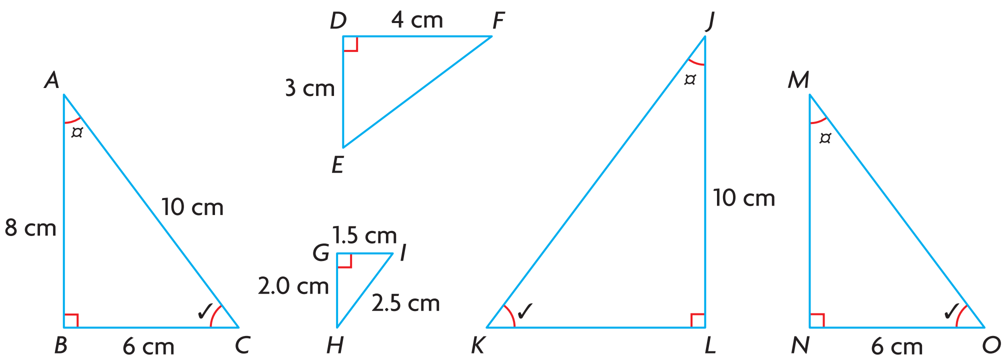
CHECK Your Understanding

1. Is $\triangle ABC \sim \triangle DEF$? Justify your answer.



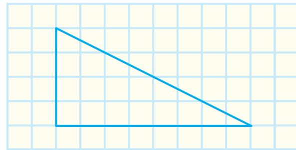
2. a) Which triangle is congruent to $\triangle ABC$?

b) Which triangles are similar to, but not congruent to, $\triangle ABC$?



3. a) Use grid paper to draw the triangle at the right. Then enlarge it by a factor of 3.

b) Reduce the original triangle by a factor of $\frac{1}{2}$.



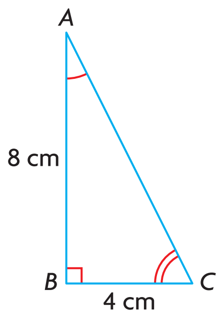
PRACTISING

4. i) For each pair of right triangles, determine whether the triangles are congruent, similar, or neither.

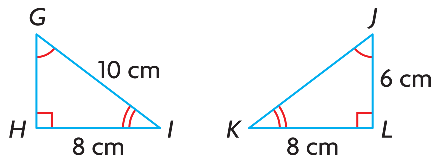
K

ii) If the triangles are congruent, identify the corresponding angles and sides that are equal. If the triangles are similar, identify the corresponding angles that are equal, and calculate the scale factor that relates the smaller triangle as a reduction of the larger triangle.

a)



c)



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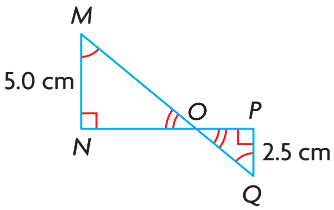
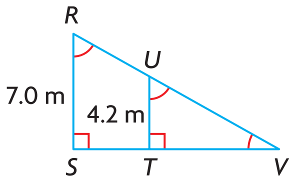
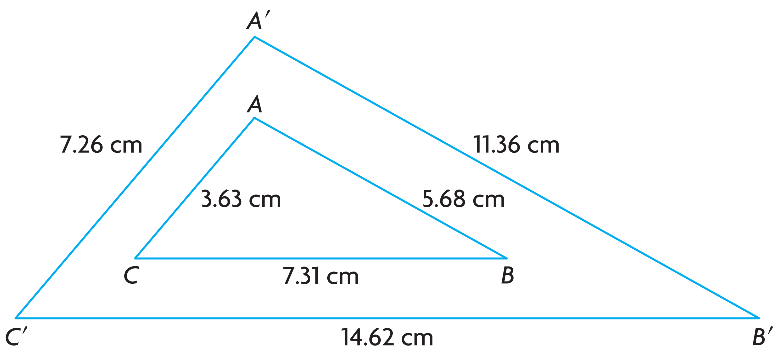


d)



5. Are these two triangles similar? Explain how you know.



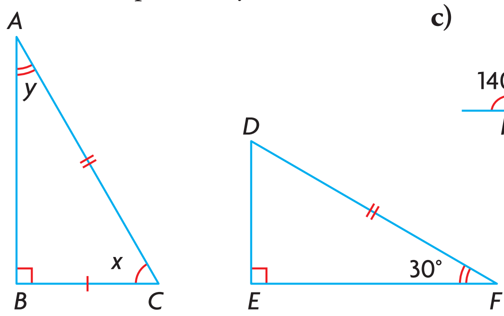
6. Suppose that $\triangle PQR \sim \triangle LMN$ and $\angle P = 90°$.

a) What angle in $\triangle LMN$ equals 90°? How do you know?

b) If $MN = 13$ cm, $LN = 12$ cm, $LM = 5$ cm, and $PQ = 15$ cm, what are the lengths of PR and QR?

7. Determine the value of each lower-case letter. If you cannot determine a value, explain why.

a)



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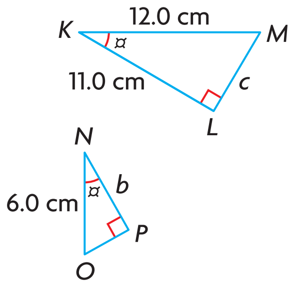
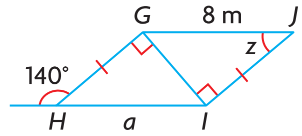
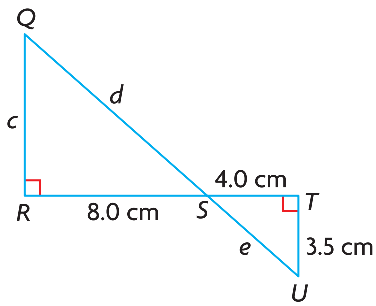


c)

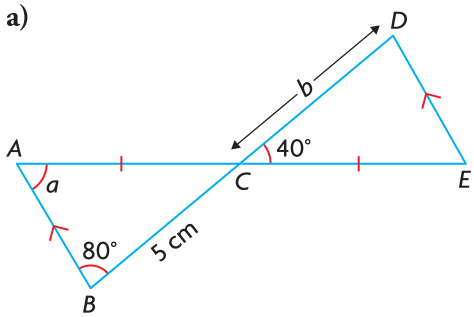


d)

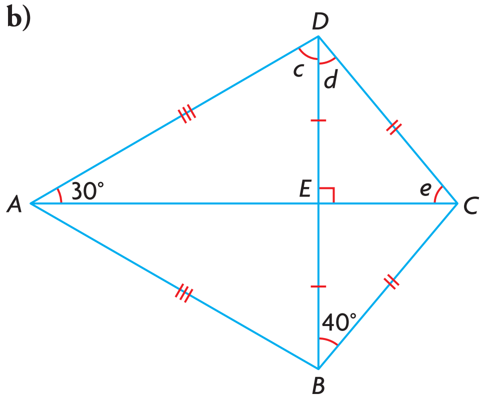


8. Determine the value of each lower-case letter.

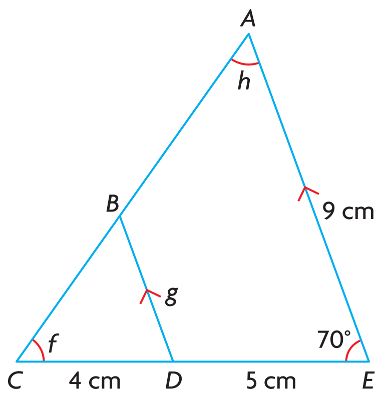
a)



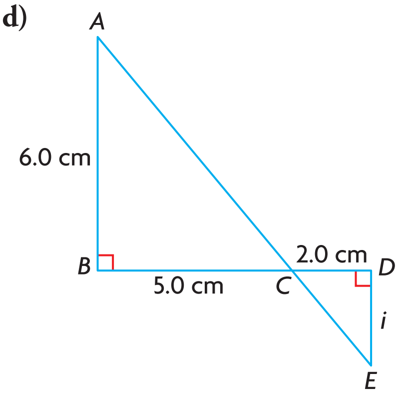
b)



c)



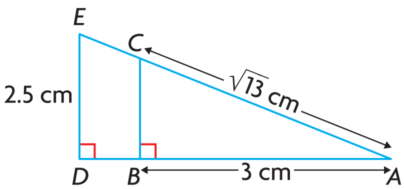
d)



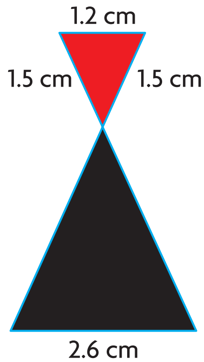
9. Draw three different triangles. Only two of the triangles must be similar.

10. Which type(s) of triangles will always be similar: right, isosceles, or equilateral?

11. Determine the length of *DB*.
T



12. An environmental club is designing a logo using triangles, as shown
A at the right. If the top and bottom lines of the logo at the right are parallel, determine the perimeter of the logo.



13. A tree that is 3 m tall casts a shadow that is 2 m long. At the same time, a nearby building casts a shadow that is 25 m long. How tall is the building?

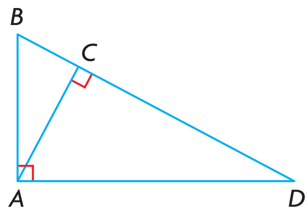
14. If two isosceles triangles have one pair of equal angles, are they similar?
C Explain.

15. Create a flow chart to summarize your thinking process when you are determining whether two triangles are congruent or similar.

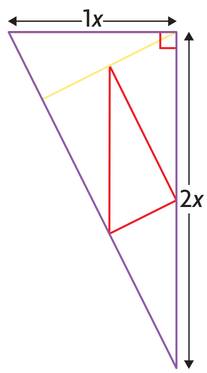
Extending

16. Follow these steps to prove the Pythagorean theorem using properties of similar triangles:

- First show that the two smaller triangles at the right are similar to the larger triangle.
- Then use the ratios of corresponding sides to prove the theorem.



17. A right triangle can be partitioned into five smaller congruent triangles, which are all similar to the original right triangle, as shown at the right. Determine how this can be done.

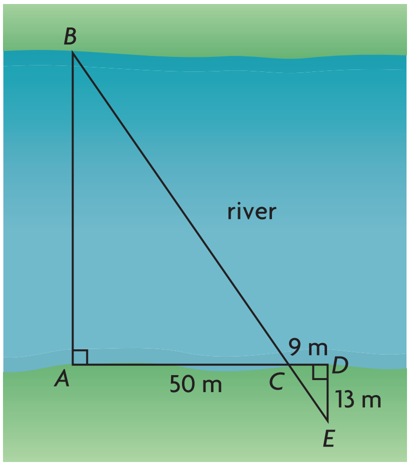


18. This trapezoid is an example of self-similarity: the trapezoid is made of four smaller similar trapezoids. Create your own example of self-similarity, using a different quadrilateral.

7.2 Solving Similar Triangle Problems

YOU WILL NEED

- ruler



GOAL

Solve problems using similar triangle models.

LEARN ABOUT the Math

A new bridge is going to be built across a river, but the width of the river cannot be measured directly. Surveyors set up posts at points A, B, C, D, and E. Then they took measurements relative to the posts.

? What is the width of the river?

EXAMPLE 1 **Selecting a similar triangle strategy to solve a problem**

Use the surveyors' measurements to determine the width of the river.

Marnie's Solution

$\angle BAC$ and $\angle EDC$ both equal 90°.
$\angle ACB = \angle DCE$
So, $\triangle ABC \sim \triangle DEC$.

$\triangle ABC$ and $\triangle DEC$ are right triangles. $\angle ACB$ and $\angle DCE$ are opposite angles. Since two pairs of corresponding angles in the triangles are equal, the triangles are similar.

$$\frac{AB}{DE} = \frac{AC}{DC}$$
$$\frac{AB}{13} = \frac{50}{9}$$

I set up a proportion to determine AB, the width of the river.

$$13\left(\frac{AB}{13}\right) = 13\left(\frac{50}{9}\right)$$
$$AB \doteq 72.2$$

I solved for AB.

The width of the river is about 72 m.

Reflecting

A. Why do you think the surveyors set up the posts the way they did?

B. Why do you think the surveyors measured two sides in $\triangle DEC$ but only one side in $\triangle ABC$?

C. What are the benefits of using similar triangles in this situation?

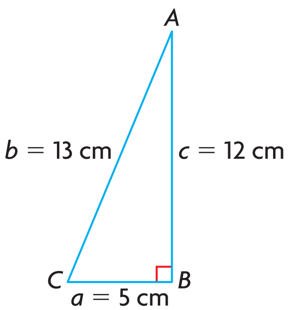
APPLY the Math

EXAMPLE 2 Solving a problem using a scale diagram

Andrea is a landscape designer. She is working on a backyard that is in the shape of a right triangle. She needs to cover the yard with sod and then fence the yard. She starts by drawing a scale diagram using the scale 1 cm represents 6.25 m. She marks the dimensions of the yard on her drawing as 5 cm, 12 cm, and 13 cm. A roll of sod covers about 0.93 m^2. How many rolls of sod does Andrea need? What length of fencing does she need?

Jordan's Solution

Let the lengths of the yard be a, b, and c.



I drew a scale diagram like Andrea's using the given information.

I named each side using the same letter as the opposite vertex, but in lower-case. Since I know that the yard is a right triangle, the longest side must be opposite the 90° angle.

$$\frac{1 \text{ cm}}{6.25 \text{ m}} \rightarrow \frac{1 \text{ cm}}{625 \text{ cm}}$$

I converted 6.25 m in the scale to centimetres, using 100 cm = 1 m. This allowed me to calculate all the dimensions in centimetres.

$a = (5)(625)$ $\quad c = (12)(625)$

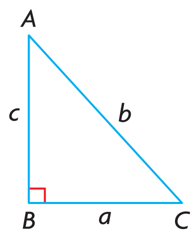
$a = 3125$ $\quad c = 7500$

Therefore, a is 3125 cm and c is 7500 cm.

To calculate the area of the yard, I needed the length of the base (side a) and the height (side c). I used my scale factor to calculate these dimensions of the actual yard.

Communication *Tip*

The vertices of a triangle are usually labelled with upper-case letters. The side that is opposite each angle is labelled with the corresponding lower-case letter.



$$A = \frac{1}{2}bh$$

I calculated the area of the yard.

$$A = \left(\frac{1}{2}\right)(ac)$$
$$= \frac{1}{2}(3125)(7500)$$
$$= 11\,718\,750$$
$$= 11\,718\,750 \div (100 \times 100)$$
$$= 1171.875$$

Since the area covered by a roll of sod is given in square metres, I converted the area of the yard into square metres. To do this, I divided by the area of a 100 cm by 100 cm square, which is 1 m^2.

The area is about 1172 m^2.

$$\text{Number of rolls} = 1172 \div 0.93$$
$$\doteq 1260.2$$

Andrea needs 1261 rolls of sod.

I divided the area of the yard by 0.93 to determine the number of rolls that Andrea needs. I rounded up since you can't buy part of a roll.

$$b = (13)(625)$$
$$= 8125$$

I used the scale factor to calculate the length of side *b*.

The length of side b is 8125 cm.

$$P = a + b + c$$
$$= 3125 + 8125 + 7500$$
$$= 18\,750$$
$$= 18\,750 \div 100$$
$$= 187.5$$

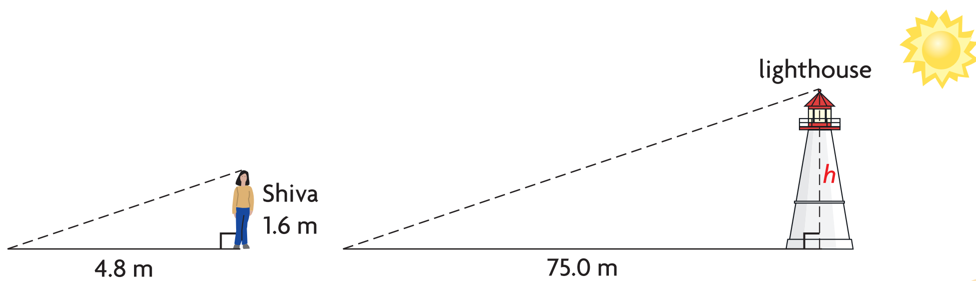
The perimeter is the sum of *a*, *b*, and *c*.

I divided my answer by 100 to convert the perimeter to metres.

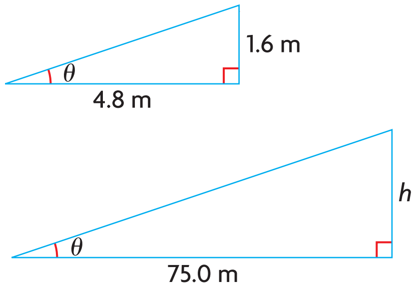
Andrea needs 187.5 m of fencing.

EXAMPLE 3 **Connecting** similar triangles to objects and their shadows

Shiva is standing beside a lighthouse on a sunny day, as shown. She measures the length of her shadow and the length of the shadow cast by the lighthouse. Shiva is 1.6 m tall. How tall is the lighthouse?



Ken's Solution



First, I had to show that the triangles are similar.

The **angle of elevation**, θ, of the Sun is equal in both diagrams. I assumed that both Shiva and the lighthouse were perpendicular to the ground, so both triangles are right triangles.

h is the height of the lighthouse.

The triangles are similar because two pairs of corresponding angles are equal.

$$\frac{h}{1.6} = \frac{75.0}{4.8}$$

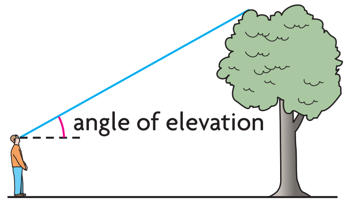
$$h = 1.6\left(\frac{75.0}{4.8}\right)$$

$$h = 25.0$$

The lighthouse is 25.0 m tall.

I set up a proportion of corresponding side lengths in the two triangles. Then I solved my proportion to calculate the height of the lighthouse.

angle of elevation (angle of inclination)
the angle between the horizontal and the line of sight when looking up at an object



Communication *Tip*

The symbols θ and α are the Greek letters *theta* and *alpha*. These symbols are often used to represent the measure of an unknown angle.

In Summary

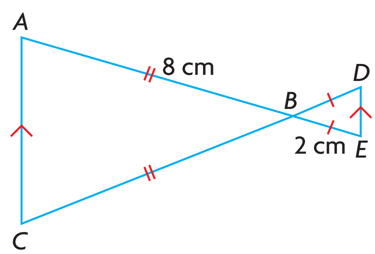
Key Idea

- Similar triangles can be used to determine lengths that cannot be measured directly. This strategy is called indirect measurement.

Need to Know

- The ratios of the corresponding sides in similar triangles can be used to write a proportion. Unknown side lengths can be determined by solving the proportion.
- If $\triangle ABC \sim \triangle DEF$ and the scale factor is $n = \frac{AB}{DE}$, then the length of any side in $\triangle ABC$ equals n multiplied by the length of the corresponding side in $\triangle DEF$.

CHECK Your Understanding



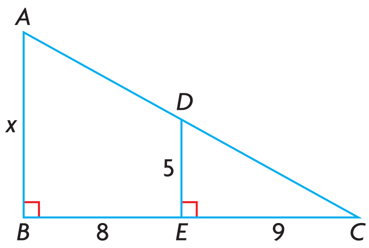
1. a) Explain why you can conclude that $\triangle ACB \sim \triangle EDB$ in the diagram at the right.

b) Determine the scale factor that relates these triangles.

2. Miki is standing in a parking lot on a sunny day. He is 1.8 m tall and casts a shadow that is 5.4 m long.

a) Draw a scale diagram that can be used to measure the angle at which the Sun's rays hit the ground.

b) Determine the length of the shadow cast by a nearby tree that is 12.2 m tall.



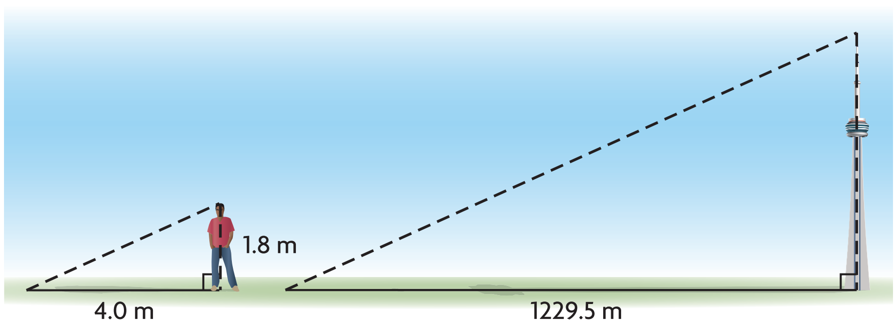
3. a) Identify the two similar triangles in the diagram at the left. Explain how you know that these triangles are similar.

b) Determine the value of x.

PRACTISING

4. Brian is standing near the CN Tower on a sunny day.

K **a)** Brian's height and his shadow form the perpendicular sides of a right triangle. What does the hypotenuse represent?



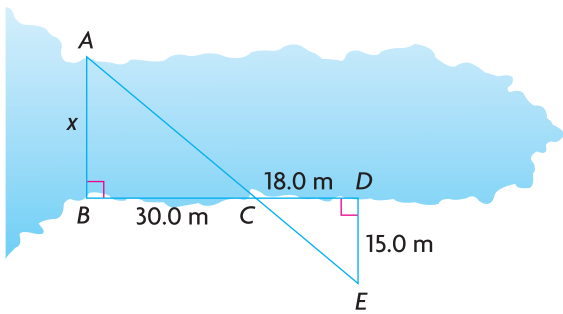
b) The CN Tower casts a shadow that is 1229.5 m long. Explain why the triangle representing the height of the CN Tower and its shadow is similar to the triangle representing Brian's height and his shadow.

c) Determine the scale factor that relates the two triangles.

d) Use the scale factor to determine the height of the CN Tower.

5. How wide is this bay?

A

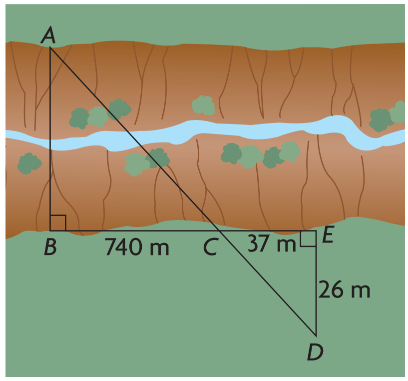


6. On June 30, 1859, Jean François Gravelet crossed the Niagara Gorge on a tightrope. Since he could not measure the distance across the gorge directly to determine the length of rope he would need, he used indirect measurement.

a) Explain why $\triangle DEC$ is similar to $\triangle ABC$.

b) What ratio might Gravelet have used to determine the scale factor of the two triangles?

c) Calculate the distance across the gorge.



7. A 3.6 m ladder is leaning against a wall, with its base 2 m from the wall.

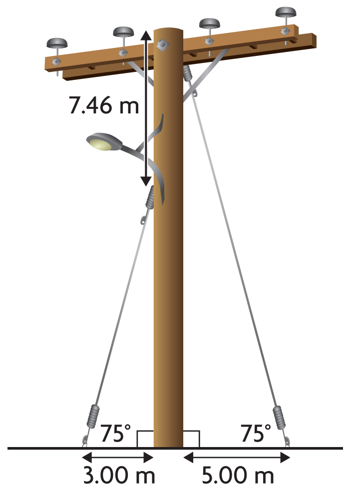
C **a)** Draw a scale diagram to represent this situation.

b) Suppose that a 2.4 m ladder is placed against the wall, parallel to the longer ladder. Explain why the triangles that are formed by the ground, the wall, and the two ladders are similar.

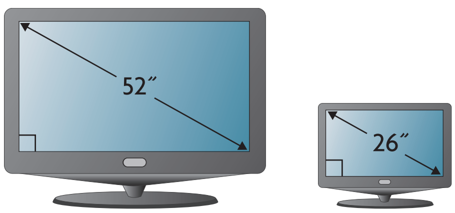
c) How far up the wall will each ladder reach?

8. Tyler, who is 1.8 m tall, is walking away from a lamppost that is 5.0 m tall. When Tyler's shadow measures 2.3 m, how far is he from the lamppost?

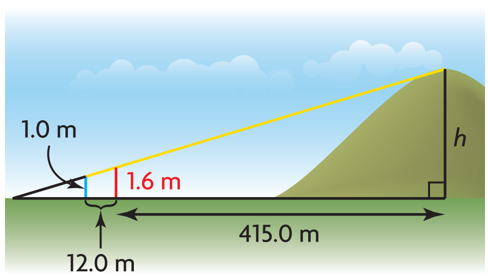
9. A telephone pole is supported by a guy wire, as shown in the diagram at the right, which is anchored to the ground 3.00 m from the base of the pole. The guy wire makes a 75° angle with the ground and is attached to the pole 7.46 m from the top. Another guy wire is attached to the top of the pole. This guy wire also makes an angle of 75° with the ground 5.00 m from the base of the pole. Determine the height of the pole.



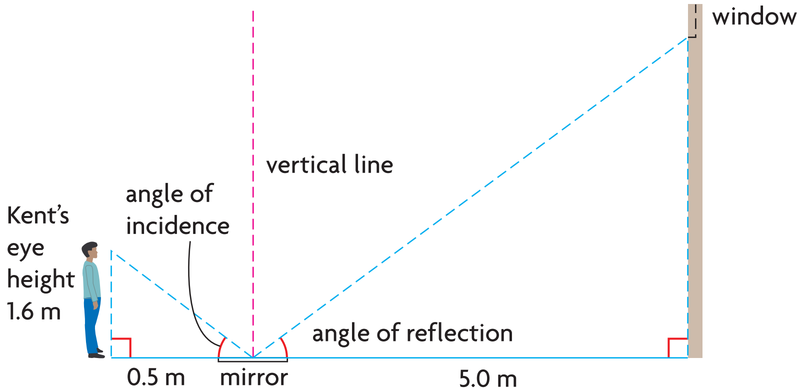
10. The salesclerk at TV-Rama says that the area of a 52 in. plasma screen is four times as large as the area of a 26 in. screen. Television screens are measured on the diagonal. Is the salesclerk correct? Explain.



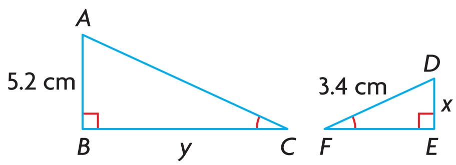
11. Surveyors need to determine the height of a hill. They set
T up a laser measuring device on a pole that is 1.0 m tall and shine the laser toward the top of a second pole, which is 1.6 m tall. Then they adjust the distance between the two poles until the laser hits the top of the longer pole and the top of the hill. The 1.6 m pole is 415.0 m from the centre of the hill. The two poles are 12.0 m apart. Determine the height of the hill.



12. Kent uses a mirror to determine the height of Julie's window. He knows that the angle of incidence equals the angle of reflection when light is reflected off a mirror. How high is the window?



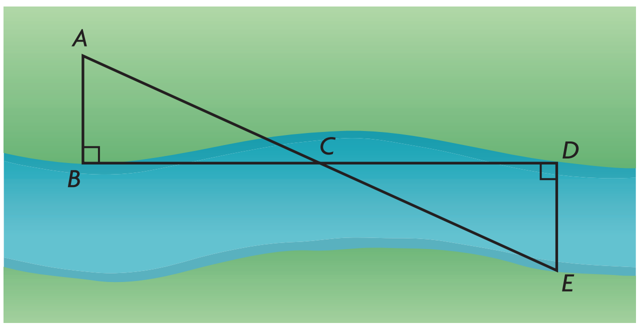
13. Two lengths in two similar triangles are given.



a) Which lengths do you need to know, other than x and y, to determine x and y?
b) Explain how you would use these other lengths to determine x and y.

Extending

14. Determine the width of this river, if $AB = 96$ m, $AC = 204$ m, and $BD = 396$ m.



15. A square photo, with an area of 324.00 cm^2, is in a square frame that has an area of 142.56 cm^2. The dimensions of the photo and the frame are proportional.
a) Determine the scale factor that relates the dimensions of the photo and the frame.
b) Determine the width of the frame.

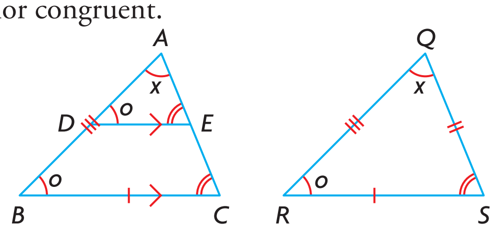
7 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How do you know whether two triangles are similar, congruent, or neither?

A: Since congruent triangles are the same size and shape, all the corresponding side lengths and angle measures are equal. Since similar triangles are the same shape but different sizes, the corresponding angles are equal and the corresponding side lengths are proportional. If the corresponding angles of two triangles are not equal, then the triangles are neither similar nor congruent.

$\triangle ADE \sim \triangle ABC$
$\triangle ADE \sim \triangle QRS$
$\triangle ABC \cong \triangle QRS$



Study Aid

- See Lesson 7.1, Examples 1 and 2.
- Try Mid-Chapter Review Question 1.

Q: How can the properties of similar triangles be used to calculate an unknown side length in a triangle?

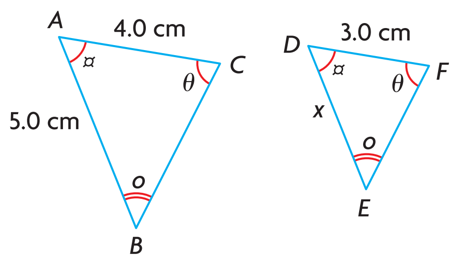
A: After you determine that two triangles are similar, you can set up a proportion using corresponding sides. Solving the proportion gives the unknown side length.

Study Aid

- See Lesson 7.1, Example 3, and Lesson 7.2, Examples 1 to 3.
- Try Mid-Chapter Review Questions 3, and 5 to 10.

EXAMPLE

$\triangle ABC \sim \triangle DEF$. Determine the value of x.



Solution

$$\frac{AB}{DE} = \frac{AC}{DF}$$

AB corresponds to *DE*, and *AC* corresponds to *DF*. Since the triangles are similar, the ratios of these sides are equal.

$$\frac{5.0}{x} = \frac{4.0}{3.0}$$

Solve for *x*.

$$x\left(\frac{5.0}{x}\right) = x\left(\frac{4.0}{3.0}\right)$$

$$5.0 = \frac{4.0x}{3.0}$$

$$(3.0)(5.0) = 4.0x$$

$$\frac{15.0}{4.0} = x$$

$$3.75 = x$$

x is about 3.8 cm.

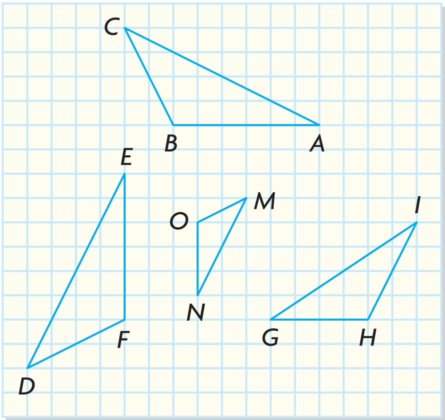
PRACTICE Questions

Lesson 7.1

1. Name the triangles that are

a) congruent to $\triangle ABC$

b) similar to $\triangle ABC$



2. $\triangle ABC \sim \triangle RST$. Complete each statement.

a) $\angle ABC = \blacksquare$

b) $\angle BCA = \blacksquare$

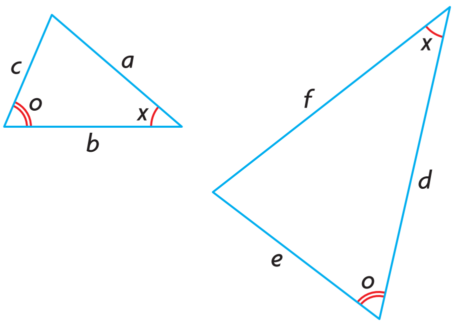
c) $\dfrac{AB}{RS} = \blacksquare$

d) $\triangle STR \sim \blacksquare$

e) $\dfrac{ST}{BC} = \blacksquare$

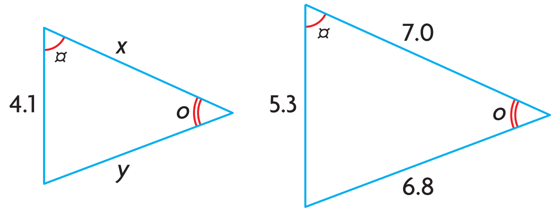
f) $\angle SRT = \blacksquare$

3. Write a proportion for the corresponding side lengths in these similar triangles.



4. Peter says, "If you know the measures of two angles in each of two triangles, you can always determine if the triangles are similar." Is this statement true or false? Explain your reasoning.

5. Determine the values of x and y.

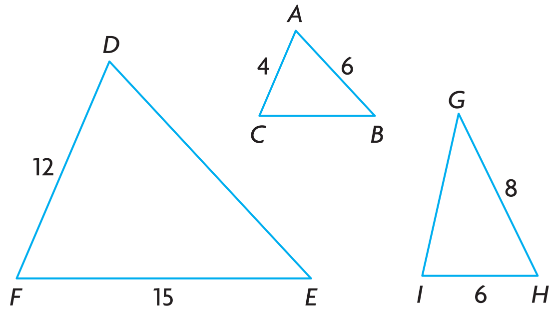


6. For $\triangle ABC \sim \triangle DEF$:

a) Determine the length of BC.

b) Determine the length of DE.

c) Is $\triangle GHI \sim \triangle DEF$? Explain.



Lesson 7.2

7. Nora, who is 172.0 cm tall, is standing near a tree. Nora's shadow is 3.2 m long. At the same time, the shadow of the tree is 27.0 m long. How tall is the tree?

8. A right triangle has side lengths 6 cm, 8 cm, and 10 cm. The longest side of a larger similar triangle measures 15 cm. Determine the perimeter and area of the larger triangle.

9. Connie placed a mirror on the ground, 5.00 m from the base of a flagpole. She stepped back until she could see the top of the flagpole reflected in the mirror. Connie's eyes are 1.50 m above the ground and she saw the reflection when she was 1.25 m from the mirror. How tall is the flagpole?

10. Cam is designing a new flag for his hockey team. The flag will be triangular, with sides that measure 0.8 m, 1.2 m, and 1.0 m. Cam has created a scale diagram, with sides that measure 20 cm, 30 cm, and 25 cm, to take to a flag maker. Did Cam create his scale diagram correctly?

7.3 Exploring Similar Right Triangles

GOAL

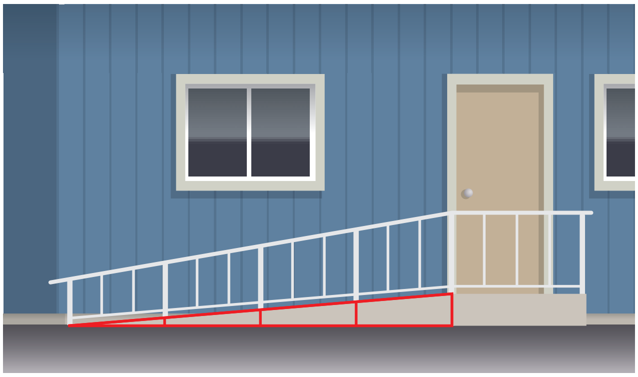
Explore the connection between the ratios of the sides in the same triangle for similar triangles.

YOU WILL NEED

- dynamic geometry software, or ruler and protractor

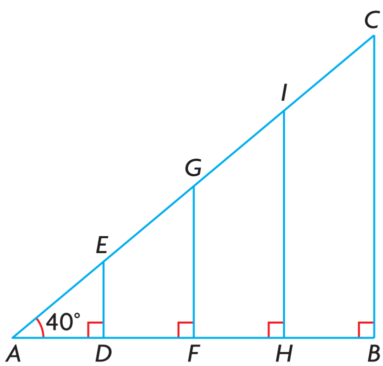
EXPLORE the Math

Mark noticed a common design element in the wheelchair ramp that was installed at his home and the skateboard ramp at a park. The ground, the ramp, and the vertical supports formed a series of right triangles.



? How are the ratios of the sides in similar right triangles related?

A. Using dynamic geometry software, or a protractor and a ruler, construct a diagram like the one shown. Make sure that $\angle A = 40°$. Also make sure that all the vertical lines are perpendicular to AB.

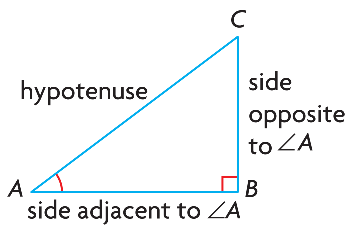


B. Explain why the four right triangles in your diagram are similar.

C. For each triangle in your diagram, measure the lengths of the **opposite side** and **adjacent side**, as well as the hypotenuse. Record these values in a table like the one at the top of the next page. Calculate each ratio to two decimal places.

opposite side

the side that is directly across from a specific acute angle in a right triangle; for example, BC is the opposite side in relation to $\angle A$



adjacent side

the side that is part of an acute angle in a right triangle, but is not the hypotenuse; for example, AB is the adjacent side in relation to $\angle A$ above.

Triangle	Side Opposite to $\angle A$	Side Adjacent to $\angle A$	Hypotenuse	Trigonometric Ratios		
				$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
$\triangle ABC$	$BC =$ ■	$AB =$ ■	$AC =$ ■	$\frac{BC}{AC} =$ ■	$\frac{AB}{AC} =$ ■	$\frac{BC}{AB} =$ ■
$\triangle ADE$						
$\triangle AFG$						
$\triangle AHI$						

Tech | ***Support***

For help constructing triangles, measuring angles and sides, and calculating using dynamic geometry software, see Appendix B-25, B-26, B-29, and B-28.

D. Describe the relationships in your table.

E. Do you think the relationships you described for part D would change if $\angle A$ were changed to a different measure? Make a conjecture, and then test your conjecture by creating a new diagram using a different acute angle for $\angle A$. Use the measure of this angle to repeat part C.

F. Compare your results with your classmates' results. Summarize what you discovered.

Reflecting

G. In each triangle, the ratio $\frac{\text{opposite}}{\text{adjacent}}$ is equivalent to the slope of AC. Explain why.

H. Did the relationships involving the ratios of the sides in similar right triangles depend on the size of $\angle A$? Explain.

In Summary

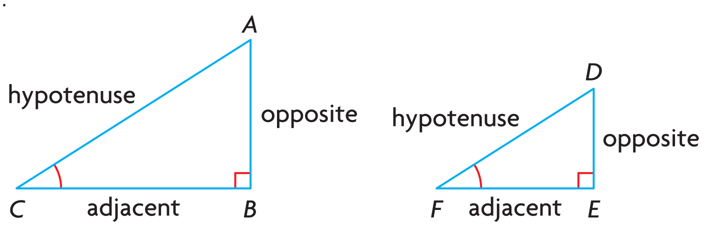
Key Idea

- In similar right triangles, the following ratios are equivalent for the corresponding acute angles:

$$\frac{\text{opposite}}{\text{adjacent}} \rightarrow \frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{\text{opposite}}{\text{hypotenuse}} \rightarrow \frac{AB}{AC} = \frac{DE}{DF}$$

$$\frac{\text{adjacent}}{\text{hypotenuse}} \rightarrow \frac{BC}{AC} = \frac{EF}{DF}$$

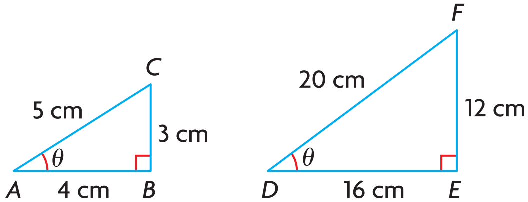


Need to Know

- The slope of a line segment is related to the angle that the line segment makes with the x-axis.
- If two lines make the same angle with the x-axis, they have the same slope.

FURTHER Your Understanding

1. a) For each triangle, state the opposite side and adjacent side to θ and the hypotenuse.



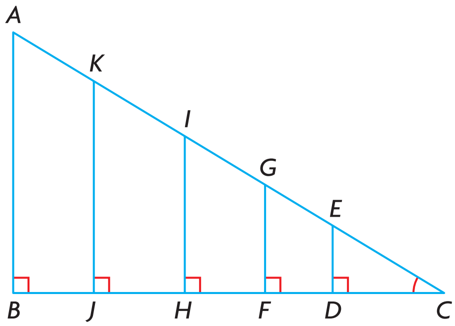
b) For $\triangle ABC \sim \triangle DEF$, show that these ratios are equal for θ in both triangles.

i) opposite : hypotenuse

ii) adjacent : hypotenuse

iii) opposite : adjacent

2. a) Identify the triangles that are similar to $\triangle ABC$.



b) State each ratio for all the triangles. Use $\angle C$ when identifying the opposite and adjacent sides.

i) opposite : hypotenuse

ii) adjacent : hypotenuse

iii) opposite : adjacent

c) State all the ratios for part b) that are equal.

3. a) Part of a road rises 8 m over a run of 120 m. What is the rise over a run of 50 m if the slope remains constant?

b) Compare the slopes for part a). Explain why these slopes are the same.

4. A moving truck has two ramps, 3 m long and 4 m long, for loading and unloading. Which ramp creates a greater angle of elevation with the ground? Include a diagram in your answer.

7.4 The Primary Trigonometric Ratios

GOAL

Determine the values of the sine, cosine, and tangent ratios for a specific acute angle in a right triangle.

LEARN ABOUT the Math

Nadia wants to know the slope of a ski hill. Her trail map shows that the hill makes an angle of 18° with the horizontal. Her friend suggests that she use **trigonometry** to calculate the slope.

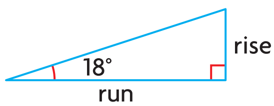
trigonometry
the branch of mathematics that deals with the properties of triangles and calculations based on these properties

? How can the 18° angle be used to determine the slope of the ski hill?

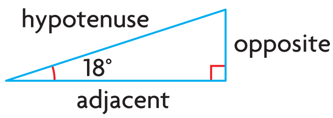
EXAMPLE 1 **Connecting** an angle to the ratios of the sides in a right triangle

Use the angle of the ski hill to determine the slope of the ski hill.

Nadia's Solution



I drew a diagram to represent this situation.



I knew that the $\frac{\text{rise}}{\text{run}}$ ratio is the same in any right triangle with an 18° angle. So I divided the length of the side opposite to the 18° angle by the length of the side adjacent to it.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 18° = \frac{\text{rise}}{\text{run}}$$

I knew that I could describe the ratio $\frac{\text{opposite}}{\text{adjacent}}$ using **tangent**.

$$\tan 18° \doteq 0.3249$$

I used my calculator to determine tan 18°.

The slope of the ski hill is approximately 0.32.

Communication *Tip*
Opposite and adjacent sides are named relative to a specific acute angle in a triangle.

tangent
the ratio of the length of the opposite side to the length of the adjacent side for either acute angle in a right triangle; the abbreviation for tangent is *tan*

Tech *Support*
When using a scientific calculator to calculate the tangent ratio, first make sure that you are in degree mode.

Reflecting

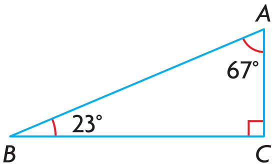
A. How did Nadia use the properties of similar triangles when she used the tangent ratio to solve the problem?

B. Why did Nadia use the tangent of the 18° angle instead of the 72° angle to solve the problem?

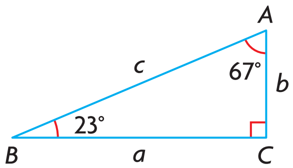
APPLY the Math

EXAMPLE 2 **Connecting** each trigonometric ratio to an acute angle

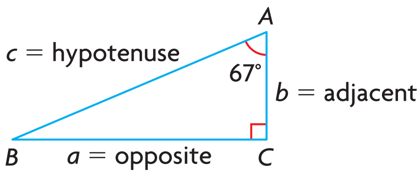
Determine the values of the tangent, **sine**, and **cosine** ratios for $\angle A$ and $\angle B$ to four decimal places.



Paula's Solution



First, I labelled each side using the lower-case letter that matched the angle opposite the side.



I named the sides relative to $\angle A$ as opposite, adjacent, and hypotenuse.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$
$$\sin 67° \doteq 0.9205$$
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$
$$\cos 67° \doteq 0.3907$$
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$
$$\tan 67° \doteq 2.3559$$

I determined the three **primary trigonometric ratios** for $\angle A$.

To four decimal places, sin A is 0.9205, cos A is 0.3907, and tan A is 2.3559.

sine

the ratio of the length of the opposite side to the length of the hypotenuse for either acute angle in a right triangle; the abbreviation for sine is *sin*

cosine

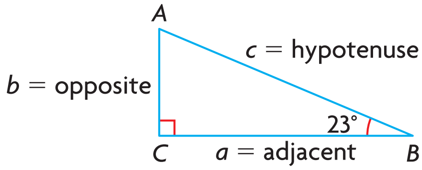
the ratio of the length of the adjacent side to the length of the hypotenuse for either acute angle in a right triangle; the abbreviation for cosine is *cos*

primary trigonometric ratios

the basic ratios of trigonometry (sine, cosine, and tangent)

Tech *Support*

When using a scientific calculator to calculate the primary trigonometric ratios, press SIN, COS, or TAN and then enter the angle, or enter the angle and then press SIN, COS, or TAN, depending on your calculator.



I renamed the sides relative to ∠B as opposite, adjacent, and hypotenuse.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c}$$
$$\sin 23° \doteq 0.3907$$
$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c}$$
$$\cos 23° \doteq 0.9205$$
$$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$
$$\tan 23° \doteq 0.4245$$

I determined the three primary trigonometric ratios for ∠B.

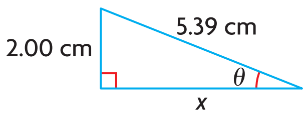
To four decimal places, sin B is 0.3907, cos B is 0.9205, and tan B is 0.4245.

Communication *Tip*

When necessary, trigonometric ratios are usually expressed with four decimal places of accuracy. This is done to calculate the angles with enough precision. Carrying as many digits as possible until the final step in a calculation reduces the possibility of variations in answers due to rounding.

EXAMPLE 3 **Connecting** an acute angle to the sides in a right triangle

Determine the measure of θ to the nearest degree, using each primary trigonometric ratio.



Jim's Solution

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin \theta = \frac{2.00}{5.39}$$

I started by determining θ using the sine ratio. If I knew the angle, its sine would be $\frac{2.00}{5.39}$.

$$\theta = \sin^{-1}\left(\frac{2.00}{5.39}\right)$$
$$\theta \doteq 21.8°$$
$$\theta \doteq 22°$$

Since I knew the sine and not the angle, I had to use the inverse of sine, which is $\sin^{-1}$.

$$x^2 + 2.00^2 = 5.39^2$$
$$x^2 + 4.00 = 29.0521$$
$$x^2 = 29.0521 - 4.00$$
$$x^2 = 25.0521$$
$$x = \sqrt{25.0521}$$
$$x \doteq 5.01$$

Since the tangent and cosine ratios both involve the adjacent side, x, I calculated the length of this side using the Pythagorean theorem.

inverse

the reverse of an original statement; for example, if $x = \sin \theta$, the inverse is $\theta = \sin^{-1} x$

Tech *Support*

When using a scientific calculator to calculate the inverse sine, press [2ND] [SIN] and enter the ratio, or enter the ratio and press [2ND] [SIN].

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{5.01}{5.39}$$

$$\theta = \cos^{-1}\left(\frac{5.01}{5.39}\right)$$

$$\theta \doteq 21.6°$$

$$\theta \doteq 22°$$

To determine the angle using the cosine ratio, I had to use the inverse of cosine, $\cos^{-1}$. My answer is the same as the one I calculated using sine.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{2.00}{5.01}$$

$$\theta = \tan^{-1}\left(\frac{2.00}{5.01}\right)$$

$$\theta \doteq 21.8°$$

$$\theta \doteq 22°$$

To determine the angle using the tangent ratio, I had to use the inverse of tangent, $\tan^{-1}$.

All the primary trigonometric ratios gave me the same answer. Next time, I know that I only have to use one of them to determine θ.

Tech *Support*

For help using a TI-83/84 graphing calculator to calculate trigonometric ratios and determine angles, see Appendix B-12. If you are using a TI-*n*spire, see Appendix B-48.

In Summary

Key Ideas

- The primary trigonometric ratios for $\angle A$ are $\sin A$, $\cos A$, and $\tan A$.
- If $\angle A$ is one of the acute angles in a right triangle, the primary trigonometric ratios can be determined using the ratios of the sides.

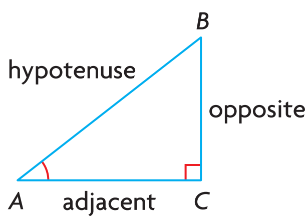
Need to Know

- If $\angle A$ is one of the acute angles in a right triangle, the three primary trigonometric ratios for $\angle A$ can be written as

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

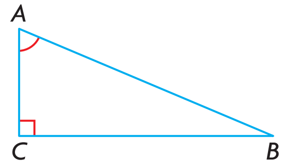
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



- Using the Pythagorean theorem, $\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2$ in any right triangle.

CHECK Your Understanding



1. a) Which side is opposite to $\angle A$?
b) Which side is adjacent to $\angle A$?
c) Which side is the hypotenuse?

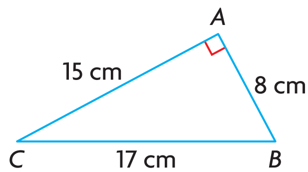
2. Determine the value of each ratio to four decimal places.
a) $\tan 34°$ **b)** $\sin 78°$ **c)** $\cos 49°$ **d)** $\sin 12°$

3. Determine the measure of θ to the nearest degree.
a) $\sin \theta = 0.5$ **c)** $\cos \theta = 0.5$
b) $\tan \theta = 1$ **d)** $\sin \theta = 0.8660$

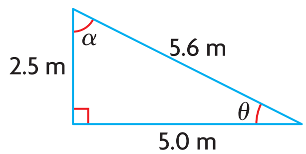
PRACTISING

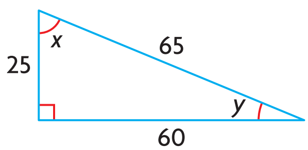
4. $\triangle XYZ$ is a right triangle, with $\angle X = 90°$.
a) Sketch $\triangle XYZ$. Label the sides using lower-case letters.
b) Write the ratios for $\sin Y$, $\cos Y$, and $\tan Y$ in terms of x, y, and z.

5. Determine each ratio, and write it as a decimal to four decimal places.
a) $\sin C$ **d)** $\tan C$
b) $\cos C$ **e)** $\cos B$
c) $\tan B$ **f)** $\sin B$



6. Decide whether each statement is true or false. Justify your decision.
a) $\sin \theta = 0.4$
b) $\tan \alpha = 2$
c) $\cos \alpha \doteq 0.8929$
d) $\cos \theta \doteq 0.8929$



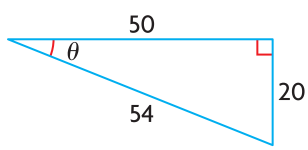


7. a) Calculate the measure of x in the diagram at the left to the nearest degree, using one of the primary trigonometric ratios.
b) Do you need to use a primary trigonometric ratio to determine the measure of y? Explain.

8. Solve for x, and express your answer to one decimal place.

a) $\cos 45° = \frac{x}{6}$ **c)** $\tan 75° = \frac{x}{20}$ **e)** $\cos 60° = \frac{15}{x}$

b) $\sin 62° = \frac{x}{14}$ **d)** $\tan 80° = \frac{12}{x}$ **f)** $\sin 45° = \frac{10}{x}$

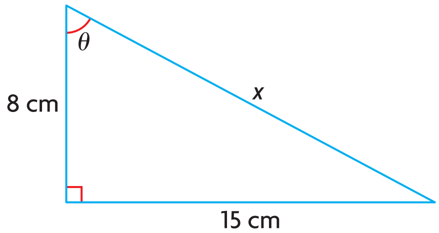


9. Identify the primary trigonometric ratio for θ that is equal to each
K ratio for the triangle at the left.

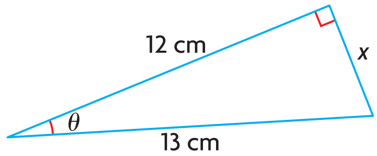
a) $\frac{50}{54}$ **b)** $\frac{20}{50}$ **c)** $\frac{20}{54}$

10. Determine the length of x. Then state the primary trigonometric ratios for θ.
A

a)



b)



11. Determine the measure of θ, to one decimal place, for each triangle in question 10.

12. Determine the measure of θ, to one decimal place.

a) $\sin \theta = \frac{2}{5}$

b) $\cos \theta = \frac{4}{9}$

c) $\tan \theta = 3$

d) $\sin \theta = \frac{1}{2}$

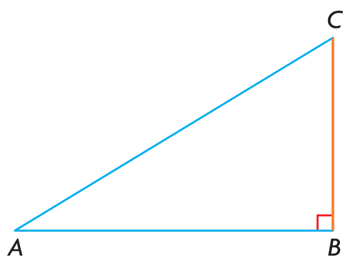
13. Does $\cos 60° = \frac{1}{2}$ mean that the side adjacent to the 60° angle measures 1 unit and the hypotenuse measures 2 units? Explain.
C

14. Draw two different right triangles for which $\tan \theta = 1$. Determine the measurements of all the sides and angles. Then compare the two triangles.
T

15. **a)** Could the orange side in $\triangle ABC$ at the right be considered an adjacent side when determining a trigonometric ratio? Explain.

b) Could the orange side be considered an opposite side when determining a trigonometric ratio? Explain.

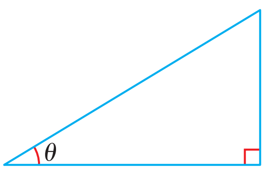
c) Could the orange side be considered the hypotenuse when determining a trigonometric ratio? Explain.



Extending

16. For what value of θ does $\sin \theta = \cos \theta$? Include a diagram in your answer.

17. Explain why the value of $\tan \theta$ increases as the measure of θ increases.



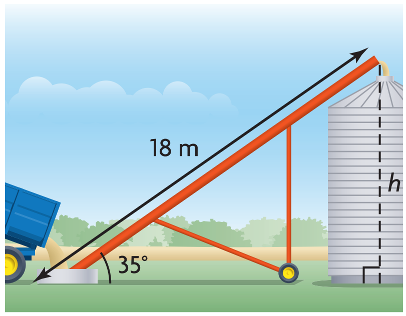
7.5 Solving Right Triangles

GOAL

Use primary trigonometric ratios to calculate side lengths and angle measures in right triangles.

LEARN ABOUT the Math

A farmers' co-operative wants to buy and install a grain auger. The auger would be used to lift grain from the ground to the top of a silo. The greatest angle of elevation that is possible for the auger is 35°. The auger is 18 m long.

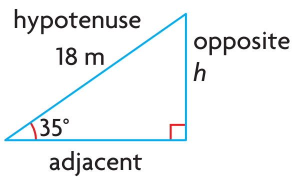


? What is the maximum height that the auger can reach?

EXAMPLE 1 **Solving a problem** for a side length using a trigonometric ratio

Calculate the maximum height that the auger can reach.

Hong's Solution



I drew a diagram to model the problem. The height is the length of the side that is opposite the 35° angle. I named the other sides relative to the 35° angle.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 35° = \frac{h}{18}$$

Because I knew the length of the hypotenuse, I used the sine ratio. The sine of 35° equals the opposite side, or height, divided by the hypotenuse.

$$18\,(\sin 35°) = 18\left(\frac{h}{18}\right)$$

$$18\,(\sin 35°) = h$$

$$10 \doteq h$$

I multiplied both sides by 18 and evaluated 18 (sin 35°) using a calculator. I rounded my answer using the degree of accuracy in the other measures.

The maximum height that the auger can reach is about 10 m.

Tech | *Support*

For help using a TI-83/84 graphing calculator to calculate trigonometric ratios, see Appendix B-12. If you are using a TI-*n*spire, see Appendix B-48.

Reflecting

A. If the height of the grain auger is increased, what happens to the sine, cosine, and tangent ratios for the angle of elevation? Explain.

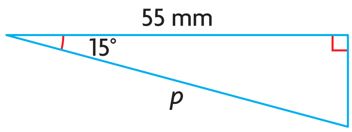
B. Why can you use either the sine ratio or the cosine ratio to calculate the maximum height?

C. Explain why Hong might have chosen to use the sine ratio.

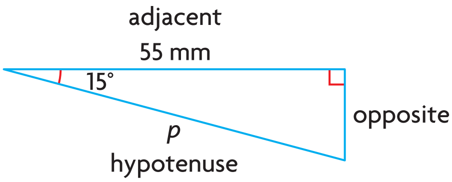
APPLY the Math

EXAMPLE 2 Connecting the cosine ratio with the length of the hypotenuse

Determine the length of p.



Mandy's Solution



The 55 mm side is adjacent to the 15° angle. I named the rest of the sides in the triangle relative to the 15° angle.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 15° = \frac{55}{p}$$

Because I knew the adjacent side and had to determine the hypotenuse, I used the cosine ratio. The cosine of 15° equals the adjacent side divided by the hypotenuse.

$$p(\cos 15°) = p\left(\frac{55}{p}\right)$$

$$p(\cos 15°) = 55$$

$$\frac{p(\cos 15°)}{\cos 15°} = \frac{55}{\cos 15°}$$

$$p = \frac{55}{\cos 15°}$$

$$p \doteq 57$$

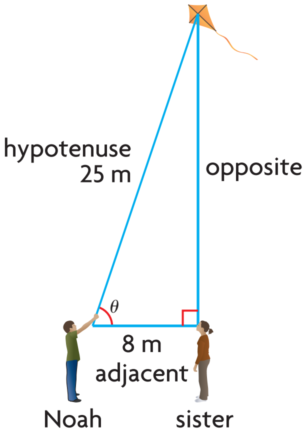
I multiplied both sides by p. Then I divided both sides by cos 15° to solve for p. I rounded my answer to the nearest millimetre.

The length of p is about 57 mm long.

EXAMPLE 3 **Connecting** the cosine ratio with an angle measure

Noah is flying a kite and has released 25 m of string. His sister is standing 8 m away, directly below the kite. What is the angle of elevation of the string?

Jacob's Solution



I drew a right triangle showing Noah, his sister, and the kite. I labelled the triangle with the information that I knew and named the sides of the triangle relative to the angle of elevation.

$$\cos\theta = \frac{8}{25}$$

Because I knew the lengths of the adjacent side and the hypotenuse, I used the cosine ratio.

$$\theta = \cos^{-1}\left(\frac{8}{25}\right)$$

I used the inverse cosine to determine the angle.

$$\theta \doteq 71°$$

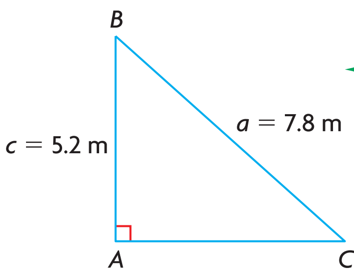
I rounded my answer to the nearest degree.

The angle of elevation of the string is about 71°.

EXAMPLE 4 **Selecting a trigonometric strategy** to solve a triangle

Solve $\triangle ABC$, given $\angle A = 90°$, $a = 7.8$ m, and $c = 5.2$ m.

Chloe's Solution



I drew the triangle and labelled it with the measurements that I knew.

Communication *Tip*

To solve a triangle means to determine all unknown angle measures and side lengths.

$$\cos B = \frac{c}{a}$$

I started with $\angle B$. Since side c is adjacent to $\angle B$ and side a is the hypotenuse, I used the cosine ratio.

$$\cos B = \frac{5.2}{7.8}$$

$$\angle B = \cos^{-1}\left(\frac{5.2}{7.8}\right)$$

$$\angle B \doteq 48°$$

I rounded my answer to the nearest degree.

$$\angle C \doteq 180° - 90° - 48°$$

$$\angle C \doteq 42°$$

I knew the sum of the angles in a triangle is 180°. I used this to determine $\angle C$.

$$b^2 + c^2 = a^2$$

$$b^2 + 5.2^2 = 7.8^2$$

$$b^2 + 27.04 = 60.84$$

$$b^2 = 60.84 - 27.04$$

$$b^2 = 33.80$$

$$b = \sqrt{33.80}$$

$$b \doteq 5.8$$

I used the Pythagorean theorem to solve for b. I could have used the sine or tangent ratios instead.

In $\triangle ABC$, $\angle B \doteq 48°$, $\angle C \doteq 42°$, and $b \doteq 5.8$ m.

In Summary

Key Idea

- Trigonometric ratios can be used to calculate unknown side lengths and unknown angle measures in a right triangle. The ratio you use depends on the information given and the quantity you need to calculate.

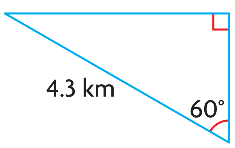
Need to Know

- To determine the length of a side in a right triangle using trigonometry, you need to know the length of another side and the measure of one of the acute angles.
- To determine the measure of one of the acute angles in a right triangle using trigonometry, you need to know the lengths of two sides.

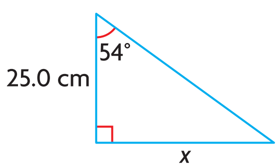
CHECK Your Understanding

1. Solve for x, to one decimal place, using the indicated trigonometric ratio.

a) cosine

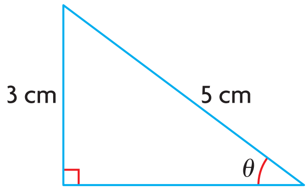


b) tangent

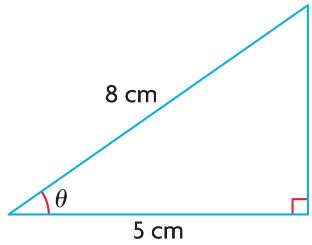


2. Determine the value of θ, to the nearest degree, in each triangle.

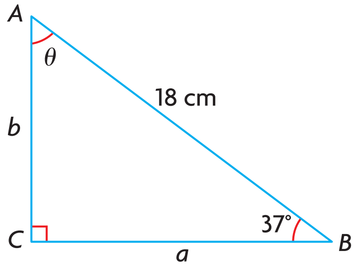
a)



b)



3. Solve $\triangle ABC$.



PRACTISING

4. Solve for $\angle A$ to the nearest degree.

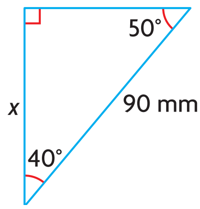
a) $\sin A = 0.9063$ b) $\cos A = \frac{4}{5}$

5. For each triangle,

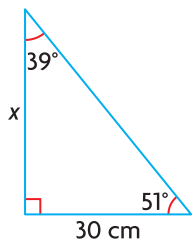
K i) state two trigonometric ratios you could use to determine x

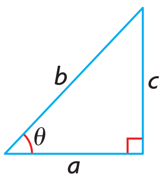
ii) determine x to the nearest unit

a)



b)



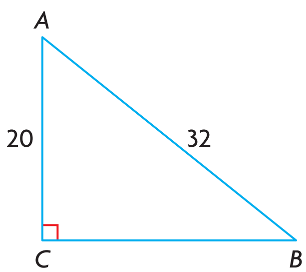


6. For each pair of side lengths, calculate the measure of θ to the nearest degree for the triangle at the left.

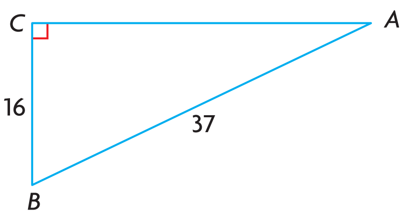
a) $a = 10$ and $c = 10$ b) $b = 12$ and $c = 6$ c) $a = 9$ and $b = 15$

7. Using trigonometry, calculate the measures of $\angle A$ and $\angle B$ in each triangle. Round your answers to the nearest degree.

a)



b)

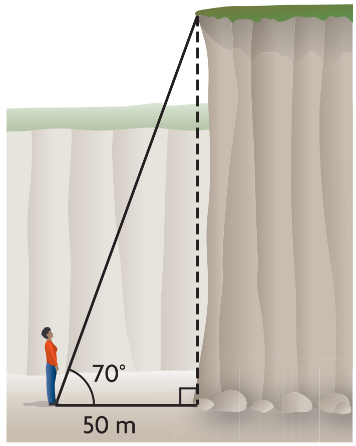


8. Calculate the measure of the indicated angle, to the nearest degree, in each triangle.

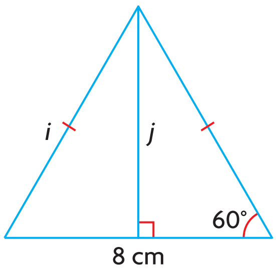
a) In $\triangle ABC$, $\angle C = 90°$, $a = 11.3$ cm, and $b = 9.2$ cm. Calculate $\angle A$.

b) In $\triangle DEF$, $\angle D = 90°$, $d = 8.7$ cm, and $f = 5.4$ cm. Calculate $\angle F$.

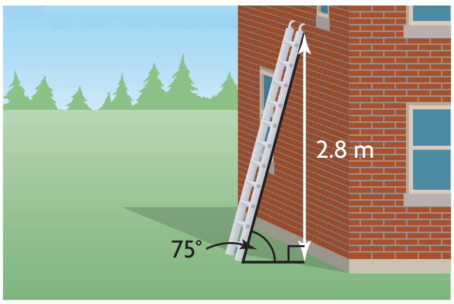
9. Janice is getting ready to climb a steep cliff. She needs to fasten herself to a rope that is anchored at the top of the cliff. To estimate how much rope she needs, she stands 50 m from the base of the cliff and estimates that the angle of elevation to the top is 70°. How high is the cliff?



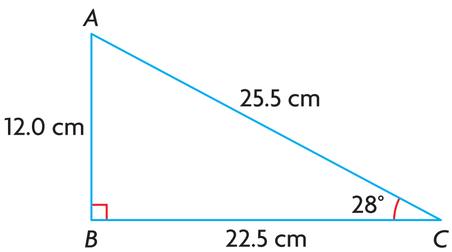
10. Solve for i and j.
T



11. A ladder leans against a wall, as shown. How long is the ladder,
A to the nearest tenth of a metre?



12. Kelsey made these notes about $\triangle ABC$. Determine whether
C each answer is correct, and explain any errors.



a) $\sin A = \frac{12.0}{25.5}$

b) $\angle A = 62°$

c) $\cos C = \frac{25.5}{22.5}$

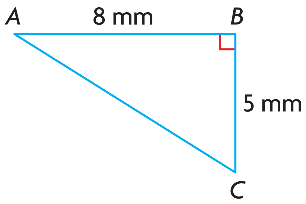
d) $\tan A = 1.875$

e) $\sin C = \frac{24}{51}$

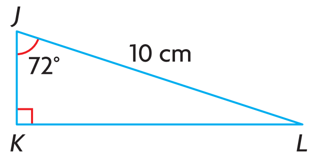
f) $\tan C = 0.53$

13. Solve each triangle. Round the measure of each angle to the nearest degree. Round the length of each side to the nearest unit.

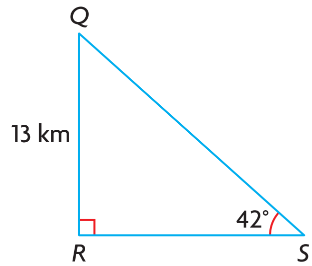
a)



b)



c)



14. For a ladder to be stable, the angle that it makes with the ground should be no more than 78° and no less than 73°.

a) If the base of a ladder that is 8.0 m long is placed 1.5 m from a wall, will the ladder be stable? Explain.

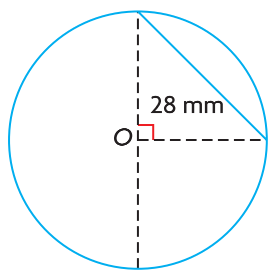
b) What are the minimum and maximum safe distances from the base of the ladder to the wall?

15. **a)** Create a mind map that shows the process of choosing the correct trigonometric ratio to determine an unknown measure in a right triangle.

b) Does the process differ depending on whether you are solving for a side length or an angle measure? Explain.

Extending

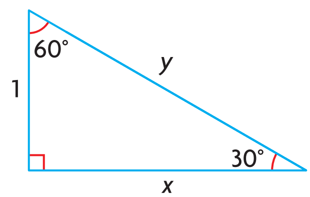
16. Determine the diameter of the circle, if *O* is the centre of the circle.



17. **a)** Determine the exact value of *x* in the triangle at the right using trigonometry.

b) Determine the exact value of *y* using the Pythagorean theorem.

c) Determine the sine, cosine, and tangent ratios of both acute angles. What do you notice?



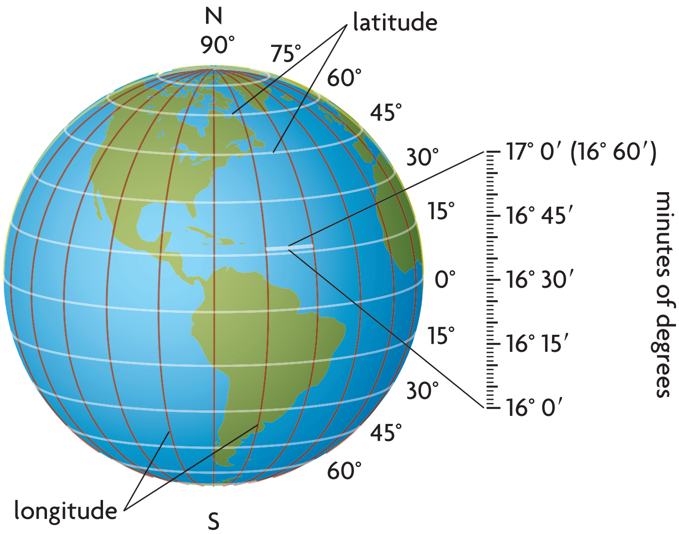
18. **a)** Draw a right isosceles triangle.

b) Calculate the sine and cosine ratios for one of the acute angles. Explain your results.

Curious | Math

A Unit That Did Not Measure Up

Because sailors and pilots are not travelling on land, they use nautical miles to measure distances. Originally, a nautical mile (M) was the distance across one minute of latitude. One minute (′) is $\frac{1}{60}$ of a degree (°).



Scientists later discovered that Earth bulges at the equator and is flatter at the poles. So, the nautical mile is shorter as you approach the equator and longer as you approach each pole.

1. **a)** Write 25°16′ as a decimal.
b) Write 48.30° using degrees and minutes.

2. Determine the length, M, in metres, of the original nautical mile at each location. Use the formula $M = 1852.27 - 9.45 (\cos 2\theta)$, where θ is the latitude in degrees.
a) Kingston, Ontario: latitude 44°15′
b) Yellowknife, Northwest Territories: latitude 62°28′
c) Alert, Nunavut: latitude 82°30′

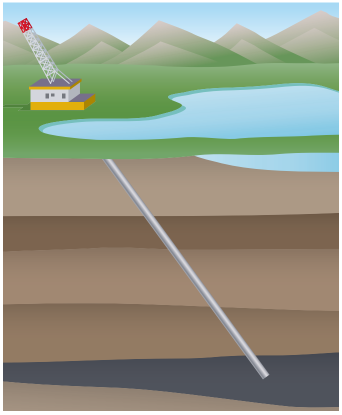
3. The nautical mile was internationally redefined in 1929 as being exactly 1852 m. Explain why this value might have been chosen.

4. What does the expression $9.45 (\cos 2\theta)$ mean in the formula $M = 1852.27 - 9.45 (\cos 2\theta)$?

7.6 Solving Right Triangle Problems

GOAL

Use the primary trigonometric ratios to solve problems that involve right triangle models.



LEARN ABOUT the Math

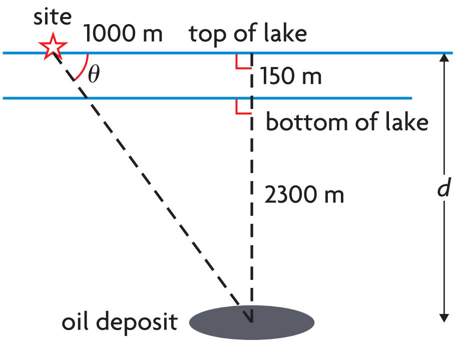
Jackie works for an oil company. She needs to drill a well to an oil deposit. The deposit lies 2300 m below the bottom of a lake, which is 150 m deep. The well must be drilled at an angle from a site on land. The site is 1000 m away from a point directly above the deposit.

? At what angle to Earth's surface should Jackie drill the well?

EXAMPLE 1 **Solving a problem** using a right triangle model

Determine the angle at which the well should be drilled.

Jackie's Solution



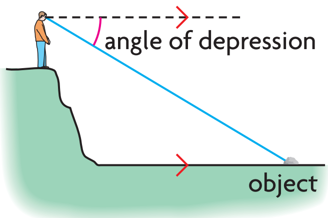
I drew a diagram that shows where the lake, oil deposit, and drill site are located.

I added a line to show the **angle of depression** to the deposit.

I labelled the angle of depression θ.

angle of depression (angle of declination)

the angle between the horizontal and the line of sight when looking down at an object



$$d = 150 + 2300$$
$$= 2450$$

I calculated the length of the side that is opposite angle θ by adding the two given vertical distances.

$$\tan \theta = \frac{2450}{1000}$$
$$\tan \theta = 2.45$$
$$\theta = \tan^{-1}(2.45)$$
$$\theta \doteq 68°$$

To calculate θ, I used my calculator. Since I knew the opposite and adjacent sides to θ, I used the inverse tangent.

The well should be drilled at an angle of about 68°.

Reflecting

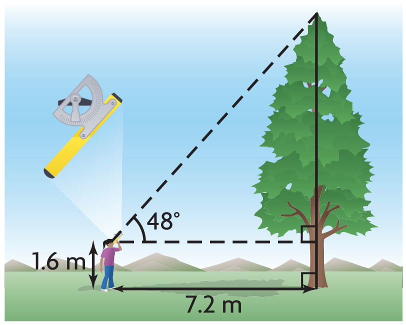
A. How does an angle of depression relate to an angle of elevation?

B. How could Jackie calculate the distance from the oil deposit to the drill site?

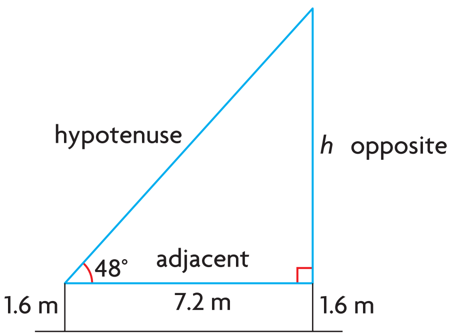
APPLY the Math

EXAMPLE 2 Solving a problem using a clinometer

Ayesha is a forester. She uses a clinometer (a device used to measure angles of elevation) to sight the top of a tree. She measures an angle of 48°. She is standing 7.2 m from the tree, and her eyes are 1.6 m above ground. How tall is the tree?



Joan's Solution



I drew a diagram to model the problem. The height is the length of the side that is opposite the 48° angle. I named the rest of the sides in the triangle relative to the 48° angle.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 48° = \frac{h}{7.2}$$

Because I knew the adjacent side, I used the tangent ratio. The tangent of 48° equals the opposite side, or height, divided by the adjacent side.

$$7.2\,(\tan 48°) = 7.2\left(\frac{h}{7.2}\right)$$

I multiplied both sides by 7.2 and evaluated.

$$7.2\,(\tan 48°) = h$$

$$8.00 \doteq h$$

$$\text{tree height} = 1.6 + 8.0$$
$$= 9.6$$

I added the distance from the ground to Ayesha's eyes to the height of the triangle to calculate the height of the tree.

The tree is about 9.6 m tall.

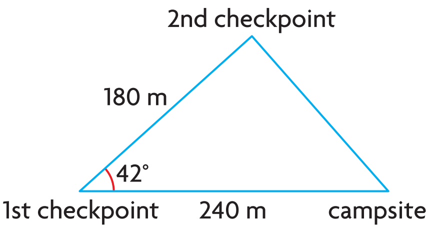
EXAMPLE 3 **Solving an area problem** when the height is unknown

A group of students are on an outdoor education trip. They leave their campsite and travel 240 m before reaching the first orienteering checkpoint. They turn, creating a 42° angle with their previous path, and travel another 180 m to get to the second checkpoint. They turn again and travel the shortest possible path back to their campsite. What area of the woods did their triangular route cover?

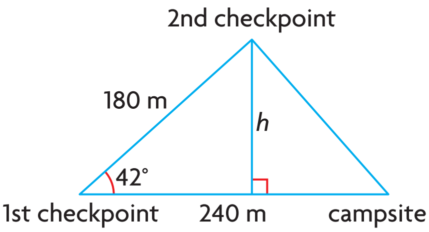
Safety *Connection*

A compass, map, first-aid kit, and signal device are important pieces of equipment when hiking in the woods.

Hugo's Solution



I created a diagram to represent the situation.



I had to determine the height of the triangle to calculate its area. I drew a line perpendicular to the 240 m side. I labelled this line as *h*.

$$\sin 42° = \frac{h}{180}$$
$$180\,(\sin 42°) = h$$
$$120.4 \doteq h$$

In the right triangle that contains the 42° angle, *h* is the side opposite this angle. I knew the hypotenuse, so I used the sine ratio to solve for *h*.

The height of the triangle is about 120.4 m.

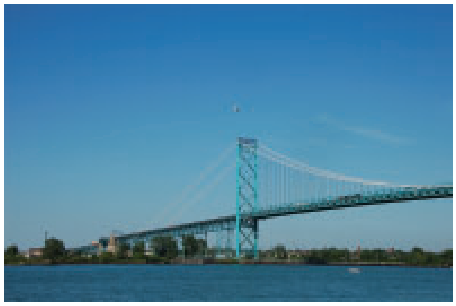
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(240)(120.4)$$
$$A = 14\,448$$

I calculated the area.

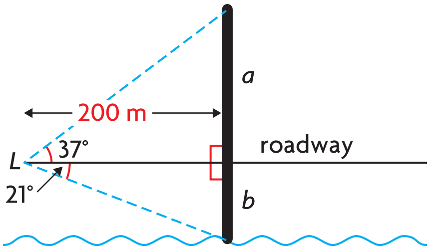
The area of the woods that the triangular route covered was about 14 448 m^2.

EXAMPLE 4 Solving a problem using two right triangles

Lyle stood on land, 200 m away from one of the towers on a bridge. He reasoned that he could calculate the height of the tower by measuring the angle to the top of the tower and the angle to its base at water level. He measured the angle of elevation to its top as 37° and the angle of depression to its base as 21°. Calculate the height of the tower from its base at water level, to the nearest metre.



Jenna's Solution



I started by drawing a diagram of the bridge and tower, and labelling the given angles. I split the height of the tower in two. I named the upper part of the tower (above roadway) *a* and the lower part of the tower (below roadway) *b*. This created two right triangles.

$$\tan 37° = \frac{a}{200}$$
$$200(\tan 37°) = a$$
$$150.7 \doteq a$$

I had to determine the side that is opposite the 37° angle in the top triangle. I knew the adjacent side, so I used the tangent ratio.

$$\tan 21° = \frac{b}{200}$$
$$200(\tan 21°) = b$$
$$76.8 \doteq b$$

I had to determine the side that is opposite the 21° angle in the bottom triangle. I knew the adjacent side, so I used the tangent ratio again.

$$\text{height} = a + b$$
$$= 150.7 + 76.8$$
$$= 227.5$$

I calculated the height of the tower by adding *a* and *b*.

The tower is about 228 m tall from its base at water level.

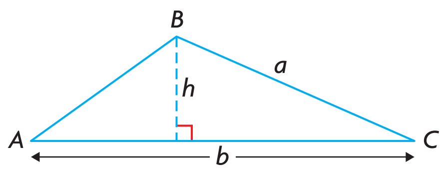
In Summary

Key Idea

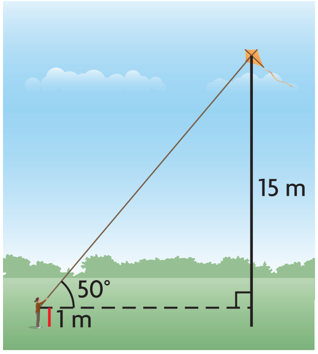
- If a problem involves calculating a side length or an angle measure, try to represent the problem with a model that includes right triangles. If possible, solve the right triangles using the primary trigonometric ratios.

Need to Know

- To calculate the area of a triangle, use the sine ratio to determine the height. For example, suppose that you know a, b, and $\angle C$ in the triangle at the right. To calculate the height, you can use $\sin C = \frac{h}{a}$, so $h = a(\sin C)$. Area of the triangle $= \frac{1}{2} \times b \times a(\sin C)$.

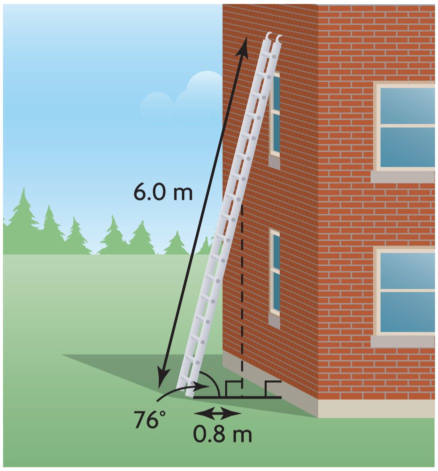


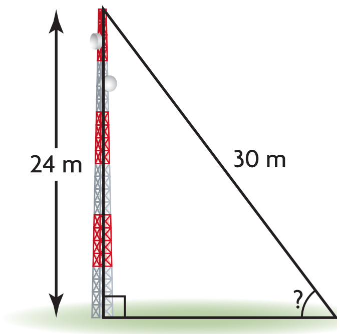
CHECK Your Understanding



1. Isabelle is flying a kite on a windy day. When the kite is 15 m above ground, it makes an angle of 50° with the horizontal. If Isabelle is holding the string 1 m above the ground, how much string has she released? Round your answer to the nearest metre.

2. Bill was climbing a 6.0 m ladder, which was placed against a wall at a 76° angle. He dropped one of his tools directly below the ladder. The tool landed 0.8 m from the base of the ladder. How far from the top of the ladder was Bill?





3. A guy wire is attached to a cellphone tower as shown at the left. The guy wire is 30 m long, and the cellphone tower is 24 m high. Determine the angle that is formed by the guy wire and the ground.

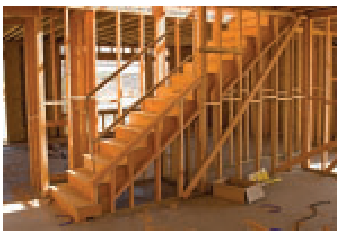
PRACTISING

4. A tree that is 9.5 m tall casts a shadow that is 3.8 m long.
K What is the angle of elevation of the Sun?

5. The rise of a rafter drops by 3 units for every 5 units of run. Determine the angle of depression of the rafter.

6. A building code states that a set of stairs cannot rise more than 72 cm for each 100 cm of run. What is the maximum angle at which the stairs can rise?

7. A contractor is laying a drainage pipe. For every 3.0 m of horizontal pipe, there must be a 2.5 cm drop in height. At what angle should the contractor lay the pipe? Round your answer to the nearest tenth of a degree.

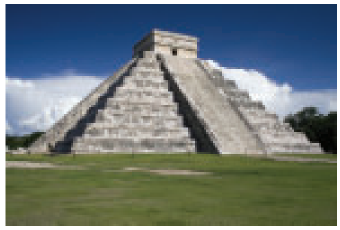


Career *Connection*

Jobs in construction include designer, engineer, architect, project manager, carpenter, mason, electrician, plumber, and welder.

8. Firefighters dig a triangular trench around a forest fire to prevent the fire from spreading. Two of the trenches are 800 m long and 650 m long. The angle between them is 30°. Determine the area that is enclosed by these trenches.

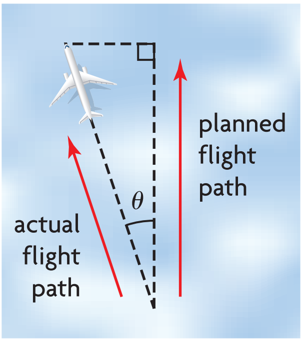
9. A Mayan pyramid at Chichén-Itzá has stairs that rise about 64 cm for every 71 cm of run. At what angle do these stairs rise?



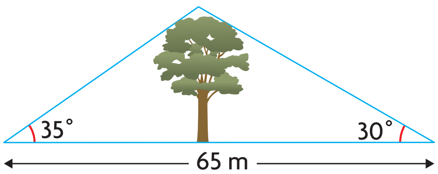
History Connection

Chichén-Itzá, in the Yucatan peninsula of Mexico, was part of the Mayan civilization. The pyramid called El Castillo, or the castle, is a square-based structure with four staircases and nine terraces.

10. After 1 h, an airplane has travelled 350 km. Strong winds, however, have caused the plane to be 48 km west of its planned flight path. By how many degrees is the airplane off its planned flight path?



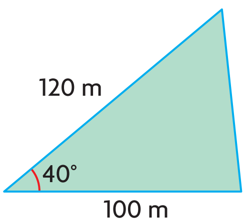
11. **A** Angles were measured from two points on opposite sides of a tree, as shown. How tall is the tree?



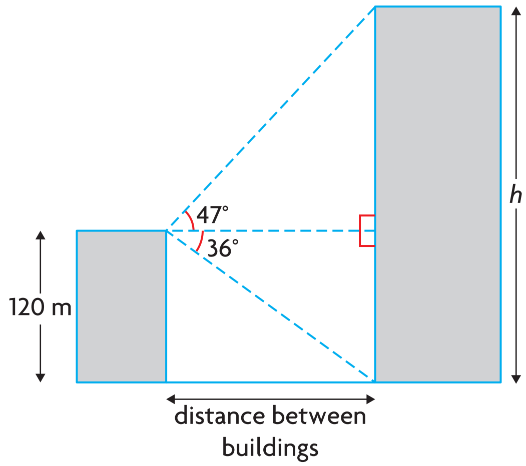
12. Determine the angle between the line $y = \frac{3}{2}x + 4$ and the x-axis.

13. **T** A bridge is going to be built across a river. To determine the width of the river, a surveyor on one bank sights the top of a pole, which is 3 m high, on the opposite bank. His optical device is mounted 1.2 m above the ground. The angle of elevation to the top of the pole is 8.5°. How wide is the river?

14. **C** Élise drew a diagram of her triangular yard. She wants to cover her yard with sod. Explain how you could calculate the cost, if sod costs $\$1.50/m^2$.



15. A video camera is mounted on top of a building that is 120 m tall. The angle of depression from the camera to the base of another building is 36°. The angle of elevation from the camera to the top of the same building is 47°.



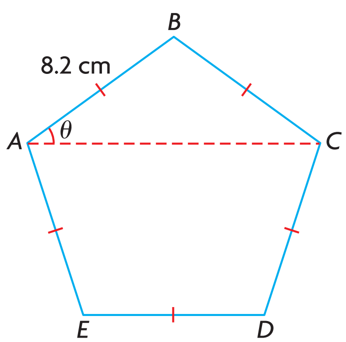
a) How far apart are the two buildings? Round your answer to the nearest metre.
b) How tall is the building viewed by the camera? Round your answer to the nearest metre.

16. An isosceles triangle has a height of 12.5 m (measured from the unequal side) and two equal angles that measure 55°. Determine the area of the triangle.

17. To photograph a rocket stage separating, Lucien mounts his camera on a tripod. The tripod can be set to the angle at which the stage will separate. This is where Lucien needs to aim his lens. He begins by aiming his camera at the launch pad, which is 1500 m away. The stage will separate at 20 000 m. At what angle should Lucien set the tripod?

18. Explain the steps you would use to solve a problem that involves a right triangle model and the use of trigonometry.

Extending



19. Each side length of regular pentagon *ABCDE* is 8.2 cm.
a) Calculate the measure of θ to the nearest degree.
b) Calculate the length of diagonal *AC* to the nearest tenth of a centimetre.

20. Determine the acute angle at which $y = 2x - 1$ and $y = 0.5x + 2$ intersect.

7 Chapter Review

FREQUENTLY ASKED Questions

Q: What are the primary trigonometric ratios, and how do you use them?

A: The primary trigonometric ratios for $\angle A$ in $\triangle ABC$ are

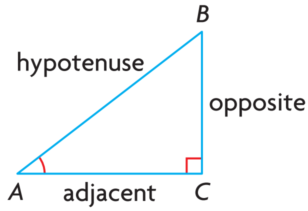
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

To calculate an angle or a side using a trigonometric ratio, follow these steps:

- Label the sides of the triangle relative to either an acute angle you know or the angle you want to calculate.
- Use the appropriate trigonometric ratio to write an equation that involves the angle or side you want to calculate.
- Solve your equation.

Study *Aid*

- See Lesson 7.4, Examples 1 to 3.
- Try Chapter Review Questions 5 to 10.



Q: How do you know when to use the inverse trigonometric ratios?

A: Use $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ when you need to determine the measure of an angle and you know the value of a ratio of two sides in a right triangle.

Study *Aid*

- See Lesson 7.4, Example 3, and Lesson 7.5, Example 3.
- Try Chapter Review Questions 5 b), 7, and 8 b).

Q: What strategies can you use to solve a problem that involves a right triangle model?

A1: Draw a diagram to model the problem. If you know the measure of one acute angle and the length of one side, follow these steps:

- Determine the third angle by subtracting the right angle and the other known angle from 180°.
- Calculate the two unknown side lengths using trigonometric ratios. Alternatively, calculate one unknown side length using a trigonometric ratio and solve for the last side length using the Pythagorean theorem.

A2: Draw a diagram to model the problem. If you know two side lengths but neither acute angle, follow these steps:

- Use inverse trigonometric ratios to calculate one of the missing angles.
- Calculate the third angle by subtracting the angle you found and the right angle from 180°.
- Calculate the third side using a trigonometric ratio or the Pythagorean theorem.

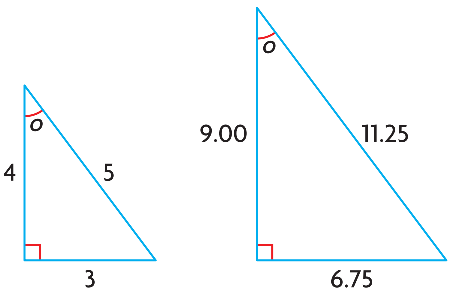
Study *Aid*

- See Lesson 7.5, Examples 1 to 4, and Lesson 7.6, Examples 1 to 4.
- Try Chapter Review Questions 11 to 17.

PRACTICE Questions

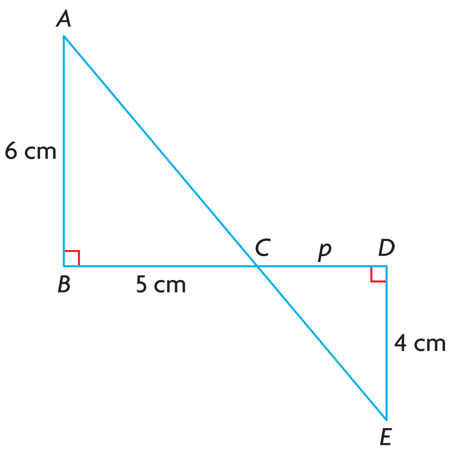
Lesson 7.1

1. Determine whether these triangles are similar. If they are similar, write a proportion statement and determine the scale factor.

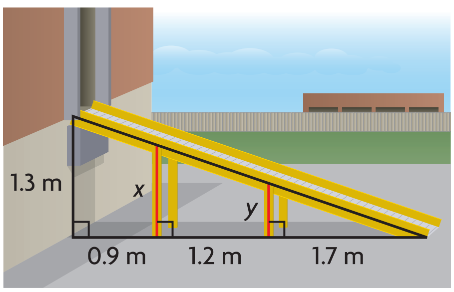


Lesson 7.2

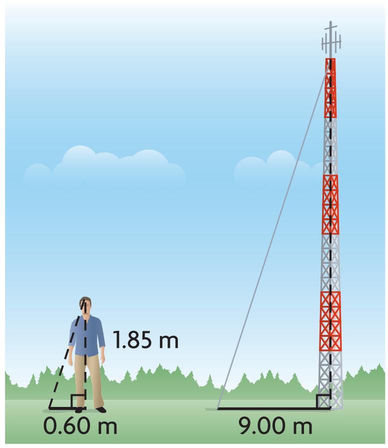
2. State whether the triangles in the diagram are similar. Then determine p.



3. Calculate the heights of the two ramp supports, x and y. Round your answers to the nearest tenth of a metre.

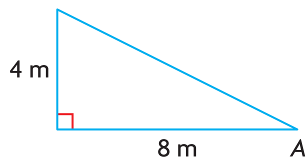


4. Brett needs to support a radio tower with guy wires. Each guy wire must run from the top of the tower to its own anchor 9.00 m from the base of the tower. When the tower casts a shadow that is 9.00 m long, Brett's shadow is 0.60 m long. Brett is 1.85 m tall. What is the length of each guy wire that Brett needs?



Lesson 7.4

5. a) Determine the three primary trigonometric ratios for $\angle A$.



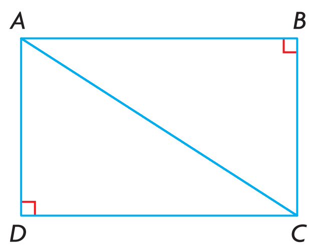
b) Calculate the measure of $\angle A$ to the nearest degree.

6. Determine x to one decimal place.

a) $\tan 46° = \dfrac{x}{14.2}$ **b)** $\cos 29° = \dfrac{17.3}{x}$

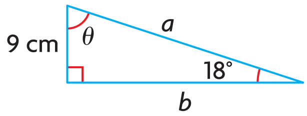
Lesson 7.5

7. $ABCD$ is a rectangle with $AB = 15$ cm and $BC = 10$ cm. What is the measure of $\angle BAC$ to the nearest degree?



8. In $\triangle PQR$, $\angle R = 90°$ and $p = 12.0$ cm.
   a) Determine r, when $\angle Q = 53°$.
   b) Determine $\angle P$, when $q = 16.5$ cm.

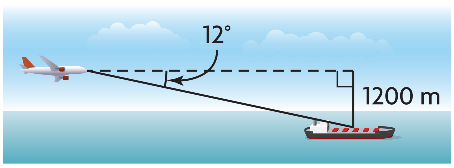
9. Solve this triangle.



10. Maria needs to load cars onto a transport truck. She is planning to drive up a ramp, onto the truck bed. The truck bed is 1.5 m high, and the maximum angle of the slope of the ramp is 35°.
    a) How far is the rear of the truck from the point where the ramp touches the ground?
    b) How long should the ramp be? Round your answer to one decimal place.

Lesson 7.6

11. A search-and-rescue airplane is flying at an altitude of 1200 m toward a disabled ship. The pilot notes that the angle of depression to the ship is 12°. How much farther does the airplane have to fly to end up directly above the ship?

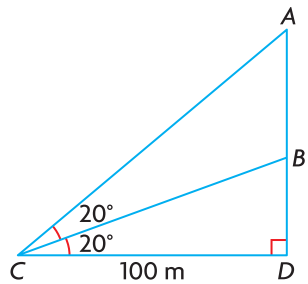


12. The angle of elevation from the top of a 16 m building to the top of a second building is 48°. The buildings are 30 m apart. What is the height of the taller building?

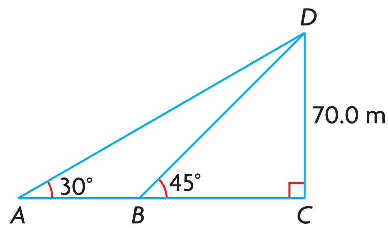
13. A cyclist pedals his bike 6.5 km up a mountain road, which has a steady incline. By the time he has reached the top of the mountain, he has climbed 1.1 km vertically. Calculate the angle of elevation of the road.

14. Two watch towers at an historic fort are located 375 m apart. The first tower is 14 m tall, and the second tower is 30 m tall.
    a) What is the angle of depression from the top of the second tower to the top of the first tower?
    b) The guards in the towers simultaneously spot a suspicious car parked between the towers. The angle of depression from the lower tower to the car is 7.7°. The angle of depression from the higher tower is 6.3°. Which guard is closer to the car? Explain how you know.

15. Calculate the length of AB using the information provided. Show all your steps.



16. A swimmer observes that from point A, the angle of elevation to the top of a cliff at point D is 30°. When the swimmer swims toward the cliff for 1.5 min to point B, he estimates that the angle of elevation to the top of the cliff is about 45°. If the height of the cliff is 70.0 m, calculate the distance the swimmer swam.



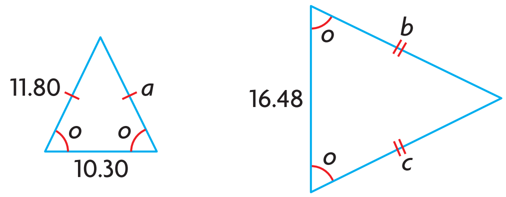
17. A plane takes off in a straight line and travels along this line for 10 s, when it reaches a height of 300 m. If the plane is travelling at 60 m/s, at what angle is the plane ascending?

7 Chapter Self-Test

Process *Checklist*

- ✔ Questions 2 and 5: Did you visualize or sketch a diagram that **represents** the information accurately?
- ✔ Question 7: Did you **communicate** your thinking with words and a diagram that **connect** the situation with trigonometry?
- ✔ Questions 8 and 9: Did you **reflect** on the relationship between the given information and the questions asked as you solved the problems?

1. Determine the indicated side lengths in the triangles.



2. Two trees cast a shadow when the Sun is up. The shadow of one tree is 12.1 m long. The shadow of the other tree is 7.6 m long. If the shorter tree is 5.8 m tall, determine the height of the taller tree. Round your answer to the nearest tenth of a metre.

3. Determine each unknown value. Round your answer to one decimal place.

a) $\sin 28° = \frac{x}{5}$

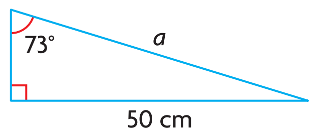
b) $\cos 43° = \frac{13}{y}$

c) $\tan A = 7.1154$

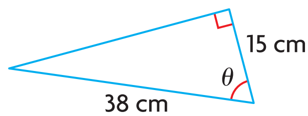
d) $\cos B = \frac{7}{9}$

4. Determine the length of the indicated side or the measure of the indicated angle.

a)



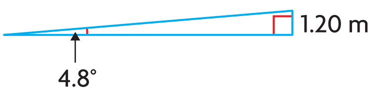
b)



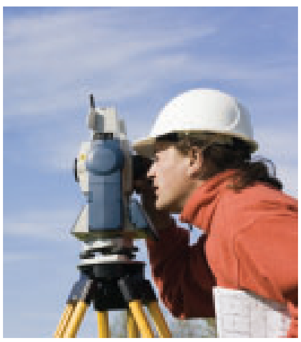
5. Solve each triangle.

a) In $\triangle ABC$, $\angle A = 90°$, $\angle B = 14°$, and $b = 5.3$ cm.

b) In $\triangle DEF$, $\angle F = 90°$, $d = 7.8$ mm, and $e = 6.9$ mm.



6. A ramp has an angle of elevation of 4.8° and a rise of 1.20 m, as shown at the left. How long is the ramp and what is its run? Round your answers to the nearest hundredth of a metre.



7. Surveyors need to determine the width of a river. Explain how they can do this without crossing the river. Use a diagram to illustrate your answer.

8. Jane is on the fifth floor of an office building 16 m above the ground. She spots her car and estimates that it is parked 20 m from the base of the building. Determine the angle of depression to the nearest degree.

9. A pilot who is heading due north spots two forest fires. The fire that is due east is at an angle of depression of 47°. The fire that is due west is at an angle of depression of 38°. What is the distance between the two fires, to the nearest metre, if the altitude of the airplane is 2400 m?

7 Chapter Task

What's the Height of Your School?

Suppose that your school is about to have its annual "Spring Has Sprung" concert. To advertise the concert, the student council wants to make a banner. The banner will hang from the roof of the school, down to the ground. No one on the student council knows the height of the school. You say that you can calculate the height, using only these materials:

- protractor
- drinking straw
- string
- clear tape
- bolt
- tape measure

? How can you use these materials to determine the height of your school?

A. Determine how you can make a clinometer using the protractor, drinking straw, string, clear tape, and bolt. Make drawings of your design. Ask a classmate to review your drawings and suggest changes.

B. Assemble your clinometer. Use it to measure the angle of elevation to an object whose height you know or can measure.

C. Test the accuracy of your clinometer using trigonometry. If necessary, move the string directly across from 90° on the protractor.

D. Decide on the site where you will determine the angle of elevation to the roof of your school. Use the tape measure to measure the distance from this site to the base of the wall of your school.

E. Which trigonometric ratio will you use to calculate the height? Compare your answer with your classmates' answers. Suggest reasons for any differences.

F. Prepare a report that explains how your clinometer works and how you used it to calculate the height of your school.

Task *Checklist*

- ✔ Did you include a diagram of your clinometer and an explanation of how it works?
- ✔ Did you include a diagram to show how you determined the height of your school?
- ✔ Did you explain the process you used to determine this height?
- ✔ Did you clearly summarize your procedure and results?